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PAPER – II : OPTICS

SC-117

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General theory of image formation : Cardinal points of an optical system, general relationship thick lens, combination of two thin lenses, Nodal slide and Newton's formula, Huygen's and Ramsden's eyepieces Aberration in images : Chromatic aberration, achromatic combination of lenses in contact and separated lenses. Monochromatic aberration and their reduction, crossed lens.

CHAPTER-2 : INTERFERENCE

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UNIT

1

GEOMETRICAL OPTICS

STRUCTURE

- Optical Path
- Fermat's Principle
- Cardinal Points
- Cardinal Points of a Lens System
- Newton's Formula
- Equivalent Focal Length of Thin Lenses Combination
- Nodal Slide
- Aberration
- Achromatism
- Spherical Aberration
- Eye-Piece
- Huygen's Eye-Piece
- Ramsden's Eye-Piece
- Comparison of Ramsden's and Huygen's Eye-piece
  - Summary
  - Student Activity
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LEARNING OBJECTIVES

After going this unit you will learn :

- Rectilinear propagation of light, law of refraction and law of reflection.
- Coincidence of principal and nodal points.
- Position of cardinal points.
- Monochromatic aberration and chromatic aberration.
- Conditions of achromatism of two thin lenses.
- Conditions of achromatism of combination of two lenses.
- Image formation and equivalent focal length.

• 1.1 OPTICAL PATH

When any ray of light travels 'd' distance in any medium of refractive index μ, the product μd is called the optical path or in other words we define the optical paths.

"An optical path is defined as the product of geometrical distance and refractive index of medium."

If medium is air, then it represents the light would travel in the same time in which it travels the distance 'd' in the medium. Then

Optical path = Velocity of light in space × time (travelling)

When a ray of light travels in a different medium, whose refractive indexes are $\mu_1, \mu_2, \mu_3, \ldots, \mu_p$ etc. and the distances are $d_1, d_2, d_3, \ldots, d_p$ then

$\therefore$ Optical path $= d = \Sigma\mu d$

where $\mu_1 d_1 + \mu_2 d_2 + \ldots + \mu_p d_p = \Sigma\mu d$

• 1.2 FERMAT'S PRINCIPLE

In 1658, Fermat's enunciated the principle of least time for the path followed by light radiations. This law covers all the following laws :

(i) The rectilinear propagation of light

(ii) The law's of reflection of light.

(iii) The laws of reflection of light.

According to this principle, **"The path actually taken by a ray of light in passing from one point to the other is the path of least time."**

However in some cases, it has been shown that the time taken by light is not minimum but maximum.

Therefore, in the modified form, Fermat's principle of least time is known as Fermat's principle of stationary time or Fermat's principle of Extremum Path.

According to this principle, **"The path taken by a ray of light in passing from one point to the other (through any number of reflections and refractions) is the path of minimum or maximum time."**

The fundamental laws of rectilinear propagation, reflection and refraction can be derived from this principle.

(I) Rectilinear Propagation of Light :

According to this, when the ray of light passes through from different mediums, therefore the rectilinear propagation of light between two points of medium in form of a straight line.

(a) Homogeneous Medium : In a homogeneous medium, the velocity of light are constants.

The time taken by the rays of light going from one point to other points is proportional to paths. According to the Fermat's principle in fig. (1), the light passes through the ACB points.

Let us consider the ds for the smallest part of the light path. Then the path between the points A and B is given by

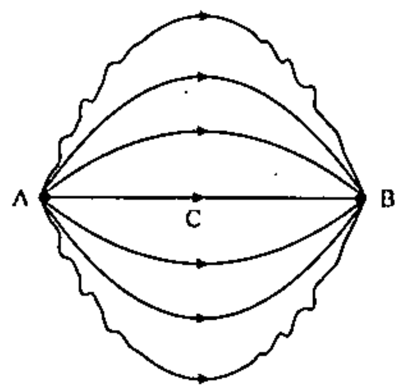


Fig. 1.

$$\int_A^B ds = \text{minimum} \qquad \text{(for least time)}$$

or $$\boxed{\delta\int_A^B ds = 0} \qquad \text{(according to Fermat's principle)}$$

which is required condition.

(b) Non-Homogeneous Medium : The velocity of light in non-homogeneous medium is changed at different points.

According to Fermat's principle

$$\int_{t_A}^{t_B} dt = \text{minimum}$$

or $$\int_{t_A}^{t_B} \frac{ds}{v} = \text{minimum}$$

or $$\int_A^B \frac{\mu ds}{C} = \text{minimum} \qquad \left[\because \frac{ds}{v} = \frac{nds}{c}\right]$$

or $$\int_A^B \frac{\mu ds}{C} = 0$$

or $$\boxed{\int_A^B \frac{\mu ds}{C} = 0}$$

where μ is the refractive index of medium and v is the velocity of light in a medium.

Thus, the light rays travel along straight line paths in a non-homogeneous medium.

(II) Law of Refraction

In the following fig. (2), suppose a ray of light QO is incident from any point Q, in a refractive index μ_1 or raer medium on surface XY, which is refracted to another point Q' in μ_2 denser medium, along the path QOQ'. Let h and h' be the lengths of the perpendiculars drawn from Q and Q' to the refracting surface. And angle of incidence is $\angle QON = i$, and angle of refraction $\angle N'QQ' = r$.

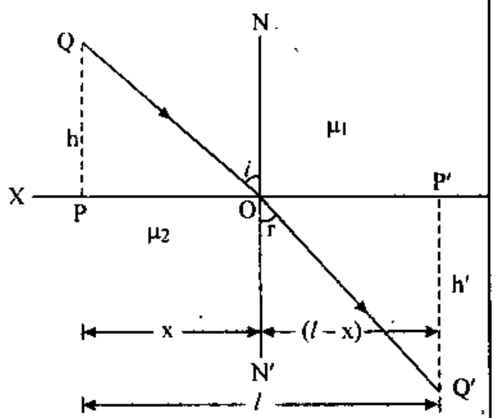


Fig. 2.

From fig. (2) say

$$PP' = l,\ PO = x \text{ and } OP' = l - x.$$

Let v_1 is the velocity of light in a raer medium and v_2 is the velocity of light in a denser medium. If t is the time taken by the rays of light from Q to Q'

$$\therefore \quad t = \frac{QO}{v} + \frac{OQ'}{v'} \quad \text{... (1)}$$

from above fig. (2),

$$OQ = (h^2 + x^2)^{1/2}$$

and $$OQ' = [h'^2 + (l - x)^2]^{1/2}$$

$\therefore$ Equation (1) becomes

$$t = \frac{(h^2 + x^2)^{1/2}}{v} + \frac{[h^{1/2} + (l - x)^2]^{1/2}}{v'} \quad \text{... (2)}$$

According to Fermat's least principle, $\frac{dt}{dx} = 0$

$\therefore$ By equation (2), $$\frac{dt}{dx} = \frac{x}{v(h^2 + x^2)^{1/2}} - \frac{(l - x)}{v'\{h'^2 + (l - x)^2\}^{1/2}} = 0$$

or $$\frac{x}{v(h^2 + x^2)^{1/2}} = \frac{(l - x)}{v'\{h'^2 + (l - x)^2\}^{1/2}} \quad \text{... (3)}$$

But $$\frac{x}{v(h^2 + x^2)^{1/2}} = \frac{PO}{QO} = \sin i$$

and $$\frac{(l - x)}{v'(h'^2 + (l - x^2)^{1/2}} = \frac{P'Q}{OQ'} = \sin r \quad \text{.... (4)}$$

$\therefore$ From equations (3) and (4),

$$\left(\frac{1}{v}\right)\sin i = \left(\frac{1}{v'}\right)\sin r \quad \text{... (5)}$$

But $$C = v\mu_1, \text{ and } C = v'\mu_2$$

$\therefore$ Equation (5) becomes

$$\left(\frac{\sin i}{\sin r}\right) = \frac{\mu_2}{\mu_1} = {}_1\mu_2 \quad \text{... (6)}$$

or $$\mu_1 \sin i = \mu_2 \sin r$$

which is the Snell's law.

(III) Law of Reflection

Suppose a ray of light QO, from any point Q is incident on plane mirror PP' and is reflected towards the point Q' at angle r. If angle of incidence $\angle QON = i$ and angle of reflection $\angle NOQ' = r$. Let h and h' be the lengths of the perpendicular drawn from Q and Q' on PP', as shown in fig. (3).

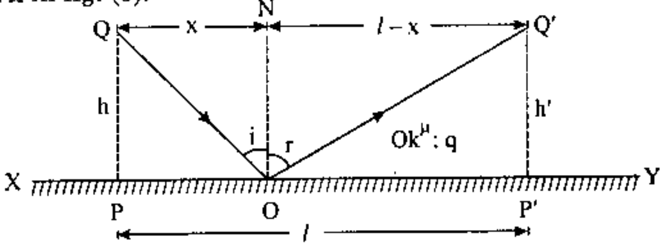


Fig. 3.

If v is the velocity of light in a medium (space). Then the time (t) taken by the rays of light from Q to Q' is given by

$$t = \frac{QO + OQ'}{v} \quad \text{... (1)}$$

according to above fig. (3),

$$QO = (h^2 + x^2)^{1/2}$$

and
$$OQ' = \{h'^2 + (l-x)^2\}^{1/2}$$

By equation (1),

$$t = \frac{1}{v}\left[(h^2 + x^2)^{1/2} + \{h'^2 + (l - x^2)\}^{1/2}\right] \quad \text{... (2)}$$

According to least Fermat's principle,

i.e.,
$$\frac{dt}{dx} = 0$$

$\therefore$ Equation (1) becomes

$$\frac{dt}{dx} = \frac{1}{v}\left[\frac{x}{(h^2 + x^2)^{1/2}} - \frac{l - x}{(h'^2 + (l-x)^2)^{1/2}}\right] = 0$$

or
$$\frac{x}{(h^2 + x^2)^{1/2}} = \frac{(l-x)}{(h'^2 + (l-x)^2)^{1/2}} \quad \text{... (3)}$$

But from above fig. (3),

$$\sin i = \frac{PO}{QO} = \frac{x}{(h^2 + x^2)^{1/2}}$$

and
$$\sin r = \frac{OP'}{OQ'} = \frac{(l-x)}{(h'^2 + (l - x^2))^{1/2}}$$

$\therefore$ Equation (3) becomes

$$\sin i = \sin r$$

or
$$\boxed{i = r} \quad \text{... (4)}$$

which is the law of reflection.

• 1.3 CARDINAL POINTS

Gauss in 1841 proved that if in an optical system the positions of specific points be known, the system may be treated as a single (one) unit. The position and size of the image on an object may then directly obtained by simple formulae for thin lenses as used. However, system may be complicated. Thus, we used the thick lenses as similar to coaxial lens system. These points are **known as Gauss or cardinal points.** There are six types viz.,

(i) two focal points

(ii) two principal points

(iii) two nodal points.

(i) Focal Points and Planes : In following fig. (4), let us consider the convergent lens optical system having its axis XX'. The first focal point F_1 is situated on axis XX' in the object space so that rays starting from it become parallel to axis after refraction through the optical system. A plane normal to the principal axis and passing through this point is **called the Ist focal plane.** It means the object point on the axis for which the images point lies at infinity. **It's called as Ist focal point**.

The IInd focal point (F_2) is situated on axis in the image space so that the ray parallel to axis XX' from the object space focus at this point. The plane normal to the axis and passing through this point (F_2) is known as **IInd focal planes** or "the image point on the axis for which the object point lies at infinity". It is **known as IInd focal point.**

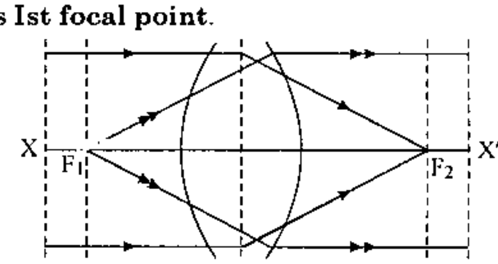


Fig. 4. Focal points and planes

Thus, the focal points are a pair of points lying on the axis of the system conjugate to points at infinity.

(ii) Principal or Unit Points and Planes, *Ist Point and Planes* : Let rays pass through F_1 as F_1C_1. It emerges as $A'B'$. Produce $A'B'$ backward and F_1C_1, Forward to meet at M_1. Pass a plane M_1H_1 at right angles to axis by M_1. The plane is called **Ist principal** or **unit plane**. Its intersection with the axis gives **Ist principal point or unit point (H_1)**.

IInd Points and Planes : A ray AB parallel to axis XX' is incident on system. It emerges as C_2F_2. *F_2 is the second focal point. Produce AB onwards and* C_2F_2 backward. These intersect at M_2 . Pass a plane through M_2 normal to axis. Plane intersection with axis gives IInd principal point (unit points) H_2, plane M_2H_2 **is called second principal or unit plane** as shown in fig. (5) or the two conjugate planes characterised by unit, +ve transverse magnification are **called unit planes**. Their intersections with the principal axis gives two points known as unit points.

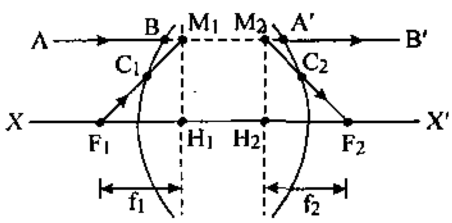


Fig. 5. Principal points (H_1, H_2) with planes

Focal Lengths : The distance of first focal point F_1 to Ist principal point H_1 is known as **Ist focal length f_1** and the distance of IInd focal point F_2 to IInd principal point is known as **IInd focal length f_2** as shown in adjoining fig. (5).

or $\quad H_1F_1 = f_1$ and $H_2F_2 = f_2$

if the medium of the system is same then

$$f_1 = + f_2.$$

(iii) Nodal Points and Planes : Two points on the principal axis of the optical system characterised by unit, angular magnification are known as **Nodal points.** Their respective planes at right angles to the axis are called as **Nodal planes. It is** measured by focal points.

They are such that an incident ray directed towards the one Nodal point emerges parallel to its original direction through the other Nodal point. In above fig. (6), N_1 and N_2 be Nodal points and emergent ray N_2Q is parallel to the incident ray PN_1, the planes through N_1 and N_2 and normal to axis are called Nodal planes.

(iv) Formation of Image in Lens System Using Cardinal Points :

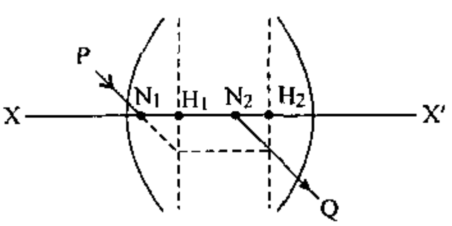


Fig. 6. Nodal planes and points

Following rules, which follow from the properties of cardinal points, may be useful in considering image formation.

(i) A ray parallel to the principal axis passes through F_2.

(ii) A ray passing through F_1 emerges parallel to the principal axis.

(iii) A ray meeting first principal plane at height (h), emerges through the second principal plane from the same height (h) and on the same sides.

(iv) A ray directed towards a Nodal point emerges through the other Nodal point parallel to the previous ray.

• 1.4 CARDINAL POINTS OF A LENS SYSTEM

There are six points of a lens system viz.

(i) First and second focal points.

(ii) First and second Gauss or principal points.

(iii) First and second Nodal points.

We, therefore define them and outline thier properties as below :

(i) Focal Points : "The focal points are a pair of points lying on the principal axis and conjugate to points at infinity."

The first and second focal points are the points situated on the principal axis. In the object space so that rays starting from it and parallel to principal axis after refraction through the optical lens system. It known as **Ist focal points** and a plane normal to principle axis and passing through this point is known as **Ist focal plane**.

If the rays parallel to principal axis (for image) from the object space focus at this point is **known as second focal point. The plane normal to the principal axis and passing through this point is called IInd focal plane.**

(ii) Principal Points : "Two conjugate planes characterised by unit, positive transverse magnification are known as **unit or principal planes.** Their intersection with unit axis (principal axis) give up two points known **as principal points**" or **"there are a pair of points (conjugate) on the principal axis having unit positive linear transverse magnification (+ 1)."**

(iii) Nodal Points : "Two points on the principal axis of the optical lens system characterised by unit, angular magnification (positive) are known as **Nodal points.**" (Their respective planes at $\frac{\pi}{2}$ angle to principal axis are known as Nodal planes).

In fig. (7), let us consider XX' is the principal axis of an optical system. The principal and focal points of the optical system are H_1, H_2 **and** F_1, F_2 respectively. Consider a point N_1 distance x_1 from F_1 (from fig. $F_1N_1 = x_1$) so that an object ray directed towards it as M_1N_2 emerges as M_2N_2. *i.e.*, N_1 is imaged as N_2 by system. Let the distance of N_2 from F_2 points be x_2. Let the Ist focal length $H_1F_1 = f_1$ and IInd focal length $H_2F_2 = f_2$. According to figure f_1, x_1, f_2 and x_2 are all +ve.

Now, considering angular magnification, we have

$$\gamma = \frac{\tan(-\theta_2)}{\tan(-\theta_1)} = \frac{\dfrac{M_2H_2}{H_2N_2}}{\dfrac{M_1H_1}{H_1N_1}}$$

$$\gamma = \frac{H_1N_1}{H_2N_2} = \frac{f_1 + x_1}{f_2 + x_2} \qquad [\because \; M_1H_1 = M_2H_2] \quad \text{... (1)}$$

For, N_1 and N_2 be Nodal points $r = 1$ *i.e.*, $\theta_1 = \theta_2$

Therefore, equation (1) becomes

$$1 = \frac{f_1 + x_1}{f_2 + x_2} = \frac{(f_1 + x_1)\, x_1}{(f_2 + x_2)\, x_1}$$

$$1 = \frac{(f_1 + x_1)\, x_1}{f_2 x_1 + x_2 . x_1} \quad \text{... (2)}$$

Now from Newton's formula $(x_1 x_2 = f^2)$ if x_1 and x_2 be the distance of two conjugate points from the focal points, then we have

$$x_1 x_2 = f_1 f_2 \quad \text{... (3)}$$

From equations (2) and (3), we have

$$1 = \frac{(f_1 + x_2)\, x_1}{f_2\,(x_1 + f_1)} = \frac{x_1}{f_1}$$

or
$$x_1 = f_2 \quad \text{... (4)}$$

Similarly, it can show that

$$x_2 = f_1 \quad \text{... (5)}$$

Equations (4) and (5) show that the

$$F_1N_1 = f_1 \text{ and } F_2N_2 = f_2$$

A ray directed towards first Nodal point N_1 emerges through the IInd Nodal point N_2 parallel to its original direction. This conclusion is used in nodal slide assembly for locating cardinal points.

According to figure,

$$H_1H_2 = H_1N_1 + N_1H_2$$

and
$$N_1N_2 = N_1H_2 + H_2N_2$$

but $$H_1N_1 = H_2N_2$$

$\therefore$ $$\boxed{H_1H_2 = N_1N_2} \quad \ldots (6)$$

Thus, the distance between two principal points is equal to the distance between two Nodal points.

Coincidence of Principal and Nodal Points

When the medium is same on both sides of the lens system, the principle points coincide with Nodal Points.

According to fig. (7), it gives

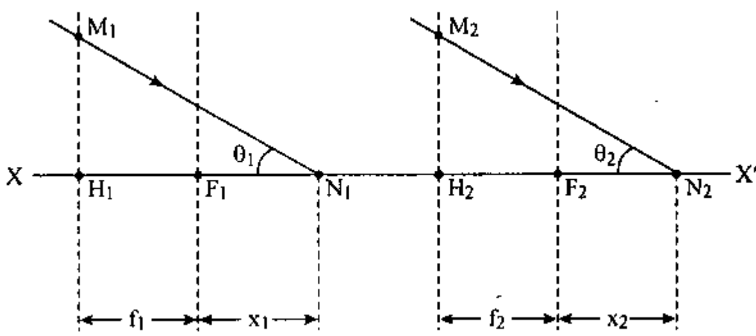


Fig. 7. Nodal points and planes

Using the equations (4) and (5),

$$H_1N_1 = f_1 + x_1 = f_1 + f_2$$
$$H_2N_2 = f_2 + x_2 = f_1 + f_2$$

$\therefore$ $$H_1N_1 = H_2N_2 = f_1 + f_2 \quad \ldots (7)$$

because medium is same then

$$f_1 = -f_2$$

$\therefore$ Then equation (1) becomes

$$H_1N_1 = H_2N_2 = 0$$

which show that the N_1 coincides with H_1 and N_2 coincides with H_2. **Hence prove that the two principal and two Nodal points are coincident in a system having same medium on both sides.**

• 1.5 NEWTON'S FORMULA

Let F_1, F_2 be the principal foci, H_1 and H_2 be the principal points and N_1 and N_2 be the Nodal points of an optical system. PQ bean object on the axis. In order to find the image of the point Q, make the following construction.

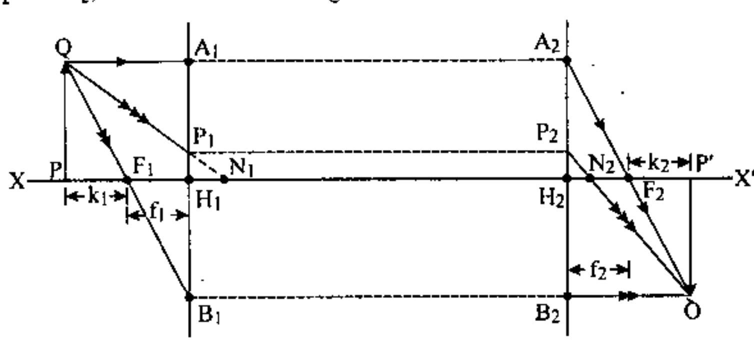


Fig. 8. Converging system forming a real image

(1) Draw a ray QA_1 parallel to the axis touching the Ist principal plane H_1 at A_1. The conjugate ray will proceed from A_2, a point on the IInd principal plane such that $A_2H_2 = A_1H_1$ and will passes through F_2.

(2) Draw another rays QF_1B_1 passing through the Ist principal focus F_1 and touching the Ist plane at B_1. It will emerge out from B_2 such that $B_2H_2 = B_1H_1$.

) Draw a third ray QP_1N_1 directed towards the Ist Nodal (N_1) point, which after tion will proceed from N_2 in a direction parallel to QN_1.

point of intersection Q' of any two of the reflected rays will give the image of Q. $Q'P'$ a perpendicular to the axis $Q'P'$ will be the image of QP.

) QPF_1 and $F_1B_1H_1$ are similar

$$\frac{B_1H_1}{QP} = \frac{H_1F_1}{PF_1}$$

$$B_1H_1 = Q'P'$$

$$\frac{Q'P'}{QP} = \frac{f_1}{k_1} \qquad \text{...(1)}$$

(Δ^s) $P'Q'F_2$ and $A_2H_2F_2$ are similar

$$\frac{Q'P'}{P'F_2} = \frac{A_2H_2}{H_2F_2} \qquad \text{But } A_2H_2 = QP$$

$$\therefore \quad \frac{Q'P'}{QP} = \frac{k_2}{f_2} \qquad \text{...(2)}$$

Equating equations (1) and (2),

$$\frac{f_1}{k_1} = \frac{k_2}{f_2}$$

$$\boxed{k_1k_2 = f_1f_2}$$

which is the Newton's formula.

• 1.6 EQUIVALENT FOCAL LENGTH OF THIN LENSES COMBINATION

Let us consider the two convex (thin) lenses L_1 and L_2 be situated at a distance 'd' (coaxially) on a principal axis XX'. A ray SA parallel to XX', is incident on lens L_1 at a height h_1, the lens deviates this ray AB through an angle δ_1. The deviated ray strikes on lens L_2 at a height h_2. The lens L_2 deviates it further through an angle δ_2. It then meets at the F_2.

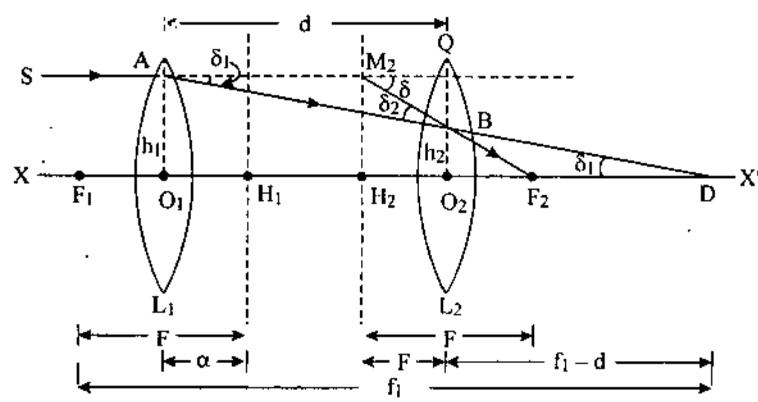


Fig. 9. Coaxial system to convex lenses

Produces SA onwards and BF_2 backwards to intersect at M_2. The total deviation produced by system is δ.

Draw a plane passing through M_2 and normal to the XX'. It meets the axis H_2. The system of lenses L_1, L_2 is replaced by an equivalent lens of focal length H_2F_2. The plane of such lens must be along the plane M_2H_2 and the distance of F_2 is referred from H_2.

Let the focal length of L_1, L_2 and combination be f_1, f_2, F respectively. For lenses of this type we have

$$\delta_1 = \frac{h_1}{f_1}, \; \delta_2 = \frac{h_2}{f_2} \;\text{ and }\; \delta = \frac{h_1}{F}$$

From Δ relationship we have

$$\delta = \delta_1 + \delta_2$$

$$\therefore \qquad \frac{h_1}{F} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \qquad \dots (1)$$

According to $\Delta^s AO_1D$ and $\Delta^s BO_2D$

$$\frac{AO_1}{O_1D} = \frac{BO_2}{O_2D}$$

$$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d}$$

or
$$h_2 = \frac{h_1}{f_1}(f_1 - d) \qquad \dots (2)$$

From equations (1) and (2),

$$\frac{h_1}{F} = \frac{h_1}{f_1} + \frac{h_1}{f_1 f_2}(f_1 - d)$$

or
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad \dots (3)$$

or
$$F = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{f_1 f_2}{\Delta} \qquad \dots (4)$$

where $(\Delta = f_1 + f_2 - d)$.

Δ is known as optical separation.

If P_1 and P_2 are the powers of lenses L_1 and L_2 respectively and P be the power of combination.

$$\because \qquad P = \frac{1}{f}$$

$\therefore$ From equation (3),

$$P = P_1 + P_2 - df_1 f_2 \qquad \dots (5)$$

Position of Cardinal Points : *Position of Second Focal Point* (F_2)—Comparing $\Delta M_2H_2F_2$ and ΔBO_2F_2 we get

$$\frac{M_2H_2}{H_2F_2} = \frac{BO_2}{O_2F_2}$$

$$\therefore \qquad O_2F_2 = \frac{BO_2 \times (H_2F_2)}{M_2H_2} = \frac{h_2}{h_1} . F$$

$$O_2f_2 = \frac{F}{h_1}\left(\frac{h_1}{f_1}(f_1 - d)\right) \qquad \text{(by using equation (2))}$$

$$O_2F_2 = F\left(1 - \frac{d}{f_1}\right) \qquad \dots (6)$$

Position of Second Principal Point (H_2) : The distance of IInd principal point H_2 from centre O_2 of L_2 lens is given by

$$H_2O_2 = H_2F_2 - O_2F_2$$

Putting the value of O_2F_2 from (6), we get

$$H_2F_2 = F - F\left(1 - \frac{d}{f_1}\right)$$

or
$$\beta = H_2O_2 = +\frac{Fd}{f_1}$$

But now use the sign convention

$$\beta = -\frac{Fd}{f_1} \qquad \dots (7)$$

Position of Ist Principal Point (H_1) : The distance of H_1 with respect to lens L_2 is similarly given by

$$\alpha = O_1H_1 = \frac{Fd}{f_2} \qquad \dots (8)$$

Position of First Focus Point (F_1): The position F_1 with respect to lens L_2 is iven by

$$O_1F_1 = H_1F_1 - H_1O_1$$

But $$H_1F_1 = F \text{ and } H_1O_1 = \frac{Fd}{f_2}$$

$$\therefore \quad O_1F_1 = F - F\frac{d}{f_2} = F\left(1 - \frac{d}{f_2}\right)$$

Now using the sign convention,

$$O_1F_1 = F\left(\frac{1-d}{f_2}\right) \qquad \text{... (9)}$$

• 1.7 NODAL SLIDE

"An apparatus (made by optical lens system) used to locate cardinal points by using the properties if Nodal points is known as **Nodal slide assembly.** It has an upright which carries a platform with two lens holders. The platform can made to move forward or backward and rotated in a horizontal plane."

Principal. "When a parallel beam of light is incident on a converging lens, an image is formed in its second focal plane on a screen. When the optical system is rotated about a vertical axis passing through its second nodal point, the image does not shift."

Let us suppose a light beam of rays parallel to the principal axis entering a lens system whose Nodal points are N_1 and N_2 shown in following fig. (10). The rays after refraction by system converge to the point I and F_2. Here the incident ray A coincident with axis passes through N_1 and N_2 to I.

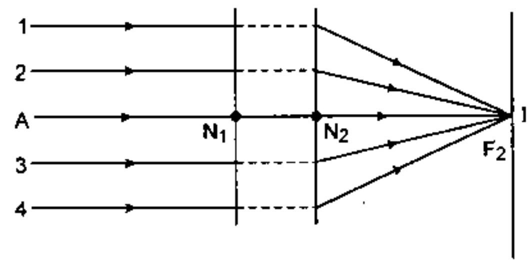


Fig. 10.

Let the system is rotated about N_2 through a small angle so that N_1 shift in position, but N_2 is unshifted shown in fig. (11). The incident ray number 2 which is directed towards N_1, must emerge along N_2I parallel to its direction of incidence. As the distance N_2I remains unshifted, the point I still lies in the second focal plane, through F_2 is now shifted to one side. Hence all the emergent rays also passes by I. Thus image still remains in the I.

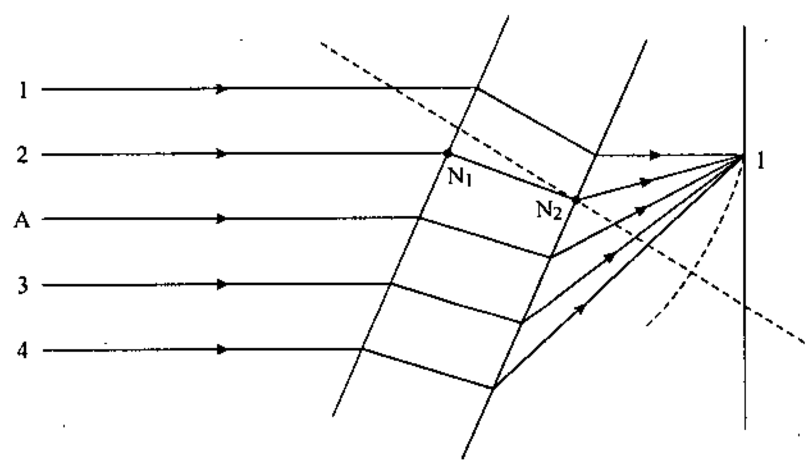


Fig. 11.

If a parallel beam incident on a system, the N_2 point can be located as the point about rotation of lens system. It produced no lateral shift in the position of the image. If medium of system is air then N_2 is also called IInd principal point.

Experimental Arrangement. We obtained the cardinal points by experiment as below :

(1) The distance of the Nodal slide from the crossed slits is adjusted to get define, image of the crossed-slits on the screen itself. In this position, the light from the crossed-slits emerges from the lens system as a parallel beam, and reflected back from the plane mirror as a parallel beam and is brought to focus again in the plane of the crossed-slits.

(2) Now, the Nodal slide is rotated about vertical axis. The image is shifted. The system is slide back and forth and the position Nodal slide apparatus. Whole is adjusted and define the image with no shift obtained. These rotation passes through N_2 and image of slit lies in IInd focal plane.

Thus psoition of N_2 and F_2 points can be read because medium is air then N_2 is defined by H_2 (IInd principal point). Its shown in fig. (12).

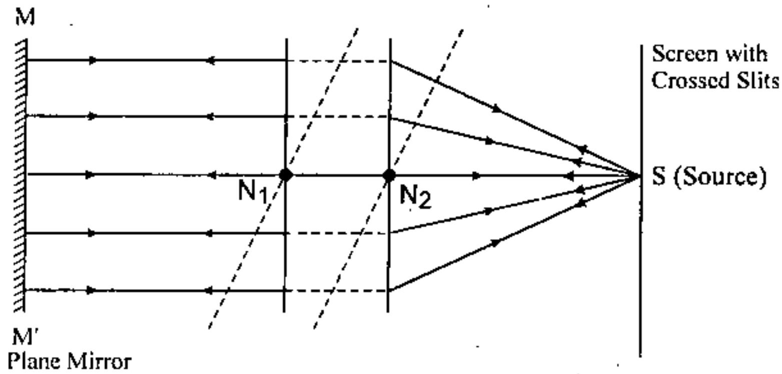


Fig. 12.

(3) Similar procedure, reversing the system relative to incident light and Ist Nodal and focal planes can be noted.

Thus, all the cardinal planes (and which intersection with axis) and cardinal points are located.

Verification :

$$\left(F = \frac{f_1 f_2}{f_1 + f_2 - d}\right)$$

Two convergence lenses are taken. Their focal lengths f_1 and f_2 are determined by separately by Nodal slide method. These lenses are then placed a known distance apart. Focal length (F) of the combination is then find by Nodal slide. F is calculated by below formula

$$F = \frac{f_1 f_2}{f_1 + f_2 - d}$$

The value of F experimentally are found to agree with in experimental error. Hence the formula stands verified.

• 1.8 ABERRATION

Defects in Images formed by a lens or combinations of lenses are known as aberrations. Because in 1855 L.V. Seidal was developed the aberrations principle, then it is called as Seidal aberrations. Basically aberrations are of two types :

(I) Monochromatic aberration.

(II) Chromatic aberration.

(I) Monochromatic aberration : Deviation or defects in images formed by lenses even if monochromatic light is incident on lenses these defects are known as **monochromatic aberrations** and further classified as given below viz., **Spherical aberrations, Coma, Astigmatism, Curvature of field, Distortion.**

(II) Chromatic aberration : Chromatic aberration is also known as colour defect and arises due to prismatic action of the lens materials (glass). A white object appears coloured when viewed through a lens due to this defect if the light of single colour is used to illuminate an object. This defect automatically disappears.

"*It is found that if the image of an object is formed by any lens under white light, it is* obtained as coloured and blurred instead of white and sharp. This defect is known as **chromatic aberration.**"

It arises because the refractive index μ of the lens material is different for different constituent colours of the white light. (or the focal length of the lens is depend upon refractive index of lens (μ) and wavelength of the light).

• 1.9 ACHROMATISM

"The images of all colours are formed in the same position and are of same size. In this condition, the chromatic aberration (longitudinal and lateral) are completely removed." It is the Ideal Position. It is however, not possible to achieve Ideal Achromatism. It is obtained by combination of two lenses satisfying the following conditions.

(a) Conditions of Achromatism of two thin lenses separated by finite distance : *Let f_1 and f_2 be the focal* lengths of two lenses separated by a distance x [in fig. 13]. The two lenses are made of the same material and μ is the refractive index of the medium. The equivalent focal length is given by

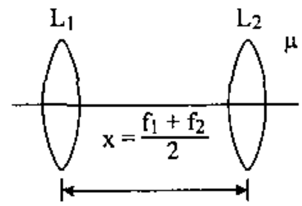


Fig. 13.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} \quad \text{... (1)}$$

Now using the method of the calculus.

Differentiating this equation (1)

$$d\left[\frac{1}{F}\right] = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2} - x\left[-\frac{df_1}{f_1^2 f_2} - \frac{df_2}{f_1 f_2^2}\right] \quad \text{... (2)}$$

But, for the combination to be achromatic.

or
$$d\left[\frac{1}{f}\right] = 0$$

Also
$$-\frac{df_1}{f_1} = -\frac{df_2}{f_2} = w$$

where w is the dispersive power of the material of the two lenses.

Substituting w in equation (2) we have

$$\therefore \quad \frac{w}{f_1} + \frac{w}{f_2} - x\left(\frac{w}{f_1 f_2} + \frac{w}{f_1 f_2}\right) = 0$$

or
$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{2x}{f_1 f_2} = 0 \quad \text{or} \quad x = \frac{f_1 + f_2}{2}$$

which is the required condition.

(b) Conditions of achromatism of combination of two lenses : Let f_1 and f_2 be the focal lengths of two lenses separated by a distance 'd'. Then equivalent focal length F is given by following formula

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \text{... (I)}$$

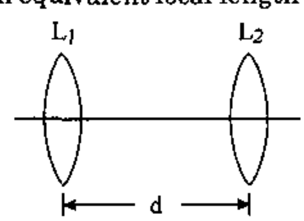


Fig. 14.

Now partial differentiating of this equation,

$$-\frac{\delta F}{F^2} = -\frac{\delta f_1}{f_1^2} - \frac{\delta f_2}{f_2^2} - d\left[-\frac{\delta f_2}{f_2^2 f_1} - \frac{\delta f_1}{f_1^2 f_2}\right]$$

or
$$\frac{\delta F}{F^2} = \frac{\delta f_1}{f_1^2} + \frac{\delta f_2}{f_2^2} - d\left[\frac{\delta f_2}{f_2^2 f_1} + \delta\frac{f_1}{f_1^2 f_2}\right] \quad \text{... (II)}$$

But for the combination to be achromatic then

$$\delta F = 0$$

Also $w_1 = \frac{\delta f_1}{f_1}$ and $w_2 = \frac{\delta f_2}{f_2}$

where w_1 and w_2 is the dispersive powers of the material of two lenses.

Putting these values, $\delta F = 0, w_1 = \frac{\delta f_1}{f_1}, w_2 = \frac{\delta f_2}{f_2}$ in equation (II), we get

$$\frac{w_1}{f_1} + \frac{w_2}{f_2} - d\left[\frac{w_2}{f_2 f_1} + \frac{w_1}{f_1 f_2}\right] = 0$$

or
$$\frac{w_1}{f_1} + \frac{w_2}{f_2} - \frac{2d}{f_1 f_2} = 0 \quad \text{... (III)}$$

Now here two cases will be arised.

Case I : When the lenses are placed in contact ($d = 0$) and material of the both lenses are not same.

From equation (III),

$$\frac{w_1}{f_1} + \frac{w_2}{f_2} = 0$$

or
$$\frac{w_1}{w_2} = -\frac{f_1}{f_2} \quad \text{...(IV)}$$

which is the required condition.

Discussion of the Condition :

(1) The equation (IV) is shown that the w_1 and w_2 are positive, and negative sign indicates that the one lens is concave and IInd lens is convex.

(2) If both lenses are made of same material, *i.e.*, $w_1 = w_2$ from equation (IV),

So
$$\frac{f_1}{f_2} = -1 \quad \text{or} \quad f_1 = -f_2$$

$$\therefore \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}, \quad \therefore \quad \frac{1}{F} = 0, \text{ or } F = \infty.$$

i.e., then the combination will behave like a plane glass plate and not a lens. So for achromatism, two lenses must necessarily be of different nature (materials). Thus to get achromatic convergent combination of two lenses in contact, the convex lens must be of crown glass and concave lens of flint glass. Its system is known as **Achromatic Doublet.**

Case II : When the both lenses are made of same materials and separated by a distance d.

if
$$w_1 = w_2 = w \quad \text{and} \quad d \neq 0$$

From the equation (IV),

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2d}{f_1 f_2}$$

or
$$d = \frac{f_1 + f_2}{2} \quad \text{... (V)}$$

which is the required condition.

Discussion of the Condition :

(1) If the lenses are of different material then they must satisfy equations (IV) and (V).

(2) Since useful combination is only that which behave as a convex system so either both the lenses must be convex lens or one concave and other convex.

1.10 SPHERICAL ABERRATION

The failure of the lens to form a point-image of an axial point-object is called as **Spherical Aberration.** The cause of spherical aberration is the spherical shape of the lens, due to which the focal length of the lens is different for different zones of the surface of lens.

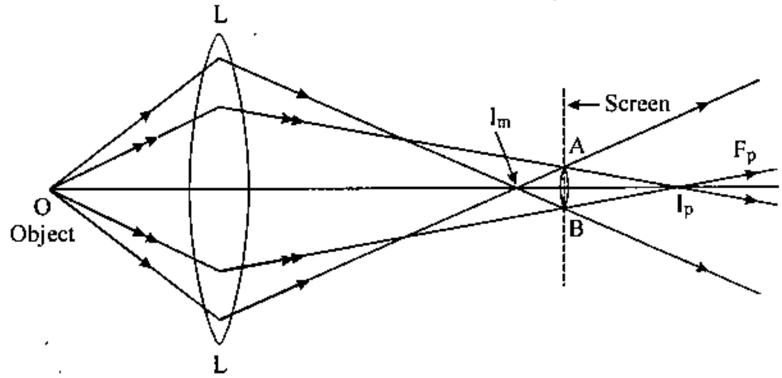


Fig. 15.

When a parallel beam of light falls on a lens in a direction parallel to the principal axis, it does not come to a single point focus. The paraxial rays (*i.e.*, the rays lying close to the axis) after refraction through the lens come to a focus F_p (Fig. 15), while the marginal rays (*i.e.*, the rays lying far from the axis) come to a focus F_m which is nearer the lens than F_p. Similarly, when a point-object O (Fig. 15) is placed on the axis of the lens, the paraxial rays come to a focus at I_p, while the marginal rays come to a focus at I_m, nearer the lens. The intermediate rays are brought to focus between I_m and I_p. The distance between I_m and I_p is a measure of **longitudinal or axial aberration.**

If a screen is placed perpendicular to the axis between I_m and I_p, the image of O is in the form of a circular disc. The size of the disc is a minimum in the position where the paraxial and marginal rays cross. It is then called the **'circle of least confusion'** and is the nearest approach to a point-image. The radius of the circle of least confusion is called **'lateral spherical aberration'**.

Defects (Spherical Aberration) be Minimized in Ordinary lenses :

In general, Spherical Aberration is minimum when the total deviation produced by system is equally divided on all refracting surfaces. Secondly system having small aperture. On this fact basis following method are used to remove aberration.

(1) By using stops : In this method either the marginal or paraxial rays are cut off. Actually the aperture of the lens is limited by using a stop. For this either the marginal or paraxial parts are blackened. Alternately a circular shutter is employed. Fig. 16 shows use of marginal and paraxial stops. However the intensity of image reduces by the use of stops.

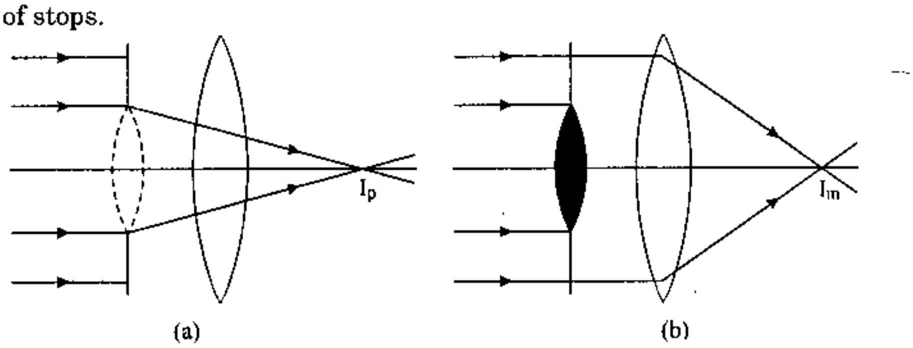


Fig. 16.

(2) For Combination of one convex and one concave lens : Spherical aberration for a convex lens is +ve and that for a concave lens is –ve. By a suitable combination of convex and concave lenses, spherical aberration can be made minimum.

(3) By using aplanatic lenses : A spherical refracting surface acts as an aplanatic surface. It forms the point image of a point object on the axis. If the distance of the point object on the axis from the centre of curvature of the surface of radius of

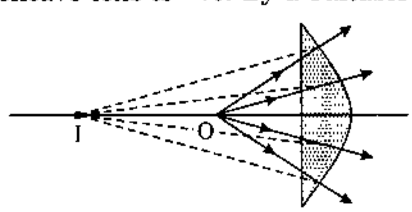


Fig. 17.

curvature R and refractive index μ be R/μ, then the distance of the point image on the axis from the centre of curvature will be $R\mu$. Such two points are known as **aplanatic points.** Meniscus lenses are frequently used as aplanatic lenses. The images formed by such lenses exhibit low aberration.

• 1.11 EYE-PIECE

Eye-Piece : Eye-Piece is a specially-designed magnifier for the coaxial system of lenses and are given a more perfect (magnified) image of an object.. Image formed obtained by a single lens of equal focal length. This system of lenses is adjusted to show minimum aberration in the finally formed images. **In this way, eye pieces are the magnifier to provide minimum aberrated image.**

Action and Construction (Image formation) : An eye-piece is a coaxial system of two convergent lenses. The lens which is towards the objective is known as **field lens (L_1)** and those towards to eye is called **L_2 eye-lens.** Two lenses are separated by a suitable distance and their relative focal lengths and placing is adjusted to exhibit

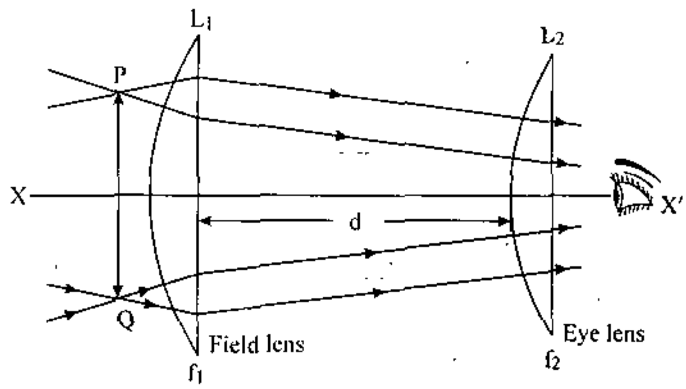


Fig. 18.

minimum aberration in the image. The field lens forms the magnified image of the object or of the image formed by the lens L_1. The image formed by lens L_1 is further magnified by the lens L_2. The utility of both lenses in an eye piece may be fomulated as given below.

Advantages over a single lens : (Merits of Eye Piece)

(i) Field of view is enlarged.

(ii) Image aberration are minimised. Spherical aberration (when $d = f_1 - f_2$) is minimised when we use the plano convex lens and chromatic aberration is minimised when $d = \frac{f_1 + f_2}{2}$.

(iii) It brings to centre of the exit pupil nearer the eye lens and increases the angular image field.

• 1.12 HUYGEN'S EYE-PIECE

(I) Construction : It consists of a two planoconvex lens (field lens and eye lens) of focal length $3f$ and f. Both the lenses are made of same material. They are placed coaxially at distance of $2f$. With their convex lens sides facting the incident light as shown in fig. 19.

The Eye-Piece satisfied the conditions of minimised aberrations as may seen below.

(II) Condition of Achromatism :

(i) For chromatic aberration : For a combination of two lenses of focal points f_1 and f_2 to be achromatic, they should be separated by distance 'd' is

$$d = \frac{(f_1 + f_2)}{2}$$

where $f_1 = 3f$, and $f_2 = f$.

Therefore eye-piece to be achromatic

$$d = \frac{3f + f}{2} = 2f \qquad \text{(distance between two lenses)}$$

Thus, the eye-piece is Free from chromatic aberrations.

(ii) For spherical aberration (minimum) : The distance between two lenses is given by

$$d = f_1 - f_2$$

Here $\quad f_1 = 3f \quad \text{and} \quad f_2 = f$

$$\therefore \quad d = 3f - f = 2f \qquad \text{(distance between two lenses)}$$

Thus, **the spherical aberration is minimum in this eye-piece.** Or thus, eye-piece is free from spherical aberration.

(III) Working (Action) or Image formation : The eye-piece adjusted for normal vision forms the final images at infinity. Hence the image I_2 formed by the lens L_1 (field lens) should lie at f to the left of L_2 (Eye-lens). For field lens (L_1), the image I_1 formed by the objective of the instruments serves as object. If its distance from the field lens is u, then from lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

We have putting $v = f$ and $f = 3f$ (for L_1)

$$\frac{1}{f} - \frac{1}{u} = \frac{1}{3f} \quad \text{or} \quad u = \frac{3f}{2}$$

Obviously, the image I_1 due to the objective lies on the same sides of the field lens as the image I_2, hence the rays coming from the objective, which were to converge at I_1, are intercepted by L_1 and focussed at I_2. The rays proceeding from I_2, are made parallel by L_2 (eye lens) as shwon in fig. 19.

(IV) Positions of the Cross Wires : Cross wires can not be used with this eye-piece to make measurements. If the measurement of the final image is required.

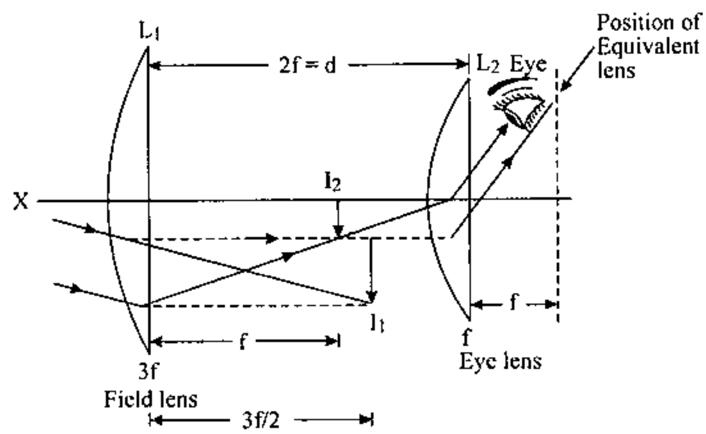


Fig. 19. Huygen's eye piece

The cross wires must be placed in the position of real image I_2, midway between the field-lens and eye-lens. In such a case, the cross wires will be magnificial by eye-lens only while the image of the object is formed by both the lenses of the eye-piece. Thus, the magnification of the image and the cross wires would not be the same.

(V) Equivalent Focal Length : The focal length of the equivalent lens is given by

$$F = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{3f \times f}{3f + f - 2f} = \frac{3f}{2}$$

The equivalent lens must be placed at a distance $3f/2$ to the right of I_1 (or at a distance f to the right of eye-lens) so that I_1 falls in its focal plane and it forms the final image at infinity as is done by the actual eye-piece.

(VI) Position of Cardinal Points :

(i) Principal Points : The distance of the first principal point H_1 from the field lens L_1 is given by

$$\alpha = L_1H_1 = \frac{Fd}{f_2} = \frac{\left(\frac{3f}{2}\right)\times 2f}{f} = 3f$$

Thus, first principal point H_1 *lies to the right of field lens* L_1 *at a distance* $3f$.

The distance of the second principal point H_2 from the eye-lens L_2 is given by—

$$\beta = L_2H_2 = -\frac{Fd}{f_1} = \frac{-\left(\frac{3f}{2}\right)\times 2f}{3f} = -f$$

Thus, H_2 *lies to the left of eye-lens* L_2 *at distance* f.

(ii) Focal Points : The distance of the first point F_1 from the field lens L_1 is —

$$L_1F_1 = -F\left(1-\frac{d}{f_2}\right) = -\frac{3f}{2}\left(1-\frac{2f}{f}\right)$$
$$= -\frac{3f}{2} + 3f = +\frac{3f}{2}$$

Thus, F_1 *lies to the right of* L_1 *at a distance* $\frac{3f}{2}$.

The distance of the second focal point F_2 from the eye-lens L_2 is

$$L_2F_2 = +F\left(1-\frac{d}{f_1}\right) = \frac{3f}{2}\left(1-\frac{2f}{3f}\right) = +\frac{f}{2}$$

Thus, F_2 *lies to the right of* L_2 *at a distance* $\frac{f}{2}$.

(iii) Nodals Points : Because medium on both of lenses (or medium in eye-piece is air) is same, then the two Nodal points N_1 and N_2, two principal points H_1 and H_2 are coincides. The respective planes pass through these points. These planes are normal to the principal axis and shown in fig. 20.

Accordingly the positions of cordinal points of Huygen's eye-piece are shown in fig. 20.

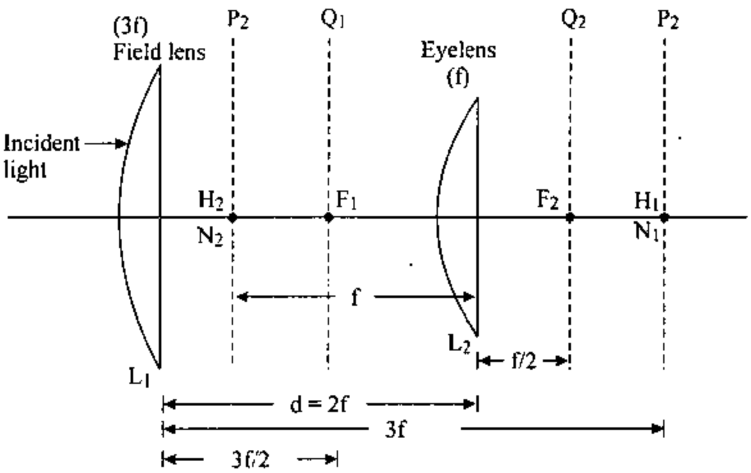


Fig. 20. Positions of cardinal points for Huygen's eye piece

• 1.13 RAMSDEN'S EYE-PIECE :

(i) Construction : A Ramsden's eye-piece has two plano-convex lenses of equal focal lengths, placed with their convex sides facing each other inwards. The distance between two lenses is equal to $\frac{2}{3}f$, where f is the focal length of their lens. Thus in Ramsden's eye-piece,

$$f_1 = f_2 = f \text{ and } d = \frac{2f}{3}.$$

The lens facing the objective is called **Field lens (L_1).** Its plane side faces the incident rays, the eye-piece is placed beyond the image formed by the objective of the instrument.

(II) Condition of Achromatism :

(i) For chromatic aberration : For a combination of two lenses of focal lengths f_1 and f_2 to be achromatic, they should be separated by distance d is

$$d = \frac{f_1 + f_2}{2} \qquad \text{where} \qquad f_1 = f_2 = f, \quad \therefore d = f.$$

This means field lens (L_1) should be placed in the focal plane of the eye lens. But in this position, any scratch/dust particles on the L_1 would be magnified and would spoil the final images. Therefore the distance between two lenses is kept $\frac{2f}{3}$. It does not causes much departure from achromatism. Thus Ramsdens, eye-piece exhibits a small chromatic aberration.

(ii) Spherical aberration : It is minimized by limiting the apertures and using the plano-convex lenses with their convex surface facing each other.

(iii) Action (Image Formation) : The eye-piece adjusted for normal vision forms the final image at infinity. Hence the image I_2 formed by the field lens (Fig. 21) should lie at a distance f to the left of eye-lens or at a distance $f/3$ to the left of field lens (because distance between the field lens and eye lens is $\left(\frac{2}{3}\right) f$. For field lens, the image I_1 formed by the objective of the instrument (in which eye-piece is fitted) serves as obejct. If distance of I_1 from field lens is u, then from lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have, on substituting $v = -\frac{f}{3}$ and $f = f$ (for field lens).

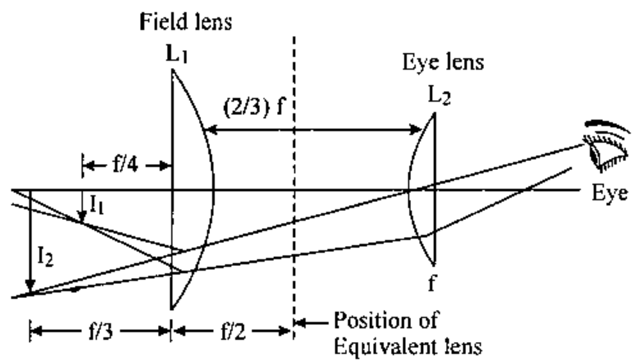


Fig. 21. Ramsden's eye piece

$$\frac{1}{-\frac{f}{3}} - \frac{1}{u} = \frac{1}{f}, \quad \text{or} \quad \frac{1}{u} = -\frac{3}{f} - \frac{1}{f} = -\frac{4}{f}, \quad \text{or } u = \frac{f}{4}$$

The $-ve$ value of u shows that the image I_1 formed by the objective lies to the left of field lens. The rays coming from I_1 appear to come from I_2 after emerging from the field lens. Since I_2 is at a distance f from eye-lens, the emergent rays from eye-lens become parallel and the final image is formed at infinity at shown in fig. 21.

(iv) Position of the cross wires : The cross wires of micrometer scale must be placed at the position where the image due to the objective is formed *i.e.*, in the position of the real image I_1. It lies at distance $f/4$ infront of the field lens. Then both, the image I_1 and cross wires (or scale placed there) will be magnified to the same extent and hence accurate measurements can be made with the help of this piece.

(v) Focal Length of the equivalent lens : The focal length of the eye piece or of equivalent lens is given by

$$F = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{f.f}{f + f - \frac{2}{3}f} = +\frac{3}{4}f$$

This equivalent lens must be placed at $\frac{3}{4}f$ to the right of real image I_1, *i.e.*, $d = \frac{3f}{4} - \frac{f}{2} = \frac{f}{2}$ to the right of L_1. Then I_1 would lie in the focal plane of the equivalent lens and it will form the final images at infinity as is done by the actual eye-piece.

(vi) Position of cardinal points :

(a) Principal points : The distance of the first principal point H_1 from the field lens (first lens) L_1 is given by

$$\alpha = L_1H_1 = +\frac{Fd}{f_2} = \frac{\left(\frac{3}{4}\right)f\left(\frac{2}{3}\right)f}{f} = +\frac{f}{2}$$

Thus, H_1 *lies to the right of* L_1 *at a distance* $\frac{f}{2}$.

The distance of the second principal point H_2 from the eye-lens L_2 is given by —

$$\beta = L_2H_2 = -\frac{Fd}{f} \quad \therefore \quad \beta = -\frac{\left(\frac{3}{4}\right)f\left(\frac{2}{3}\right)f}{f} = -\frac{f}{2}$$

Thus H_2 *lies to the left of eye-lens* L_2 *at a distance* $\frac{f}{2}$.

(b) Focal points : The distance of the first focal point F_1 from the field lens L_1 is

$$L_1F_1 = -F\left(1 - \frac{d}{f_2}\right) = -\frac{3f}{4}\left(1 - \frac{\left(\frac{2}{3}\right)f}{f}\right) = -\frac{f}{4}$$

Thus, F_1 *lies to the left of field lens* L_1 *at a distance* $f/4$.

The distance of the second focal point F_2 from the eye-lens L_2 is

$$L_2F_2 = +F\left(1 - \frac{d}{f_1}\right) = \frac{3f}{4}\left(1 - \frac{\left(\frac{2}{3}\right)f}{f}\right) = +\frac{f}{4}$$

Thus, F_2 *lies to the right of* L_2 *at a distance* $f/4$.

Accordingly the positions of cardinal points of Ramsden's eye-piece are shown in fig. 22.

(c) Nodal points : As the medium on either side of eye piece is the same (air), the Nodal points N_1 and N_2 coincide with the principal points H_1 and H_2. The respective planes are shown by dotted lines in fig. 22.

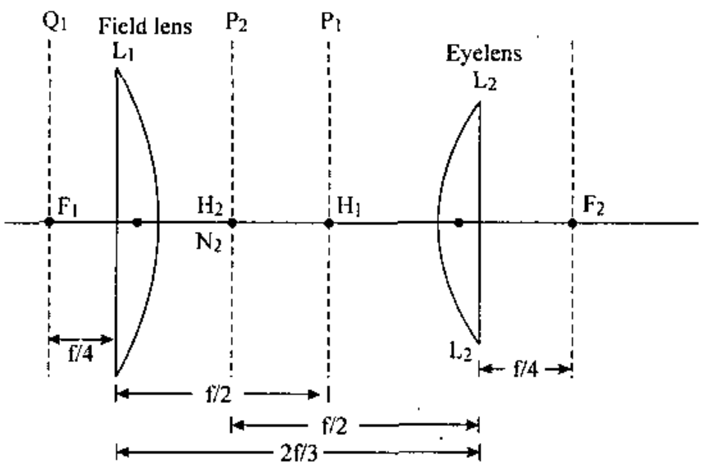


Fig. 22. Positions of cardinal points for Ramsden's eye-piece

• 1.14 COMPARISON OF RAMSDEN'S AND HUYGEN'S EYE-PIECE

S. No.	Ramsden's Eye-Piece	Huygen's Eye-Piece
1.	It does not satisfy the condition of minimum spherical aberration $d = f_1 - f_2$. Yet spherical aberration is minimised by dividing deviation on all four sources.	It satisfies the condition of minimum spherical aberration $d = f_1 - f_2$.
2.	It does not satisfy the condition of minimum chromatic aberration $d = \frac{(f_1 + f_2)}{2}$. But used for measurements of one colour at a time hence important.	It satisfies the condition of minimum chromatic aberration $d = \frac{f_1 + f_2}{2}$. Yet some colour defect exhibited.
3.	Final Image is flat.	Final Image is convex towards eye.
4.	The cross-wire are put outside the eye piece. It involves no mechanical problems.	Fitting of cross wires involves mechanical difficulty, because cross wires, if used are to be placed mid way between two lenses.
5.	It exhibits no comma, and distortion is less than in case of Huygens eye piece.	It exhibits distortion and comma.
6.	Its used in microscopes and telescopes for accurate quantitative measurements.	Its used for qualitative purpose in microscope (No measurements).

• SUMMARY

- The light rays travel along straight line paths in a non-homogeneous medium.
- $\sin i = \sin r$, *i.e.*, law of reflection.
- The focal points are a pair of points lying on the axis of the system conjugate to points at infinity.
- Two points on the principal axis of the optical system characterised by unit, angular magnification are known as nodal points.
- The failure of the lens to form a point-image of an axial point-object is called as spherical aberration.
- The lens facing the objective is called field lens.

• STUDENT ACTIVITY

1. Define optical path.

2. Define focal points and focal planes.

3. What are nodal points ?

4. Show that the distance between two principal points and between two nodal points is same.

5. Show that if medium on both sides of the lens system is same, then principal and nodal points coincides.

6. What do you mean by aberration ?

7. Define achromatism.

8. What is an eye-piece ?

• TEST YOURSELF

1. Describe Fermat's principle of least time. Prove the law of reflection and refraction at a plane surface.
2. Describe cardinal points and their planes. How an image is formed in a lens system using cardinal points ?
3. Deduce Newton's formula $f_1 f_2 = k_1 k_2$ for a coaxial combination of two thin lenses, where k_1 and k_2 are distances of object and image from the first and second focal points.
4. Derive expression for the equivalent focal length and positions of principal points and focal points of a coaxial system of two convex thin lenses separated by a distance 'd'.
5. Describe the principle of Nodal slide and its uses in verifying the equivalent focal length formula $F = \dfrac{f_1 f_2}{f_1 + f_2 - d}$.
6. Derive the condition of achromatism for two thin lenses when they are placed in contact and when they are separated by a finite distance.
7. What is spherical aberration ? How can this defect be minimized in ordinary lenses.
8. Discuss the principle of working of Huygen's eye-piece. Deduce the positions of cardinal points of Huygen's eye-piece and indicate them on diagram.
9. Discuss the construction and working of Ramsden's eye-piece.

10. The principle of least time is enunciated by :
 (a) Fermat (b) Gauss (c) Newton (d) None of these
11. In a Homogeneous medium, the velocity of the light is always :
 (a) Variable at every point (b) Constant at every point
 (c) Both of (a) and (b) are false (d) None of these
12. The velocity of light in a non-homogeneous medium always :
 (a) Changes at every point (b) Fixed at every point
 (c) Both of (a) and (b) are false (d) None of these
13. Which is true ?
 (a) $\frac{\sin i}{\sin r} = {}_1\mu_2$ (b) $i = r$ (c) Both of (a) and (b) are true (d) None of these
14. Cardinal point is enunciated by :
 (a) Fermat (b) Gauss (c) Newton (d) None of these
15. A plane normal to the principal axis and passing through F_1 point, it is known :
 (a) IInd focal plane (b) Ist focal plane (c) Nodal plane (d) None of these
16. The distance between principal point and focal point is known as :
 (a) Focal length (b) Nodal length (c) Unit length (d) None of these
17. When a ray of light is incident on a glass sheet at 90°, in presence of air. The value of refracted angle as :
 (a) 0° (b) $\pi/2$ (c) 1 (d) None of these
18. The linear transverse magnification of Nodal point is always :
 (a) +1 (b) −1 (c) ± 1 (d) None of these
19. The ratio between the length of image and length of the object is known as :
 (a) Lateral transverse magnification (b) Linear transverse magnification
 (c) Both of (a) and (b) (d) Angular magnification
20. Defect in imaged produced by a lens system is known as :
 (a) Aberration (b) Aplanatic (c) Achromatism (d) None of these
21. Spherical aberration is minimised by using a :
 (a) Telephoto lens (b) Convex lens(c) Objective lens (d) None of these
22. Coaxial system of two convergent lenses is known as :
 (a) Telescope (b) Eye-piece (c) Microscope (d) None of these
23. In an eye-piece, the lens is near to an object is known as :
 (a) Eye lens (b) Field lens (c) Both of (a) and (b) (d) None of these
24. Whose eye-piece can be used as a simple magnifier
 (a) Kellner (b) Ramsden (c) Gauss (d) Huygen's
25. If equivalent focal length of a Huygen's eye-piece is 5.0 cm. What is the focal length of the field lens :
 (a) 10 cm (b) 20 cm (c) 30 cm (d) 40 cm
26. If focal length of a convergent lens is 1.5 cm. Then the focal length of a Huygen's eyepiece is given by ?
 (a) 2.25 cm (b) 2.00 cm (c) 3.25 cm (d) 3.00 cm
27. Two thin convex lenses each of focal length, f are separated by X. For what value of X, the equivalent focal length will be negative ?
 (a) $X > 2f$ (b) $X < 2f$ (d) $2X > f$ (d) $2X < f$
28. If different zones of a lens produce different lateral magnification it is always known as :
 (a) Coma (b) Sherical aberration (c) Distortion (d) Astigmatism

ANSWERS

10. (a) 11. (b) 12. (a) 13. (c) 14. (b) 15. (b) 16. (a) 17. (b) 18. (b)
19. (c) 20. (a) 21. (a) 22. (b) 23. (b) 24. (b) 25. (a) 26. (a) 27. (a)
28. (a).

UNIT 2

INTERFERENCE

STRUCTURE

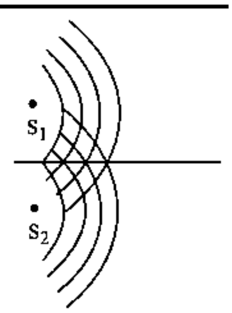
- Interference of Light
- Determination of Fringes Width Formula
- Conditions for Interference Lights
- Theory of Two Slit Interference
- Determination of Fringes Width
- Coherent Sources
- Incoherent Sources
- Expression for the Intensity at a Point in the Region of Superposition of two Waves of Same Periods (Wavelengths).
- Formation of Newton's Rings
- Raidus of Bright Rings
  - Summary
  - Student Activity
  - Test Yourself

LEARNING OBJECTIVES

After going this unit you will learn :
- Fresnel's Bi-prism; formation of coherent sources and interference fringes.
- Measurement of fringe-width ($\overline{X}$).
- Maxima, minima and fringe system.
- Michelson's interferometer and Young's Double Slit Method.
- Newton's rings experiments method and its applications.

• 2.1 INTERFERENCE OF LIGHT

When two waves of the same amplitude and frequency travel over the same path in a medium, the resultant displacement at any point is the algebraic sum of the displacements due to each waves separately. The phenomenon is called the *Interference of light*. They are of two types at some points, the intensity is maximum and minimum and the interference at these points is known as **constructive** and **destructive interference.** When two light waves are made to interfere we get alternates dark and bright bands shape, these are called interference fringes, if fringes is in circular form these called rings. Its shown in fig. (1).



• Constructive interference
∘ Destructive interference

Fig. 1.

Fresnel's Bi-prism (Construction) : The Fresnel used a biprism to show interference phenomenon. The biprism *abc* consists of two acute angled prism placed base to base. Actually its constructed as a single prism of obtuse angle of about 179°. The acute angle α on both sides is about 30°. The Biprism is shown in fig. (2).

Formation of Coherent Sources and Interference fringes (Working) : The prism is placed with its refracting edge parallel to the line source *S*. Such that *"Sa"* is normal to face "bc" of the prism, when light falls from S on lower portion of bi-prism. Its bent upwards and light falls on upper portion of prism is bent downwards and both appear to come from the virtual sources A and B. Hence, A and B acts as two coherent sources. Suppose, the distance between A and B is $2d$, if a screen is placed at C. And MN is stop to limit rays. The cones of light HAY and XBH', diverging from A and B are superposed and the interference fringes are formed in the overlapping region HH'. These may be obtained at C, in fig. (3)

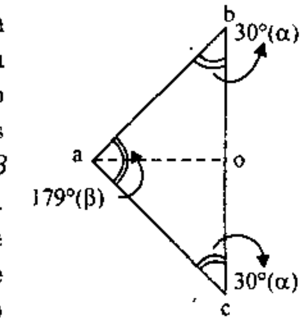


Fig. 2.

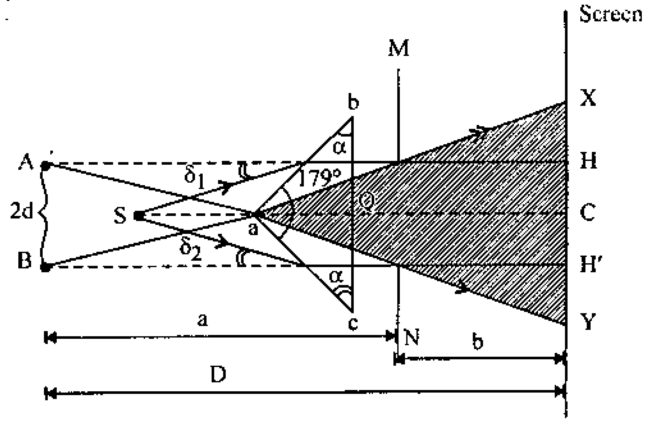


Fig. 3.

Experimental Adjustment : Before performing the experiment, the following adjustment must be made.

Adjustment

(i) Level the bed of optical bench with the help of spirit level and levelling screws.

(ii) The slit, biprism and eye-piece are adjusted at the same height the slit and the cross wire of eye-piece are made vertical.

(iii) The micrometer of eye-piece is focussed on crosswires.

(iv) With an opening provide to cover of the monochromatic source, the light is allowed to incident on the slit and bench is so adjusted that light comes straight along its lengths. This adjustment is made to avoid the loss of light intensity for the interference pattern.

(v) Place the Bi-prism upright near the slit and move the eye-piece sideway. See the two images of the slit through Bi-prism; if they are not seen, move the upright of Bi-prism right angle to the bench till they are obtained. Make the two images parallel by rotating Bi-prism in its own plane.

(vi) Bring the eye-piece near to the bi-prism and give it a rotation at right angle of the bench to obtain a patch of light. As a matter of fact, the interference fringes are obtained in this patch provided that the edge of the prism is parallel to the slit.

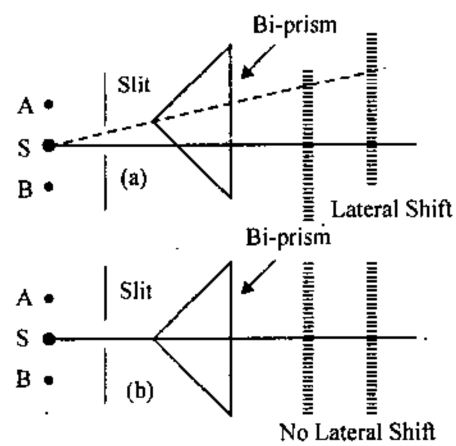


Fig. 4. (a and b)

(vii) To make the edge of the Bi-prism parallel to the slit, the Bi-prism is rotated with the help of tangent screw till a clear interference pattern is obtained. These fringes can be easily seen even with the naked eye.

(viii) The line joining the centre of the slit and the edge of the Bi-prism should be parallel to the bed of the bench. If this is not so, there will be lateral shift and the removal is most important. This is shown in fig. (4b).

(*a*) In order to adjust the system for no lateral shift, the eye-piece is moved away from Bi-prism. In this case, the fringes will move to the right of left but with the help of base screw provided with Bi-prism, it is moved at right angle to the bench in a direction to bring the fringes back to their original position.

(*b*) Now move the eye-piece towards the Bi-prism and the same adjustment is made with the help of eye-piece.

Now using the process again the again, the lateral shift is removed.

Measurement of "D" : The distance between slit and eye-piece uprights is noted. This distance gives D. The value of D is correct for the bench error.

Measurement of '2d' : In fig. (3), the light falls upon the upper portion of Bi-prism (aob) from S. It produces the deviation angle δ_1 and when light falls from S on the lower portion of prism (aoc). It produces the δ_2 deviation angles.

Also
$$SB = a.\delta_2$$
$$SA = a.\delta_1$$

the distance between A and B is
$$2d = SA + SB$$
$$\therefore \quad 2d = a(\delta_1 + \delta_2) \qquad \ldots (1)$$

We know that the
$$\delta = (\mu - 1)\, A'$$

where μ is the refractive index of lens and A' is the prism angles. If α_1 and α_2 are the angles of Bi-prism, therefore
$$\delta_1 = (\mu - 1)\,\alpha_1 \text{ and } \delta_2 = (\mu - 1)\,\alpha_2$$

From equation (1),
$$2d = a(\mu - 1)(\alpha_1 + \alpha_2)$$
if
$$\alpha_1 = \alpha_2 = \alpha \text{ then}$$
$$2d = a(\mu - 1)\, 2\alpha$$
or
$$d = \alpha a(\mu - 1)$$

where α in a radian form (π Radian $= 180°$) and 'a' is the distance between source and bi-prism.

Measurement of Fringe-width $\overline{X}$: After obtaining the fringes, the vertical cross-wire of the eye-piece is set on a bright fringe on one side of the interference pattern. The reading of the micrometer screw is taken. Then the eye-piece is moved laterally so that the vertical cross-wire coincides with successive bright fringes and the corresponding reading are noted. From these readings, the fringe-width $\overline{X}$ is found.

• 2.2. DETERMINATION OF FRINGES WIDTH FORMULA

It is the separation between two consecutive bright (or dark) interferences to fringes.

In fig. (5), A and B are the two coherent sources and D is the distance, from S (A or B) to C. The distance between A and B is $2d$. Let X be any point on the screen, distance x, from C. From the geometry of fig., we get
$$(AX)^2 = D^2 + (x - d)^2$$
and
$$(BX)^2 = D^2 + (x + d)^2$$

Hence, the paths difference at X is
$$\delta = BX - AX = [D^2 + (x + d)^2] - [D^2 + (x - d)^2]$$

or $$\delta = D\left[\left(1+\frac{x+d}{D}\right)^2\right]^{1/2} - D\left[1+\left(\frac{x-d}{D}\right)^2\right]^{1/2}$$

$$\delta = D\left[1+\frac{1}{2}\left(\frac{x+d}{D}\right)^2 - 1 - \frac{1}{2}\left(\frac{x-d}{D}\right)^2\right]$$

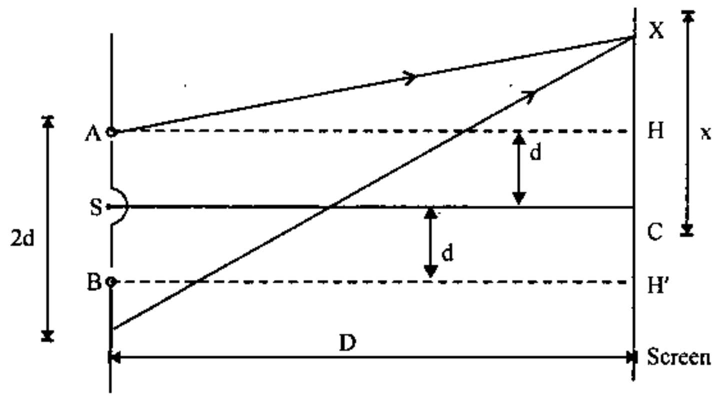


Fig. 5. Determination of fringe width

On expanding the R.H.S. by using the binomal theorem and $D >> x, d << D$ so neglect the higher terms.

$$\delta = \frac{D}{2D^2}[(x+d)^2 - (x-d)^2] = \frac{4xd}{2D} = \frac{2xd}{D}$$

Now, for X to be the centre of a bright image, the path difference δ should be an integral multiple of λ *i.e.*

$$\delta = \frac{2xd}{D} = n\lambda$$

where $n = 0, 1, 2, \ldots$

or $$x = \frac{n\lambda D}{2d}$$

Let x_n and x_{n+1} be distance of the nth and $(n+1)$th bright fringes from C. Then the fringe width is given by

$$\bar{X} = x_{n+1} - x_n$$

$$= \frac{(n+1)\lambda D}{2d} - \frac{n\lambda D}{2d}$$

$$\bar{X} = \frac{\lambda D}{2d}$$

where, $\bar{X}$ is the distance between any two dark fringes. Thus, the wave length of sodium light is

$$\boxed{\lambda = \frac{\bar{X}2d}{D}.}$$

• 2.3. CONDITIONS FOR INTERFERENCE LIGHTS

(i) The two sources of light should be monochromatic, *i.e.* the sources should have the same single frequencies (or wavelength) if the light waves are of different frequencies, the phase difference between them continuously vary. Hence, the resultant intensity at any point will change with time being alternately maximum and minimum.

(ii) The two sources should be coherent, *i.e.* they should vibrate in the same phase or the phase difference should remain constant throughout.

In a steady interference pattern, the resultant intensity at any point should remain constant with time. The resultant intensity at a point X is given by

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos\delta_1,$$

where a_1 and a_2 are the amplitudes of the interfering waves at X and δ the total phase difference at X. Hence I will remain constant if δ remains constant. Now.

$$\delta = \delta_1 + \delta_2,$$

where δ_1 is the initial phase difference between the waves and δ_2 is the phase difference at X due to the path difference and is equal to $(2\pi/\lambda) \times$ path difference.

At a given point, the path difference is constant so that δ_2. remains constant. Therefore, for δ to remain constant, the phase difference δ_1 between the waves must be constant.

(iii) The amplitudes of the interfering waves must be equal. The intensities of the bright and dark fringes are $(a_1 + a_2)^2$ and $(a_1 - a_2)^2$ respectively, where a_1 and a_2 are the amplitudes of the two waves. In case $a_1 >> a_2$ there will be very little difference between the intensities of dark and bright fringes. Therefore the contrast between them will be poor. Hence, for good contrast, $a_1 \approx a_2$. Maximum contrast is obtained when $a_1 \approx a_2$, because then dark fringes will have zero intensity.

(iv) If the interfering waves are polarised, they must be in the same state of polarisation.

• 2.4. THEORY OF TWO SLIT INTERFERENCE

In fig. (6), two coherent sources A and B are shown derived from a single monochromatic source S and a screen PQ is placed such that the distance between the planes of screen PQ and AB is equal to 'D' with SC as the principal axis of the optical system.

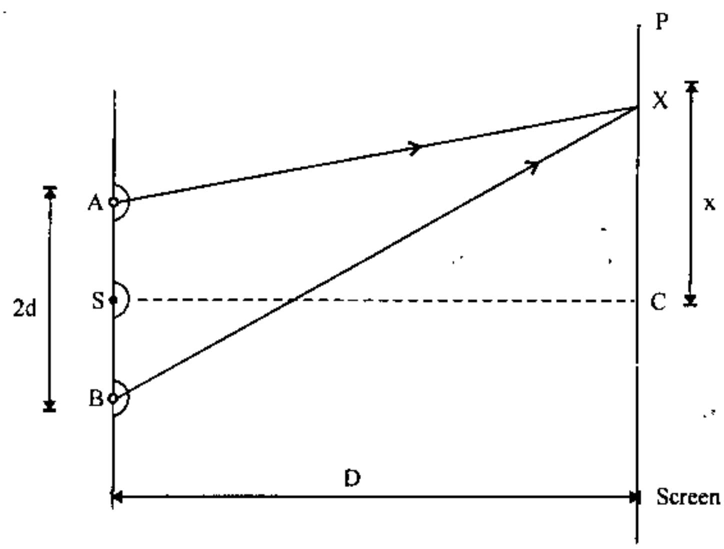


Fig. 6.

Two exactly similar waves originate at A and B to interfere and exhibit, maxima and minima on the screen let the wave to be of simple harmonic type is

$$Y = a \sin \frac{2\pi}{\lambda} (vt - x) \qquad \text{... (1)}$$

with terms having usual equal meaning. Then the effect of two waves at X point on screen respectively from A and B is given by the algebraic sum of the following versions of equation (1).

$$y_1 = a \sin \frac{2\pi}{\lambda} (vt - AX) \qquad \text{... (2)}$$

and

$$y_2 = a \sin \frac{2\pi}{\lambda} (vt - BX) \qquad \text{... (3)}$$

adding, the resultant displacement at X is

$$y = y_1 + y_2$$

$$y = 2a \cos \frac{2\pi}{\lambda} \left[\frac{BX - AX}{2} \right] \sin \frac{2\pi}{\lambda} \left[vt - \frac{BX + AX}{2} \right] \qquad \text{... (4)}$$

Evidently, the term $2a \cos \frac{2\pi}{\lambda}\left[BX - \frac{AX}{2} \right]$ is the amplitude of two waves at X. Its value depends upon the path differences $\Delta = (BX - AX)$.

(i) Maxima : The amplitude is maximum if

$$\cos \frac{2\pi}{\lambda}\left[\frac{BX - AX}{2} \right] = \pm 1$$

or $$\frac{2\pi}{\lambda}\left[\frac{BX - AX}{2} \right] = 0, \pi, 2\pi, \ldots = n\pi$$

or $$\Delta = BX - AX - n\lambda = 2n \cdot \frac{\lambda}{2} \qquad \ldots (5)$$

where $n = 0, 1, 2, \ldots$

This is the condition for X to be a maximum.

(ii) Minima : The value of amplitude is minimum provided

$$\cos \frac{2\pi}{\lambda}\left[\frac{BX - AX}{2} \right] = 0$$

or $$\frac{2\pi}{\lambda}\left[\frac{\Delta}{2} \right] = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots = (2n + 1)\frac{\pi}{2}$$

or $$\Delta = (2n + 1)\frac{\lambda}{2} \qquad \ldots (6)$$

where $n = 0, 1, 2, \ldots$

This is the condition for X to be a minimum.

(iii) Fringe system : Evidently for point C on axis $BX = AX$ and $\Delta = 0$. Hence C is called as zero order maximum if C for $n = 0$, the $\Delta = \frac{\lambda}{2}$, and we get Ist order minimum. Next follows $\Delta = \lambda$ for $n = 1$ and we get Ist order maximum. This succession continuous and we get a system of alternates maxima and minima on either side of C and system is called fringes system.

• 2.5. DETERMINATION OF FRINGES WIDTH

"The separation between two consecutive maxima of two consecutive minima on screen is known as fringe width."

Let in fig. (7), separation AB be equal to '$2d$', let D be the distance of screen PQ from the planes AB and SC as the principal axis of the system. Let the distance of a point X from C be x. Drop perpendiculars AH and BH' on PQ from respective sources A and B. Geometrically in $\Delta BH'X$,

$$(BX)^2 = (BH')^2 + (H'X)^2 = D^2 + (x + d)^2$$

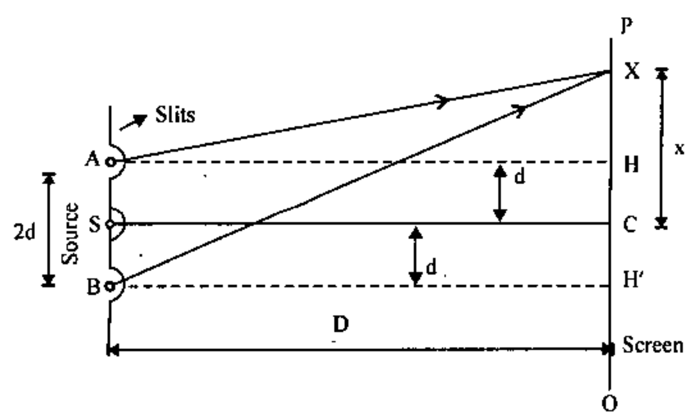


Fig. 7.

or $$(BX) = [D^2 + (x + d)^2]^{1/2}$$

$$\therefore \qquad (BX) = D\left[1 + \left(\frac{x+d}{D}\right)^2\right]^{1/2} \qquad \text{... (7)}$$

and similarly right angled triangle AHX,

$$(AX)^2 = (AH)^2 + (HX)^2 = D^2 + (x-d)^2$$

$$\text{or} \qquad (AX) = D\left[1 + \left(\frac{x-d}{D}\right)^2\right]^{1/2} \qquad \text{... (8)}$$

The path difference Δ (or δ) for point X on screen be

$$\Delta = BX - AX$$

$$\Delta = D\left[1 + \left(\frac{x+d}{D}\right)^2\right]^{1/2} - D\left[1 + \left(\frac{x-d}{D}\right)^2\right]^{1/2}$$

$$\Delta = D\left[1 + \frac{1}{2}\,\frac{1\,(x+d)^2}{D^2} - 1 - \frac{1}{2}\left(\frac{x-d}{D}\right)^2\right]$$

On expanding the R.H.S. by binomal theorem and $D >> (x+d)$ so that (neglect the higher terms),

$$\Delta = \frac{D}{2D^2}\,[(x+d)^2 - (x-d)^2]$$

$$\therefore \qquad \Delta = \frac{4xd}{2D} = \frac{2xd}{D} \qquad \text{... (9)}$$

(i) For maxima : We know that the condition of maxima

$$\Delta = \frac{2n\lambda}{2} = n\lambda \qquad \text{...(10)}$$

Hence, if X is the nth maximum then let $CX = x_n$ from equation (9) we have

$$\frac{2x_n d}{D} = \frac{2n\lambda}{2} = n\lambda$$

(ii) For minima :

$$x_n = \frac{Dn\lambda}{2d} \qquad \text{... (11)}$$

Let us consider the next $(n+1)^{th}$ minimum.

$$\text{Then} \qquad x_n + 1 = \frac{(n+1)D\lambda}{2d} \qquad \text{...(12)}$$

Using equations (11) and (12), we get

$$x_{n+1} - x_n = \frac{D\lambda}{2d}$$

This gives the separation between two consecutive maxima on the screen and known as fringe width represented by $\overline{X}$.

$$\therefore \qquad \boxed{\overline{X} = \frac{D\lambda}{2}}$$

which is shown that the $\overline{X}$ is independent of order n.

Hence, the spacing between any two consecutive bright fringes is the same or equispaced.

• 2.6. COHERENT SOURCES

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiations of the same colour or wavelengths.

• 2.7. INCOHERENT SOURCES

When the phase difference between two light waves arriving at a point vary with time in a random way, the waves-sources are called as **incoherent sources.**

Methods : The principle of obtaining coherent sources of light is given below.

"If two sources are derived from a single source by some device, then the any phase difference in one is simultaneously accompanied by the same phase-change in the other, thus the phase difference remains constant."

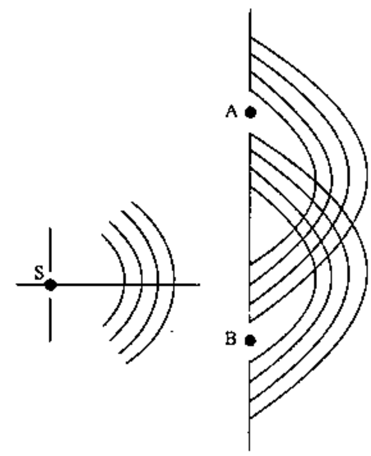


Fig. 8.

(1) Michelson's Interferometer : In this device, a single light beam is broken into two light waves perpendicular to each other, one by reflection and the other by refraction through a *half-silvered mirror*. The two beams, when reunite, produce fringes.

(2) Young's Double Slit Method : In this device [fig. (8)]. the wavefront starts from a small fine slit S backed by a strong source of light. This wavefront advances towards a screen having two narrow slits A and B. These slits are exactly symmetrically situated with respect to the slit S, from which they receive light. Thus A and B act as coherent sources.

• 2.8. EXPRESSION FOR THE INTENSITY AT A POINT IN THE REGION OF SUPERPOSITION OF TWO WAVES OF SAME PERIODS (WAVELENGTHS)

Consider a source of light S emitting waves whose wavelength is λ. And narrow pin holes A and B. A and B are equidistant from S and acts as virtual coherent sources. If a be amplitude of waves. Also phase difference between two waves at point X is given by and shown in fig. (9).

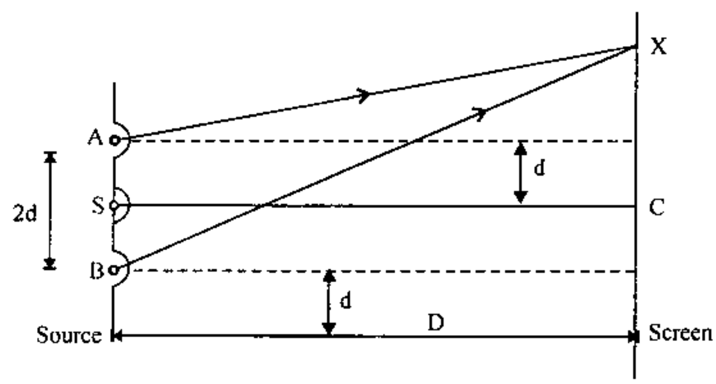


Fig. 9.

If y_1 and y_2 are the displacements

$$y_1 = a \sin \omega t$$

$$y_2 = a \sin (\omega t + \delta)$$

$$\therefore \quad y = y_1 + y_2 = a(\sin \omega t) + a \sin (\omega t + \delta)$$

$$y = a \sin \omega t\,(1 + \cos \delta) + a \cos \omega t \sin \delta \qquad \ldots (1)$$

taking $R \cos \theta = a\,(1 + \cos \delta)$

and $R \sin \theta = a \sin \delta$

$\therefore$ Equation (1) becomes

$$y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$$

$$y = R \sin (\omega t + \theta) \qquad \ldots (2)$$

which represents the equation of S.H. Vibration of amplitude is R.

$$\therefore \quad R^2 \sin^2 \theta + R^2 \cos^2 \theta$$
$$= a^2 \sin^2 \delta + a^2 (1 + \cos \delta)^2$$
$$= 2a^2 + 2a^2 \cos \delta$$
$$= 2a^2 (1 + \cos \delta)$$

or $$R^2 = 2a^2 \,.\, 2 \cos^2 \left(\frac{\delta}{2}\right) = 4a^2 \cos^2 \frac{\delta}{2}$$

Let the intensity at a point X is given by the square of the amplitudes.

$$\therefore \quad I = R^2$$

or $$I = 4a^2 \cos^2 \frac{\delta}{2} \qquad \ldots (3)$$

Cases : When phase difference $\delta = 0, 2\pi, 2(2\pi), \ldots n(2n)$

$$\therefore \quad I = 4a^2$$

Thus, the path difference is a whole number multiple of wavelength.

When $\delta = \pi, 3\pi, \ldots, (2n + 1)\pi$

$$\therefore \quad I = 0.$$

Intensity is minimum when the δ is an odd number multiple of 1/2 wavelength.

• 2.9. FORMATION OF NEWTON'S RINGS

When a plano-convex lens of large focal length is placed with its convex surface in contact with a plane glass plate, a thin wedge-shaped film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is almost zero at the point of contact and gradually increases from the point of contact outwards. When viewed by reflected monochromatic light, a system of concentric bright and dark rings is observed in the film. They are called *Newton's rings.*

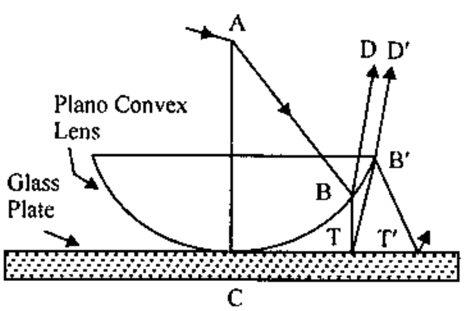


Fig. 10.

Newton's rings are the example of interference *fringes of constant thickness.* Since the thickness of air-film remains constant along a circle.

Definition : "The rings are alternate dark and bright rings observed due to the interference of light."

Mechanism of Interference

The mechanism of interference is shown in Fig. (10). An incident wave along AB is divided into two at B. The first reflected part is BD. It is reflected by the curved surface of the lens. The second reflected part is $B'D'$. These interfere to give the pattern in reflected beam. The transmitted pattern can also be seen along TT'. In order to remove the transmitted part and reflection from the surface of the plate, the lower surface of the plate is blackened.

Newton's Rings Experiments (Method)

The experimental arrangement for obtaining Newton's rings is shown in Fig. (11). A plano-convex lens L of large radius of curvature is placed with its convex surface in contact with a plane glass plate P. Light from a monochromatic source (sodium lamp) is allowed to fall on a convex lens through a broad slit which renders it into a nearly parallel beam. Now these parallel rays fall on the glass plate G which is held inclined at an angle of $\pi/4$ with the vertical. The light reflected from G falls normally on the air film enclosed between L and P. Interference fringes are formed between the rays reflected from the upper and lower surfaces of this film. These rings are concentric circles which are seen directly by a low power travelling microscope T focused on the air film.

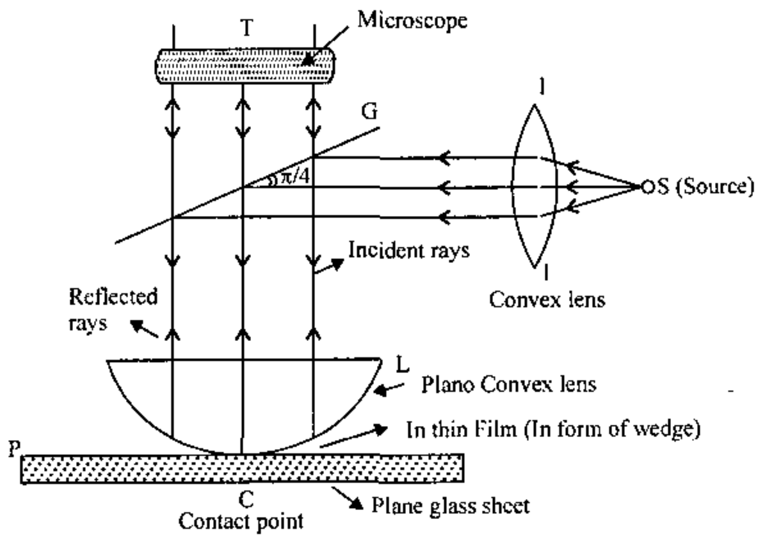


Fig. 11.

Applications of Newton's Ring Method : It is used to determinates the :

(1) Wavelengths of monochromatic light. (2) Refractive index of a liquid.

(1) Determination of wavelength (monochromatic) of light by Newton's Rings : The diameter of the nth dark ring in Newton's rings experiments is given by

$$D_n^2 = \frac{4n\lambda R}{\mu} \qquad \text{... (1)}$$

for a film of air enclosed between the plate and lens $\mu = 1$.

$$\therefore \qquad D_n^2 = 4n\lambda R \qquad \text{[from equation (1)]} \quad \text{... (2)}$$

Similarly, the diameter of the $(n + p)$ the ring is given by

$$D_{n+p}^2 = 4(n+p)\lambda R \qquad \text{... (3)}$$

Difference of equations (3) and (2) gives

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\therefore \qquad \boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}} \qquad \text{... (4)}$$

On measuring the diameters of rings and the radius of curvature R the wavelength λ can be calculated.

(2) Determination of Refractive Index of a Liquid by Newton's rings : The experiment is performed when there is air film between the plano convex lens and the

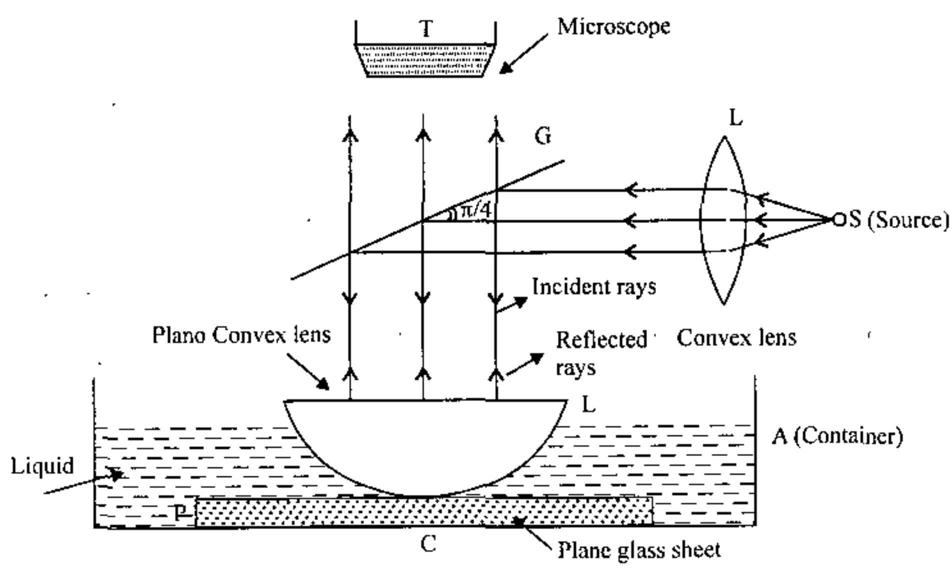


Fig. 12.

optically plane glass plate. These are kept in a metal container (A). The diameters of the nth and $(n+P)$th dark rings are determined with the help of a travelling microscope T as shown in fig. (12).

For air
$$\left.\begin{aligned}(D_{n+p})^2 &= 4(n+p)\lambda R \\ \text{and} \quad D_n^{\,2} &= 4n\lambda R\end{aligned}\right\} \quad \ldots (1)$$

The thin film of air is replaced by the liquid in container is without disturbing.

Then nth and $(n+p)$th rings diameter is obtained.

For liquid,

$$2\mu t \cos\theta = n\lambda \qquad \text{(for dark rings)}$$

or
$$2\mu t = n\lambda \qquad \text{if } \theta = 0°$$

But
$$t = \frac{r^2}{2R} \qquad \text{(where } r \text{ is radius of } L)$$

$\therefore$
$$\frac{2\mu r^2}{2R} = n\lambda$$

or
$$r^2 = \frac{n\lambda R}{\mu}$$

But
$$r = \frac{D}{2},$$

$\therefore$
$$D^2 = \frac{4n\lambda R}{\mu}$$

If D_n' is the diameter of the nth ring and D'_{n+p} is the diameter of $(n+p)$th ring.

$\therefore$
$$\left.\begin{aligned}(D'_{n+p})^2 &= \frac{4(n+p)\lambda R}{\mu} \\ \text{and} \quad (D'_n)^2 &= \frac{4n\lambda R}{\mu}\end{aligned}\right\} \quad \ldots (2)$$

or
$$(D'_{n+p})^2 - (D'_n)^2 = \frac{4\lambda RP}{\mu}$$

or
$$\mu = \frac{4\lambda RP}{(D'_{n+p})^2 - (D'_n)^2} \quad \ldots (3)$$

If P, λ, R, D'_{n+p}, and D'_n are known, then μ can be calculated. If λ is unknown, then equations (3) and (1) become

$$\mu = \frac{(D_{n+p})^2 - (D_n)^2}{(D'_{n+p})^2 - (D'_n)^2}. \quad \ldots (4)$$

• 2.10. RADIUS OF BRIGHT RINGS

Suppose the radius of the curvature of lens is 'R' and the thickness of air film is 't' at distance MN. From the point of contact O and r_n be the radius of a Newton's rings across a point M. It is shown in following fig. (13).

From figure (13),

$$MP^2 = OP \times PO'$$

or
$$r_n^2 = t.(2R - t) = 2Rt - t^2 \quad \ldots (1)$$

Since $t << R$, we can ignore the term t^2. Then

$$r_n^2 = 2Rt$$

or
$$t = \frac{r_n^2}{2R} \quad \ldots (2)$$

The condition of a bright ring is given by

$$2t = (2n-1)\frac{\lambda}{2} \quad \ldots (3)$$

or $$t = \frac{(2n-1)\lambda}{4}.$$

Since, the effective path of interfering rays from fig. (13) is

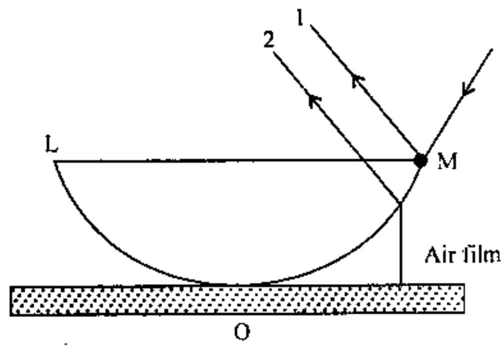


Fig. 13.

$$P = 2\mu t \cos r + \frac{\lambda}{2} \quad \dots (4)$$

But $\mu = 1$ (air refractive index) and $r = 0$

$$\therefore \quad P = 2t + \frac{\lambda}{2} \quad \dots (5)$$

If $t = 0, \Rightarrow P = \frac{\lambda}{2}$. It shows the central spot is dark the condition of maximum intensity (bright ring)

$$P = n\lambda$$

Equation (5) becomes $$n\lambda = 2t + \frac{\lambda}{2}$$

$$2t = \frac{\lambda}{2}(2n-1) \quad \dots (6)$$

for the condition of minima is given by

$$P = \frac{(2n+1)\lambda}{2} \quad \text{(Dark ring)}$$

equation (5) becomes

$$2t = n\lambda \quad \dots (7)$$

But from equation (2),

$$\frac{r_n^2}{2R} = (2n-1)\frac{\lambda}{4}$$

or $$r_n^2 = \frac{(2n-1)\lambda R}{2}$$

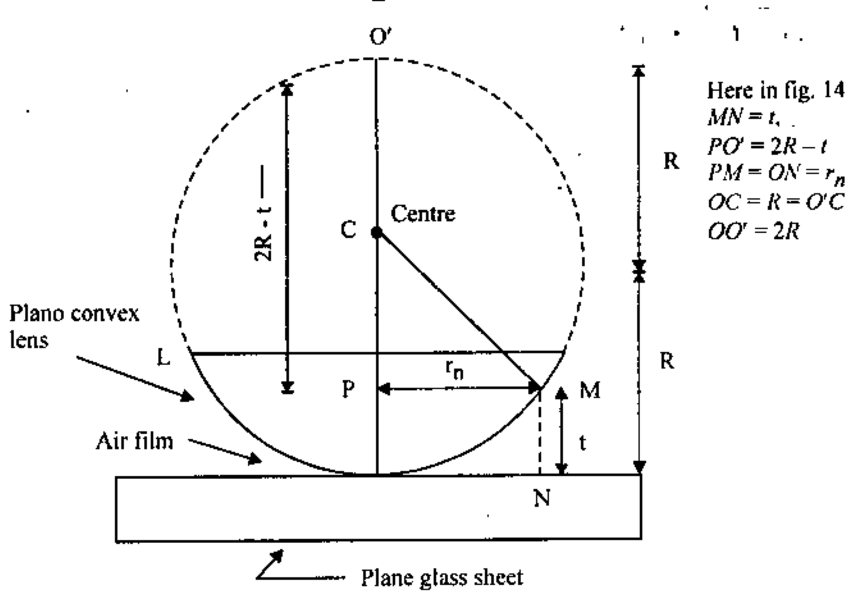


Fig. 14.

If D_n is the diameter of the nth ring (bright),

$$\therefore \quad D_n = 2r_n \Rightarrow r_n = \frac{D_n}{2} \quad \text{... (9)}$$

From equations (8) and (9),

$$D_n^2 = 2(2n-1)\lambda R$$

or

$$\boxed{D_n \propto \sqrt{2n-1}} \quad \text{... (10)}$$

where $\sqrt{2\lambda R}$ is a constant and $\sqrt{2n-1}$ is an odd number.

Thus, the diameters of bright rings are proportional to the square roots of the odd numbers (natural).

(B) Dark Ring (Radius) : The condition of a dark ring is

$$2t = n\lambda$$

But from equation (2),

$$t = \frac{r_n^2}{2R}$$

$$\therefore \quad n\lambda = \frac{r_n^2}{R}$$

$\therefore$ But from equation (9),

$$r_n = \frac{D_n}{2}$$

$$\therefore \quad \frac{D_n^2}{4R} = n\lambda$$

or

$$D_n = \sqrt{4\lambda R}\,\sqrt{n}$$

$$\boxed{D_n \propto \sqrt{n}} \quad \text{... (11)}$$

Thus, the diameters of dark rings are proportional to the square roots of natural numbers.

• SUMMARY

- When two light waves are made to interfere, we get alternates dark and bright bands shape, these are called interference fringes.
- The rings are alternate dark and bright rings observed due to the interference of light.
- The diameters of bright rings are proportional to the square roots of the odd numbers.
- The diameters of dark rings are proportional to the square roots of natural numbers.

• STUDENT ACTIVITY

1. What is interference of light?

2. Explain the conditions of interference.

3. What will happen if you use a white light source?

4. What is the effect of changing the distance between the slit and prism on the fringe width?

5. Define coherent and incoherent sources of light.

6. Prove that the diameters of dark rings are proportional to the square roots of natural numbers.

7. Prove that the diameters of bright rings are proportional to the square roots of odd numbers.

8. What will happen if white light is used in place of monochromatic light?

9. What will happen if the lens is lifted slowly up the plate?

• TEST YOURSELF

1. Describe Fresnel's bi-prism method to determine the wave-length of light.
2. Derive the expression of fringe-width formula by using bi-prism.
3. Describe the formation of interference fringes using two parallel slits (Young's experiment) obtain an expression for the fringe width.
4. Deduce an expression for the intensity at a point in the region of superposition of two waves of same periods (wavelengths)
5. Describe the formation of Newton's rings. How can these used to determine the refractive index of liquid and wavelength of sodium light?
6. The diameter of the 10th dark ring in an arrangement giving Newton's rings changes from 1.55 cm to 1.36 cm as a liquid introduced between lens and plate. Calculate the value of the μ of the liquid.
7. If a_1 and a_2 are the amplitudes of the two waves respectively. Then we obtained the maximum contrast when :

   (a) $a_1 = a_2$ (b) $a_1 \neq a_2$

   (c) $a_1 + a_2 = 0$ (d) None of the above
8. When two sources must emit radiations of the same colour. Then it is known as

   (a) Coherent sources (b) Incoherent sources

   (c) Ordinary sources (d) None of these
9. The laws of reflection and refraction could be explained on the basis of a wave theory by :

   (a) Huygen (b) Newton

   (c) Young (d) None of above
10. The concept of secondary wavelets is given by :

    (a) Huygen (b) Einstein

    (c) Newton (d) Young

ANSWERS

7. (a) **8.** (a) **9.** (a) **10.** (a)

UNIT 3

INTER FEROMETERS

STRUCTURE

- Michelson Interferometer
- Fabry-Perot Interferometer and Etalon
- Comparison of Febry-Perot interferometer with Michelson Interferometer
  - Summary
  - Student Activity
  - Exercise

LEARNING OBJECTIVES

After going this unit you will learn :

- Circular fringes and straight or localized fringes.
- Determination of wavelength of monochromatic light.
- Determination of difference between two close wavelengths.
- Formation of rings and intensity distribution.
- Condition of maximum and minimum intensity with visibility of fringes.

• 3.1 MICHELSON INTERFEROMETER

Michelson interferometer is an excellent device to show optical interference by the way of division of amplitude. An incident wave is divided into two waves of nearly equal amplitudes by partial reflection and transmission by an optically plane plate inclined at an angle of 45° to the incident wave. The divided waves travel in directions at right angles to each other and brought back together by reflections from two plane mirrors to interfere and the interference fringes are observed by a telescope. Two waves interfere after travelling distances of cm order and therefore, optical path difference may be of many wavelength, this is the ingenuity of Michelson interferometer. In cases like Newton's rings, Fresnel's biprism etc.

Construction : Michelson's (in 1881) interferometers consists of two highly polished mirrors M_1 and M_2. And mounted vertically on two arms at $\frac{\pi}{2}$ angles to each other their planes (M_2) can be slightly tilted about axes by adjusting screws at their backs. The mirror M_1 is mounted on a carriage provided with accuracy and M_1 and be moved in the direction of the arrows. (least count of the scale is 10^{-5} cm) and M_2 are fixed on scale. In fig. (1), two plane glass plates (same thickness) G_1 and G_2 parallel to each other and inclined at $\frac{\pi}{4}$ angles to M_1 and M_2. The surface of G_1 towards G_2 is partially silvered.

Working : When the parallel rays of light are incident on G_1 plate, then amplitude is equally transmitted by reflection and refraction and therefore two rays Ist and IInd are obtained. The reflected ray (1) and transmitted rays (2), travel to M_1 and M_2, mirrors respectively. After reflection at M_1 and M_2, the two rays recombine at half silvered plate G_1 and enter in a telescope T because both rays (1 and 2) are detected by reflection-refraction. Such they are coherent and hence in a position to interfere.

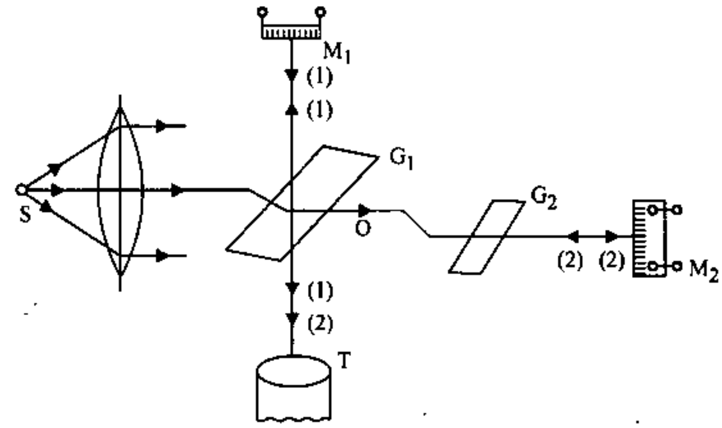


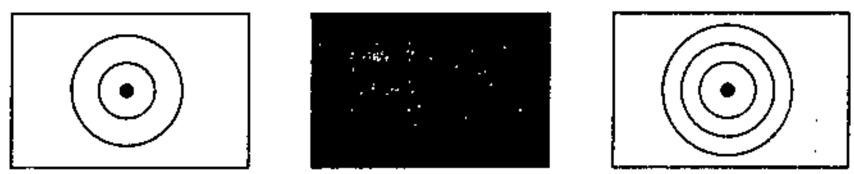
Fig. 1.

Function of plate G_2 : Let us suppose that the Plate G_2 are absence in paths of rays 2, then the ray 1 travels through the Glass plate G_1 at O. Twice (after reflection and transmission) while rays 2 does not to do so even once. Thus the paths of rays 1 and 2 in a glass are not same. To equalise the these Paths of glass G_2, which has the same thickness as G_1, is placed parallel to G_1. Here G_2 is known as the compensating plate.

Types of Fringes

(a) Circular Fringes (or Rings) : Circular rings are produced with monochromatic light in an interferometer. Here, the mirror M_1 and virtual mirror M_2' which is the image of M_2 must be parallel. (In fig. 3).

The source `S' is an extended one S_1 and S_2 are the virtual images of the source due to M_1 and M_2'. If the distance M_1M_2' is d, the distance between S_1 and S_2 are $2d$. The Paths difference between the two beams will be $2d \cos\theta$. Therefore the rays for which $2d \cos\theta = n\lambda$ will reinforce to produce maxima. These circular fringes which are due to



(a) (b) (c)

Fig. 2.

Interference with a phase difference determined by the inclination θ are called as **Haidinger's** or **Fringes of equal inclination**. When M_1 and M_2' coincide the path difference is zero and the field by view is perfectly dark. (In fig. 2b) when M_2' is nearer the eye than M_1 the fringes are shown in fig. (2a) when M_1 is further from eye than μ_1 rings shown in fig. (2c). They are shown in fig. (2)

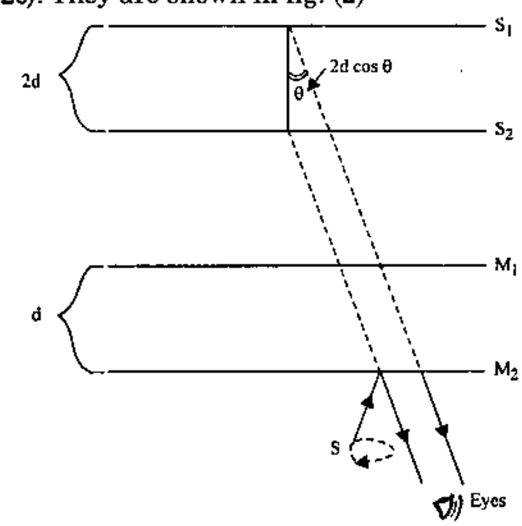
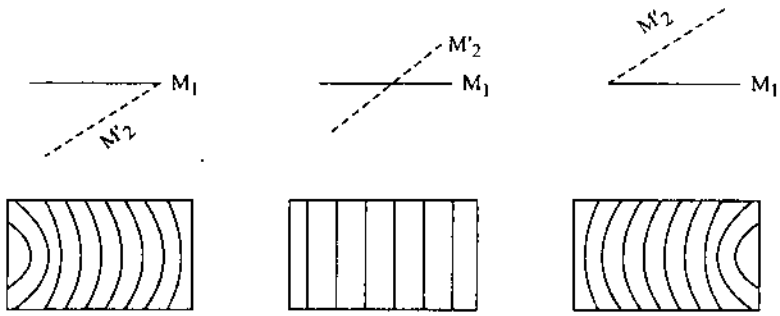


Fig. 3.

(b) Straight or Localized Fringes : When the mirror M_1 and the virtual mirror M_2' are (Image of M_2) inclined, then air film enclosed in wedge-shaped and straight rings are observed. The shape of the rings observed for various values of the difference paths as shown in fig. 4, the rings perfectly straight when M_1 actually intersects M_2' in the middle in fig. (4b). In other position as shown in fig. (4a and 4c), they are curved and are always convexd toward the thin edge of the wedge. This type of fringes are not observed for large different paths.



(a) (b) (c)

Fig. 4.

Applications

(i) It can used to determine refractive index and thickness of various thin transparent materials.

(ii) For the measurement of the standard metre in terms of wavelength of light.

(iii) The wavelength of given monochromatic source of light.

(c) Determination of wavelength of monochromatic light :

(i) Mirrors M_2 and M_1 are adjusted so that circular fringes are visible in the field of view.

(ii) With any ring at the centre, the reading of micrometer is noted. Let it be x_1.

(iii) Now mirror M_2 is kept fixed and the mirror M_1 is moved with the help of micrometer screw. The fringes appear to sink or rise due to the change of path difference. Let N fringes move and x_2 be the new reading of the micrometer. Then

$$x_2 - x_1 = x = N \cdot \frac{\lambda}{2} \quad \text{... (i)}$$

because for one fringe shift, the mirror moves through a distance equal to half the wavelength, *i.e.*, if M_1 is moved away from M_2 by $\frac{\lambda}{2}$, then $2x$ increased by λ. Thus,

$$\lambda = \frac{2(x_2 - x_1)}{N} = \frac{2x}{N} \quad \text{... (ii)}$$

(d) Determination of difference between two close wavelengths :

(i) Circular fringes are obtained first of all in the usual manner.

(ii) By adjusting the position of mirror M_1, a position is obtained when the intensity is maximum at centre. It happens when the bright fringes due to two close wavelengths (say λ_1 and λ_2) coincide. The reading of micrometer in this position is noted.

(iii) Now mirror M_1 is moved away by working the micrometer screw. The two sets of fringes get out of step and the rings have maximum indistinctness when bright fringe due to one set coincide with dark fringe due to other set. The mirror M_1 is moved further till the fringes become most distinct once again. This position till the fringes becomes most distinct once again. This position of micrometer screw is again noted.

Suppose the two lines D_1 and D_2 are of sodium light, they are very near to each other and λ_1 is wavelength of D_1 line and λ_2 is the wavelength of D_2 line. Also $\lambda_1 \neq \lambda_2$, let n_1 and n_2 be the changes in the order at the centre fixed when the M_1 is displaced through a distance 'd' between two consecutive positions of maximum distinctness of the rings.

$$\therefore \quad 2d = n_1\lambda_2 = n_2\lambda_2$$

if $\lambda_1 > \lambda_2$ then $n_2 = n_1 + 1$

$\therefore$ $$2d = n_1\lambda_1 = (n_1 + 1)\lambda_2 \quad \dots (1)$$

or $$n_1\lambda_1 = (n_1 + 1)\lambda_2$$

$$n_1 = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Putting this value in (1),

$$2d = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

or $$\boxed{\lambda_1 - \lambda_2 = \frac{\lambda^2}{2d}}$$

$$\Delta\lambda = \lambda_1 - \lambda_2$$

and $$\lambda_1 \lambda_2 = \lambda^2.$$

• 3.2. FABRY-PEROT INTERFEROMETER AND ETALON

The F.P. instruments are made of two types. In one, *the separation between two plates is kept fixed.* **It is called F.P. Etalon.** while in 2nd, *a screw is provided with either plate by which separation between plates can be changed.* **It is then called F.P. interferometer.**

Construction : It is constructed by C. Fabry and A. Perot in 1899. It is based on a principle of infinite rays interference.

It consists of two glass plates P_1 and P_2 (as shown in fig. 5) placed exactly parallel to each-other. Both plates are silvered at inner surfaces so that they reflect 80% of light incident on them. Their plate (P_2) can be slightly tight about axis by adjusting screws at their backs. The plate P_1 can be moved in the direction of arrows with more accuracy.

Formation of Rings : From fig. (5), the monochromatic lights from broad source S is made (parallel rays) incident on this system. Each ray of light, by multiple reflections and refractions by plates P_1 and P_2, produces a large number of transmitted beams. Hence, when these are focussed by a convex lens on a screen then an interference pattern is obtained on the screen.

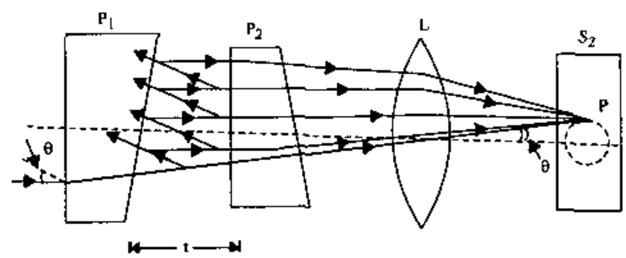


Fig. 5.

Intensity Distribution : In fig. (6), SA_1 is the incident light of system. The transmission intensity of partially part of incident rays light at P_1 by reflection (A_1 R_1) and refraction (A_1B_1). Let suppose that the reflected part is R and refracted part is B of light intensity.

$$\therefore \quad R = \frac{\text{Intensity of reflected light}}{\text{Intensity of refracted light}} \quad \text{and} \quad T = \frac{\text{Intensity of transmission light}}{\text{Intensity of incident light}}$$

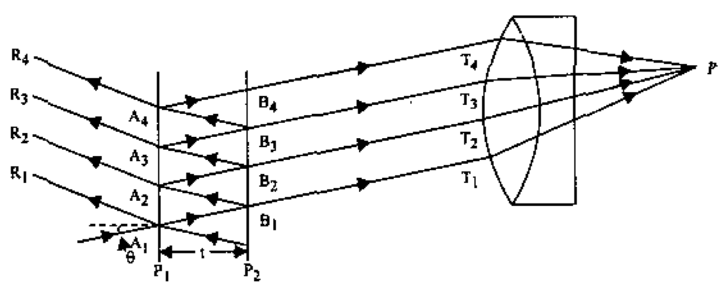


Fig. 6.

Here R is the reflection coefficient and T is the transmission coefficient. Therefore reflected part of amplitude is $\sqrt{R}$ and transmitted part of amplitude is $\sqrt{T}$ and a is amplitude of waves. The amplitudes of successive rays transmitted through the pair of plates will be :

$$B_1T_1 = \sqrt{T} \times \sqrt{T} = T$$
$$B_2T_2 = \sqrt{T} \times \sqrt{R} \times \sqrt{R} \times \sqrt{T} = TR$$
$$B_3T_3 = \sqrt{T} \times \sqrt{R} \times \sqrt{R} \times \sqrt{R} \times \sqrt{T} \times \sqrt{R} = TR^2$$

....

....

....

The phase difference between any two rays at P is

$$\lambda = \frac{2\pi}{\lambda} \times (2\mu t \cos\theta)$$

For air $\mu = 1$

$$\therefore \quad \delta = \frac{2\pi}{\lambda}(2t\cos\theta)$$

If the incident wave is represented by (if $a = 1$)

$$y = a \sin wt = \sin wt$$

then the waves reaching a point P on the screen and produces the vibration will be

$$\left.\begin{aligned} y_1 &= T\sin wt \\ y_2 &= TR(\sin(wt-\delta)) \end{aligned}\right\} \quad \text{... (1)}$$

$\therefore$ ....

....

....

By principles of superposition, the resultant vibration is given by

$$R = y_1 + y_2 + y_3 + \ldots$$
$$D\sin(wt-\psi) = T\sin wt + TR\sin(wt-\delta) + TR^2\sin(wt-2\delta) + \ldots$$

(From eq. (1) ... (2)

Now equating the coefficients of $\sin wt$ and $\cos wt$ from equation (2) we get

$$D\cos\psi = T + TR\cos\delta + TR^2\cos 2\delta + \ldots \quad \text{... (3)}$$
$$D\sin\psi = TR\sin\delta + TR^2\sin 2\delta + \ldots \quad \text{... (4)}$$

then the resultant amplitude is given by

$$I = D^2 = (D\cos\psi + iD\sin\psi)(D\cos\psi - iD\sin\psi) \quad \text{... (5)}$$

where ψ is the resultant phase.

From above equations (4) and (3),

$$D\cos\psi + iD\sin\psi = T(1 + Re^{i\delta} + R^2e^{2i\delta} + \ldots)$$
$$= T\,\frac{1}{1-Re^{i\delta}} \quad \text{... (6)}$$

and $\quad D\cos\psi - iD\sin\psi$

$$= T(1 + Re^{-i\delta} + R^2e^{-2i\delta} + \ldots)$$
$$= T\left(\frac{1}{1-Re^{-i\delta}}\right) \quad \text{... (7)}$$

From equations (5) and (7),

$$I = T^2 \cdot \frac{1}{(1-Re^{i\delta})\cdot(1-Re^{-i\delta})}$$
$$= \frac{T^2}{(1+R^2-R(e^{i\delta}+e^{-i\delta}))}$$

$$= \frac{T^2}{1 + R^2 - 2R\cos\delta} = \frac{T^2}{(1-R)^2 + 2R - 2R\cos\delta}$$

$$= \frac{T^2}{(1-R)^2 + 2R(1-\cos\delta)} = \frac{T^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}}$$

$$= \left[\frac{T^2}{(1-R)^2}\right]\left[\frac{1}{1 + \frac{4R}{1-R^2}\sin^2\frac{\delta}{2}}\right]$$

$$\boxed{I = \frac{T^2}{(1-R)^2} \cdot \frac{1}{\left[1 + F\sin^2\frac{\delta}{20}\right]}} \quad \ldots (8)$$

where $F = \left(\frac{4R}{(1-R)^2}\right)$ is called as **coefficient of Finesse.**

which is the required condition.

Condition of Maximum and Minimum Intensities with Visibility of Fringes:

For maximum intensity, if $\sin^2\frac{\delta}{2} = 0$, or $\delta = 2n\pi$

where $n = 0, 1, 2, \ldots$

Equation (8) becomes $$I_{\max} = \frac{T^2}{(1-R)^2} \quad \ldots (9)$$

For minimum intensity, if $\sin^2\frac{\delta}{2} = 1$, $\delta = (2n+1)\pi$

where $n = 0, 1, 2, \ldots$

Equation (8) remains becomes

$$I_{min} = \frac{T^2}{(1-R)^2} \cdot \frac{1}{[1+F]}$$

$$I_{min} = \frac{T^2}{(1-R)^2} \cdot \frac{(1-R)^2}{(1+R)^2} = \frac{T^2}{1+R^2} \qquad \left(\because F = \frac{4R}{1-R^2}\right) \quad \ldots (10)$$

From equations (9) and (8) we may write as :

$$I = I_{max}\frac{1}{1 + F\sin^2\frac{\delta}{2}} \quad \ldots (11)$$

and using the equations (9) and (10) we have

$$\frac{I_{max}}{I_{min}} = \frac{(1+R)^2}{(1-R)^2}$$

or visibility $$(V) = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \text{ or } V = \frac{2R}{(1+R)^2}. \quad \ldots (12)$$

• 3.3. COMPARISON OF FEBRY-PEROT INTERFEROMETER WITH MICHELSON INTERFEROMETER

The Fabry-Perot fringes differ from the Michelson's fringes in two respects :

(i) The Fabry-Perot fringes formed by multiple-beam interference are much sharper than the Michelson's fringes.

A measurement of the sharpness of a fringe is the "half-width" which is the width of the $I - \delta$ curve at the place where $I = \frac{1}{2} I_{max}$ (Fig. 7). Substituting the condition in the intensity expression,

$$I = \frac{I_{max}}{1 + \frac{4R}{(1-R)^2}\sin^2\left(\frac{\delta}{2}\right)},$$

we get $$\frac{1}{2} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\left(\frac{\delta}{2}\right)}$$

or $$\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2} = 1$$

or $$\delta = 2\sin^{-1}\frac{1-R}{2\sqrt{R}}$$

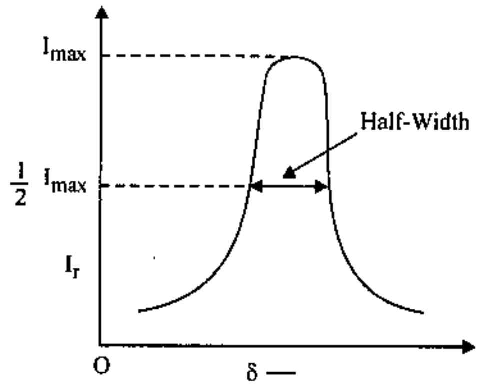


Fig. 7.

if $R = 0\cdot 8$ then

$$\delta = 2\sin^{-1}\frac{1-0\cdot 8}{2\sqrt{0\cdot 8}} \quad \text{or} \quad \delta \approx 2\sin^{-1}\frac{1}{9} = \frac{2}{9} = 0\cdot 22 \text{ radian.}$$

In Michelson's interferometer, the fringes are formed by the interference of two beams, and the intensity distribution is given by :

$$I = I_{max}\cos^2\frac{\delta}{2}.$$

For $$\frac{I}{I_{max}} = \frac{1}{2}, \text{ we have}$$

$$\delta = 2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \quad \text{or} \quad \delta = 1.57 \text{ radian,}$$

which is about 7 times the half-width of Fabry-Perot fringes.

(ii) When the light consists of two or more close wavelengths then in a F.P. interferometer each wavelength produces its own pattern, and the rings of one pattern are clearly separated from the corresponding rings of the other pattern. Hence, the instrument is very suitable for the study of the fine structure of spectral lines. In Michelson's instrument, separate patterns are not produced. The presence of two close wavelengths is judged by the alternate distinctness and indistinctness of the rings when the optical path difference is increased.

• SUMMARY

- Circular rings are produced with monochromatic light in an interferometer.
- The Fabry-Perot fringes differ from the Michelson's fringes.
- Michelson's interferometer consists of two highly polished mirrors.

• STUDENT ACTIVITY

1. Determine the wavelength of monochromatic light by using Michelson interferometer.

2. How will you determine the difference in the wavelengths of two D-lines of sodium light by Michelson's interferometer.

• TEST YOURSELF

1. Describe the construction and working of Michelson's interferometer.
2. Describe the construction and formation of rings by Febry-Perot interferometer. Discuss the intensity distribution in fringe system obtained by it.
3. Compare the fringes of Febry-Perot interferometer with those of Michelson's interferometer.
4. Etalon interferometer is invented by :
   (a) Fabry and Perot (b) Fabry and Michelson
   (c) Perot and Lummer (d) None of these
5. The ratio of the reflected and refracted light is known as :
   (a) Reflection coefficient (b) Transmission coefficient
   (c) Refraction coefficient (d) None of these
6. If a liquid is introduced between the plate and lens, the diameters of the rings are :
   (a) decreased (b) increased (c) uneffected (d) None of these
7. The intensity distribution in F.P. interferometer is equal to
   (a) $I = \dfrac{I_0}{1 - F\sin^2(\delta/2)}$ (b) $I = \dfrac{I_0}{1 + F\sin^2(\delta/2)}$
   (c) $I = \dfrac{I_0}{1 + F\cos^2(\delta/2)}$ (d) None of these
8. Interference filters work on the principle of interferometer which is :
   (a) F.P. (b) Michelson (c) Lummer (d) None of these
9. Circular fringes are observed in a Michelson's interferometer illuminated with light of wavelength 5896 Å. When the path difference between the mirrors M_1 and M_2 is $0 \cdot 3$ cm, the central fringe is bright. What should be the angular diameter of the 7th bright fringe ?
   (a) 4° (b) 3° (c) 2° (d) 5°
10. When a movable mirror of Michelson's interferometer is shifted through $0 \cdot 002945$ cm. A shift of 100 fringes is observed. What is the wavelength of light used in Å?
    (a) 5890 Å (b) 5892 Å (c) 5809 Å (d) None of these
11. White light is incident normally on a Febery-Perot interferometer with a plate separation of 4×10^{-4} cm. What the value of wavelengths for which there are interference maxima in the transmitted beam in the range 4000 to 5000 Å?
    (a) (4210, 4444, 4706) Å (b) (4000, 4400, 4700) Å
    (c) (4211, 4445, 4707) Å (d) None of these

ANSWERS

4. (a) **5.** (a) **6.** (a) **7.** (b) **8.** (a) **9.** (a) **10.** (a) **11.** (a)

UNIT

4

DIFFRACTION

STRUCTURE

- Diffraction of Light
- Fresnel's Half-Period's Zones
- Diffraction Pattern due to a Straight Edge
- Diffraction by a Narrow Wire
- Fraunhofer's Diffraction at a Single Slit
- Fraunhofer Diffraction at two Slits
- Plane Diffraction Grating
- Theory of Plane Transmission Grating
- Concave Reflection Grating
- Various Mountings
- Difference between the Grating and Prism Spectra
- Dispersive Power of Grating
- Resolving Power of an Optical Instruments
- Limit of Resolution of a Telescope
- Relation between Magnifying Power and the Resolving Power of Telescope
  - Summary
  - Student Activity
  - Exercise

LEARNING OBJECTIVES

After going this unit you will learn :

- Difference between Fresnel's and Fraunhofer's classes.
- Difference between of interference and diffraction.
- Width of diffraction bands.
- Dependence of intensity on $R_0^2 \sin^2 (\beta / \beta^2)$ and $\cos^2 \gamma$.
- Intensity distribution curve.
- Effect of increasing width of slit and distance between slits.
- Action of a diffraction grating.
- Formation of spectra.
- Row land mounting and determination of wavelength.
- Rayleigh limit resolution and resolving power of a plane transmission grating.

• 4.1 DIFFRACTION OF LIGHT

If light travels in straight lines in a transparent homogeneous medium, but when light is passed by the edge of an opaque obstacle. It bends slightly into the geometric shadow that would be cast by edge on a screen. This slight bending of light is called **diffraction of light.**

(a) Difference between Fresnel's and Fraunhofer's classes

	Fresnel's	Fraunhofer's
1.	The source, or screen, or both are at Finite distance from the diffracting elements.	The distance of the sources and screen from the diffracting elements are effectively infinite.
2.	No mirrors or lenses are used for observation.	Diffracted light is collected by a lens as in a telescope.
3.	Distances are important in this class of diffraction.	The angular inclinations are important in this type of diffraction.
4.	The wavefronts are divergent either spherical or cylindrical.	The wavefront incident on the aperture is plane which is realized by using a collimating lens.
5.	Mathematical investigation are complicated and only approximate.	Mathematical investigation are rigorous and easy.
6.	No such addition of effects is possible.	The effect of several diffracting elements can be added together.

(b) Difference between of Interference and Diffraction

(1) In the phenomenon of interference, the interaction occurs between two separate wavefronts originating from the two coherent sources. While in case of diffraction the interaction occurs between the secondary wavelets originating from different points of the exposed parts of the same wavefront.

(2) In an interference pattern, the regions of minimum intensity are usually almost perfectly dark while it is not so in a diffraction pattern.

(3) In an interference pattern, all the maxima are of same intensity but in a diffraction pattern they are varying intensity.

(4) The interference fringes are usually of the same width while the diffraction fringes are never of the same width.

• 4.2. FRESNEL'S HALF-PERIOD'S ZONES

Fresnel's applied Huygen's principle to explain the phenomenon of diffraction. According to Huygen's principle each point on wavefront **(when light travels, in space set particles in medium are vibrated then locus of the either particles in the same phase of vibration is called wavefront)** sends out secondary wavelets. Fresnels assume that these wavefronts produce interference due at any point. The light intensity at any point to wavefront. Fresnel's divided the wavefront into a number of zones for calculating the resultant intensity. These zones are called **Fresnel's half period zones.**

Cylindrical Wavefront : Suppose, a long narrow slit SS' illuminated by a monochromatic source of wavelength λ. The envelope of secondary wavelets diverging from the slit is a cylindrical surface WW'. With the slit as its axis (Fig. 1) in order to find the effect of WW' at an external point P. We divided the wavefront into a number of half period strip following the method adopted for Fresnel's half period zones.

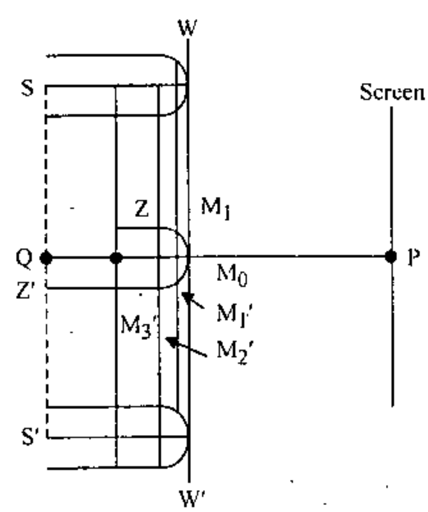


Fig. 1.

From 'P' draw a perpendicular PQ on SS'. The point of intersection MO of PQ with the wavefront is the pole of wave w.r.t. to P.

Let us now consider an equatorial band ZZ' through the line PM_0Q and suppose $M_0Q = \alpha, PM_0 = b$, with P as centre and radii

$$b + \frac{\lambda}{2}, b + \frac{2\lambda}{2}, \ldots, b + \frac{n\lambda}{2}$$

Draw, arcs cutting ZZ' in points, $M_1, M_2, \ldots$ and $M_1', M_2' \ldots$ as shown in fig. (2).

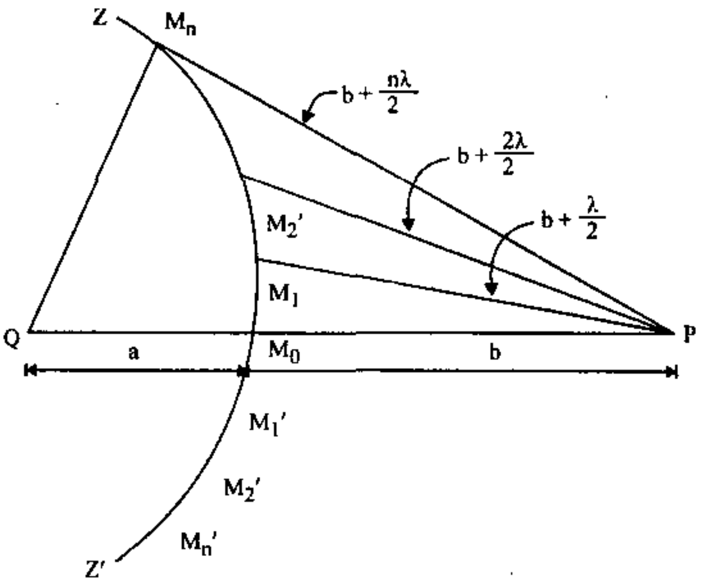


Fig. 2.

From the points $M_0, M_1, M_1', M_2, M_2'$ etc. draw lines which are parallel to the points so as to divide the wavefronts into a number of rectangular strips of equal length. But of different widths $M_0\ M_1, M_1\ M_2, \ldots,$ etc.

These points are called respectively Ist, IInd, third, etc half period strips. In this way the whole of the wavefront is divided into two exactly similar halves on the two sides of M_0. It is shown in fig. (3).

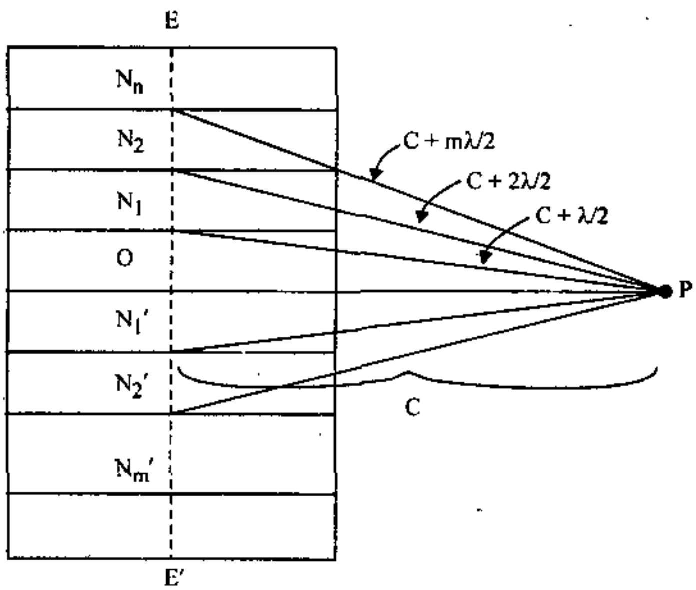


Fig. 3.

Let us now find the widths of these strips.

From fig. (2),

$$\Delta QM_n P \text{ say}$$

$$PM_n^2 = QM_n^2 + QP^2 - 2QM_n \ . OP \cos\theta_n$$

or $$\left(b + \frac{n\lambda}{2}\right)^2 = a^2 + (a+b)^2 - 2a(a+b)\cos\theta_n \qquad \text{... (1)}$$

but λ and θ_n be small, λ^2 can be neglected, then

$$b^2 + bn\lambda = a^2 + a^2 + b^2 + 2ab - 2a(a+b)\left(1 - \frac{\theta_n^2}{2}\right)$$

or $$bn\lambda = a(a+b)\,\theta_n^2$$

or $$\theta_n = \sqrt{\frac{bn\lambda}{a(a+b)}} = K\sqrt{n} \qquad \text{... (2)}$$

where $K\left\{=\left(\frac{b\lambda}{a(a+b)}\right)^{1/2}\right\}$ is a constant as a, b and λ are all constants.

Therefore

$$M_0M_n = a\theta_n = aK\sqrt{n} \qquad \text{... (3)}$$

by equation (2).

Putting $n = 1, 2, 3, \ldots$ etc, we obtain the widths of various half period strips in the following way

$$M_0\,M_1 = aK\sqrt{1} = aK$$
$$M_1\,M_2 = (M_0\,M_2 - M_0\,M_1) = 0\cdot 414\,aK$$

....

....

....

$$M_{15}\,M_{16} = (M_0\,M_{16} - M_0\,M_{15}) = 0\cdot 127\,aK$$
$$M_{16}\,M_{17} = (M_0\,M_{17} - M_0\,M_{16}) = 0\cdot 123\,aK$$

Thus, the widths of these strips decrease rapidly at Ist but more slowly as the order of strips increases. As the lengths of strips are equal, their area decreases. In order to find the effect of the entire wavefront WW' at P. Let us consider the strips EE', then centre of which is at an average distance O from P as shown in fig. 4. The strips can be divided into half period elements by drawing lines across its length such that the middle points $N_1, N_2, \ldots, N_m$ etc. of successive lines, are at distances $\left(C + \frac{\lambda}{2}\right)$, $\left(C + \frac{2\lambda}{2}\right) \ldots \left(C + \frac{n\lambda}{2}\right)$ etc. from P.

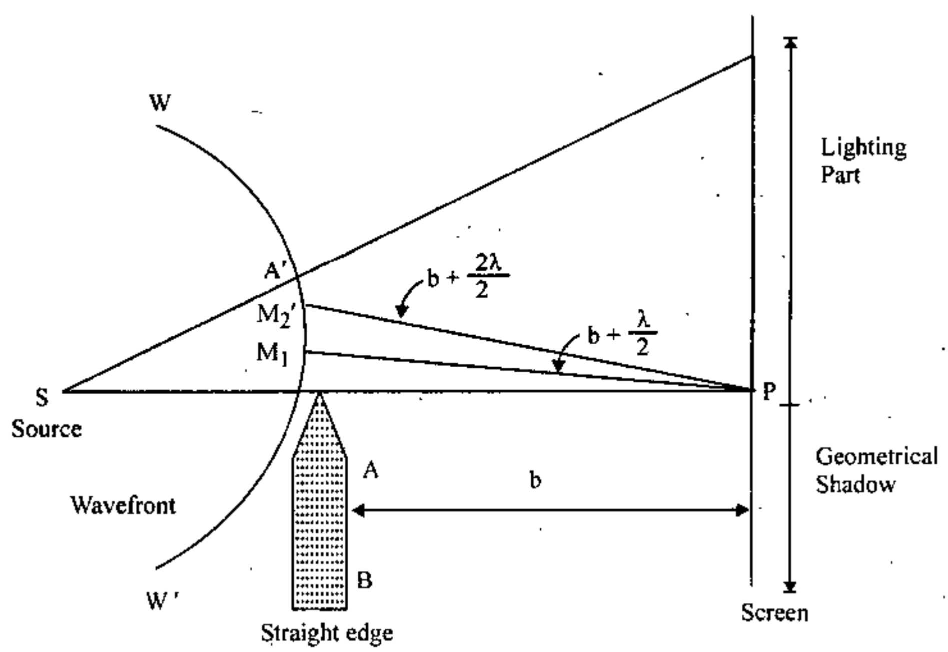


Fig. 4.

From $\Delta\, PON_m$, we have

$$\left(C+\frac{m\lambda}{2}\right)^2 = C^2 + ON_m^2$$

or $\quad ON_m = \sqrt{Cm\lambda} = K'\sqrt{m} \quad \ldots (4)$

where K' $(= \sqrt{C\lambda})$ is a constant.

Thus, width of nth element is

$$ON_m - ON_{m-1} = k'\{\sqrt{n} - \sqrt{n-1}\}$$

Let $R_1, R_2, R_3, \ldots, R_m$ be the amplitudes at P due to wavefront from Ist, IInd ... etc. half period strips on either sides of the central line through M_0.

The resultant amplitudes is given by

$$R = R_1 - R_2 + R_3 - R_4 + \ldots$$
$$= \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \left(\frac{R_3}{2} - R_4 + \frac{R_5}{2}\right) + \ldots$$

But successive amplitudes $R_1, R_2, \ldots$ etc are in decreasing order on account of

(i) gradually decreasing areas of successive strips.

(ii) incresing distance of successive strips from P.

(iii) greater obliquity of the successive strips.

Thus, in each of the above bracket, the sum of two extreme terms is roughly equal to middle term. Therefore value in bracket is almost zero. Hence,

$$R = \frac{R_1}{2}$$

Therefore resultant amplitudes at P *owing to the entire wavefront is given by*

$$R = \frac{R_1}{2} + \frac{R_1}{2} = R_1$$

• 4.3. DIFFRACTION PATTERN DUE TO A STRAIGHT EDGE

Let S be a narrow slit illuminated by a source of monochromatic light of wavelength λ. The length of the slit is perpendicular to the plane paper. AB is the straight edge and the length of the edge is parallel to the length of the slit shown in fig. (5). WW' is the incident cylindrical wavefront. P is a point on the screen and SAP is perpendicular to the screen. The screen is perpendicular to the plane of the paper. Below the point P is

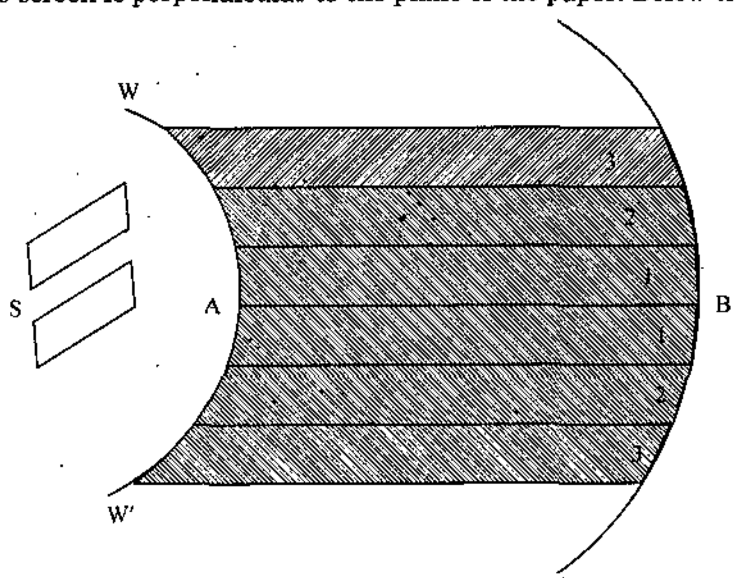


Fig. 5.

the geometrical shadow and above P is the illuminated portion. Let the distance AP be 'b'. With reference to the point P, the wavefront can be divided into a number of half period strips. In fig. (6.6), WW' is the wavefront, A is the pole of wavefront and

AM_1, M_1M_2, M_2M_3 etc measure the thickness of Ist, IInd, third etc. half period strips with the increase in order to strip then the area of the strip decreases shown in fig. (6).

In fig. (5),

$$AP = b, PM_1 = b + \frac{\lambda}{2}, PM_2 = b + \frac{2\lambda}{2} \text{ etc.}$$

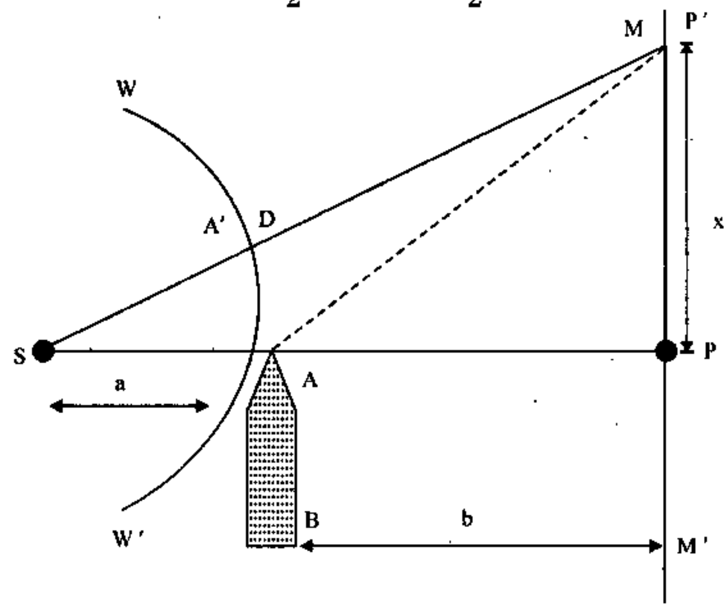


Fig. 6.

Let P' be a point on the screen in illuminated portion. In fig. (7), to calculate the resultant effect at P' due to WW'. Join S due to P'. This line meets the wavefront at point $D.D$ is the pole of wavefront with respect to point P' and the intensity P' will depend upon the number of half period strips, enclosed between A and B. The point P'

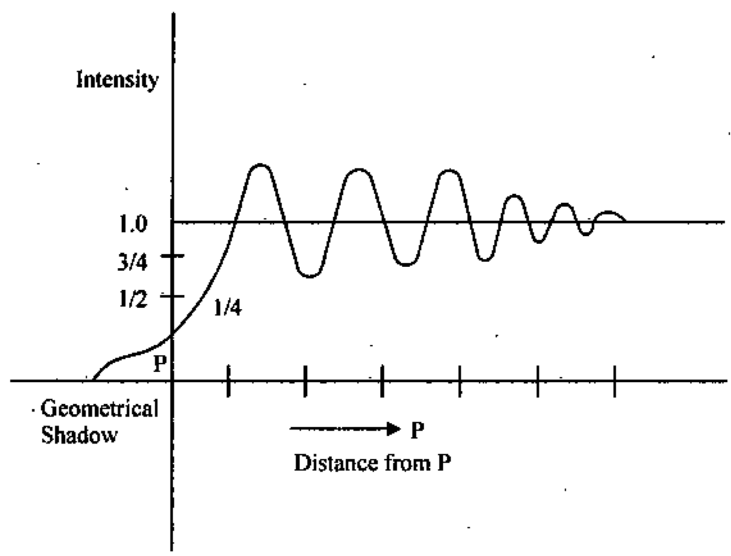


Fig. 7.

will be of maximum intensity, if the number of strips is odd in A and B portions. The intensity is minimum at P', if half period of strips is even between A and B portions.

Width of Diffraction Bands or Position of Maximum and Minimum Intensity : Let the distance between the slit and the straight edge be 'a' and the distance between straight edge and screen is 'b'.

The path difference is

$$\delta = AP' - DP' = (b^2 + x^2)^{1/2} - [SP' - SD']$$

$$= (b^2 + x^2)^{1/2} - [\sqrt{(a+b)^2 + x^2} - a]$$

$$= b\left[1 + \frac{x^2}{2b^2}\right] - (a + b)\left[1 + \frac{x^2}{2(a + b)^2}\right] + a$$

$$= b + \frac{x^2}{2b} - a - b - \frac{x^2}{2(a + b)} + a$$

$$\delta = \frac{x^2}{2}\left[\frac{1}{b} - \frac{1}{(a + b)}\right] = \frac{x^2}{2}\left(\frac{a}{b(a + b)}\right) \qquad \text{... (1)}$$

The point P' will be of **maximum intensity** if

$$\delta = (2n + 1)\frac{\lambda}{2} \qquad \text{... (2)}$$

$$(2n + 1)\frac{\lambda}{2} = x_n^2\left(\frac{a}{(a + b)\,b}\right)$$

or

$$x_n = \sqrt{\frac{(a + b)(2n + 1)\lambda b}{a}} \qquad \text{... (3)}$$

where x_n be the distance of nth bright band from P.

For minimum intensity, $\delta = 2n\,\dfrac{\lambda}{2} = n\lambda$

$\therefore$ From question (1),

$$x_n = \sqrt{\frac{2n(a + b)\,b\lambda}{a}} \qquad \text{... (4)}$$

where x_n be the distance of n dark bands from P. Thus, diffraction bands of varying intensity are observed above geometrical shadow.

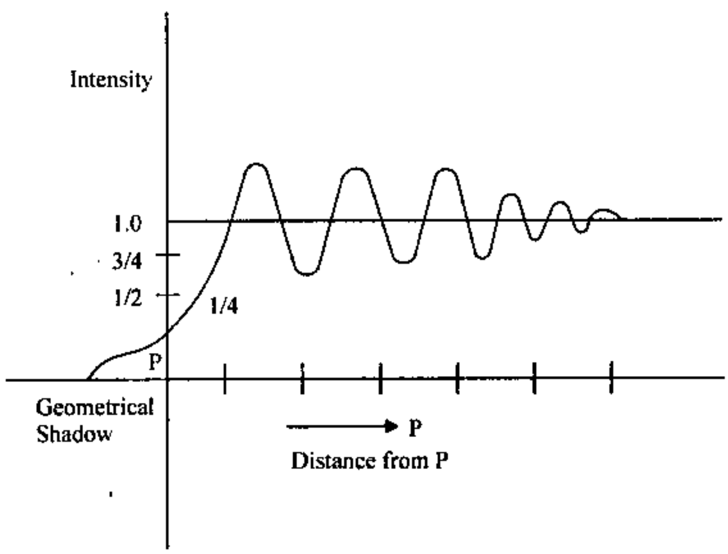


Fig. 8.

The intensity pattern distribution on the screen is represented graphically in fig. (8). In the geometrical shadow, intensity is rapidly falls. While outside it, there is a system of bright and dark bands till uniform illumination results.

Determination of wavelength

For Ist maximum, we have

$$x_1 = \sqrt{\frac{b\lambda(a + b)}{a}} \qquad \text{... (5)}$$

and for nth maximum

$$x_n = \sqrt{\frac{b(a + b)(2n + 1)\lambda}{a}} \qquad \text{... (6)}$$

Substracting equation (5) from (6), we get separation between the Ist and the nth maximum as

$$x_n - x_1 = \left[\sqrt{\frac{b(a+b)(2n+1)\lambda}{a}}\right] - \left[\sqrt{\frac{b(a+b)\lambda}{a}}\right]$$

$$x_n - x_1 = \sqrt{\frac{b(a+b)\lambda}{a}}\{(\sqrt{2n}+1)-1\}$$

or $$(x_n - x_1)^2 = \frac{b(a+b)\lambda}{a}[(\sqrt{(2n}+1)-1]^2$$

$$\lambda = \frac{(x_n - x_1)^2 a}{b(a+b)[\sqrt{2n}+1-1]^2} \quad \ldots (7)$$

which is the required formula for calculating the wavelength.

The slit, illuminated with the given monochromatic light, and the straight edge are arranged on an optical bench. The diffraction pattern is observed by means of a micrometer eye-piece on the same bench. The separation between the Ist and the nth maximum, $x_n - x_1$ is measured. The distances 'b' and 'a', are obtained by noting the position of slit, straight edge and eye-piece on the optical bench. Then λ can be determined from the last expression.

• 4.4. DIFFRACTION BY A NARROW WIRE

In fig. (9), WW' is a cylindrical wavefront of wavelength λ coming from a narrow slit S, which is normal to the plane of the paper and falling on a narrow wire of diameter AB where length is parallel to the slit. On the screen (XY), which is also perpendicular to the plane of the paper, EF be the region of geometrical shadow and above E and below F

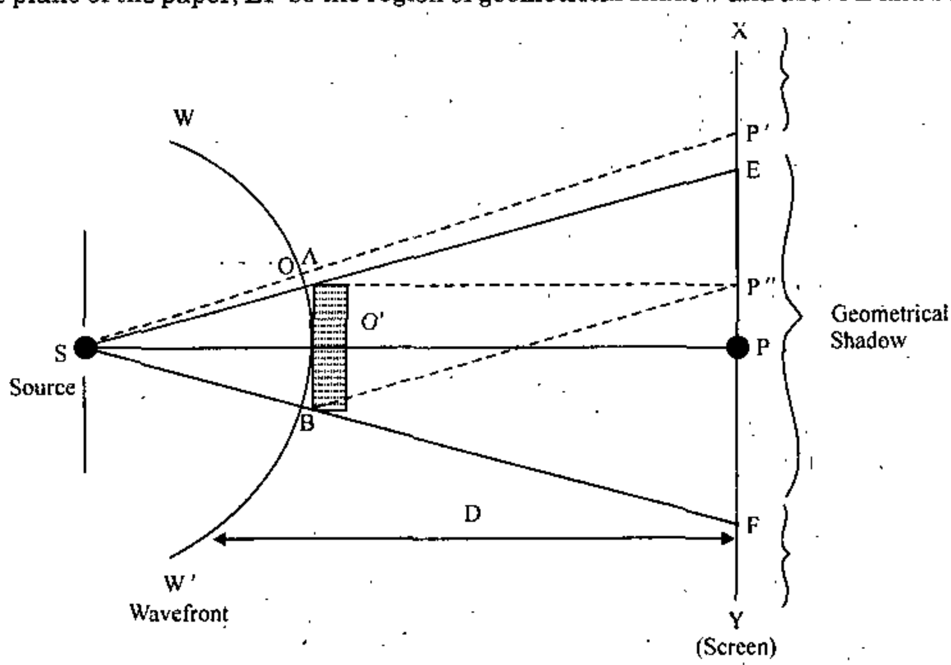


Fig. 9

the screen is illuminated. According to rectilinear propagation of light, there should be uniform illumination above E and below F while complete darkness with in EF. But actually unequally spaced bands with poor contrast and running parallel to the length of the slit are observed in the region of above E and below F. In the geometrical shadow, EF equally spaced bands running parallel to the length of the slit and wire are observed. The intensity distribution is shown in fig. (10).

Explanation : Let us suppose, a point P' in the region above E. The line SP' cuts the wavefront at O, which is the pole of the wavefront with respect to P'. To find the intensity of light at P', let the wavefront be divided into half period strips. Then P' will receive light from the entire upper half wavefront on and from those half period strips of the lower half which are contained in OA. If OA contains numbers (an odd and even) of

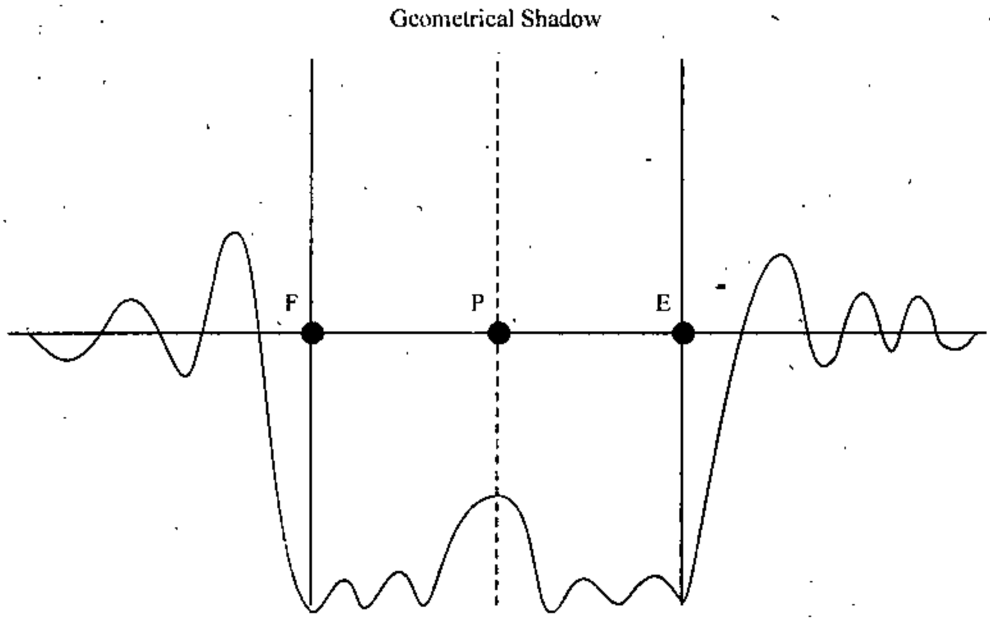


Fig. 10.

strips. There will be produced maxima and minima at P'. Thus, above E we obtain unequally-spaced bands with poor contrast similar to those of a straight edge. These are the diffraction bands, similar bands are obtained in region below F. Now suppose a point P'' in a geometrical shadow region EF. The pole corresponding to its O'. Therefore P'' receives light from the AW and BW'. The resultant amplitude at P'' due to AW and BW' which is equal to half amplitudes due to the half period strips adjacent to A and B and is in phase with it. Since, half period strips adjacent to A and B and is in phase with it. Since, half period strips adjacent to A and B lies on the same wavefront. The waves start from them in the same phase. Thus, the effect of AW and BW' at P'' is same as of two coherent sources at A and B. Hence, as in case of interference from two sources equally spaced bright and dark interference bands are obtained in the region EF. The bands width is given by

$$\overline{X} = \frac{D\lambda}{d} \qquad \text{... (1)}$$

where D is the distance between the wires and screen. And d is the diameter of used wire. If the path difference of waves which starts from AW and BW' is multiple of λ, therefore the maxima and minima are obtained at P''.

Determination of thickness of the wires : The experiment is performed on an optical bench. The slit and the wire are mounted parallel to each other on the Ist two uprights and the bands are measured by a micrometer eye-piece. The thickness of the wires is calculated by equation (1).

(B) If the thickness of the wire is gradually increased, the interference fringes gradually becomes narrower while the diffraction fringes remain unaffected. If the quite thick ($\approx$ few mm.), the inter.

4.5. FRAUNHOFER'S DIFFRACTION AT A SINGLE SLIT

Let a plane wavefront WW' of monochromatic light of wavelength λ be emitted from sources 'S' and incident normally on a narrow slit AB of width 'a'. Let the diffracted light be focussed by means of a convex lens (L), on a screen placed in the focal plane of the lens L, as shown in fig. (11). (Instead of a sharp image of the slit as may expected from geometrical optics). A central bright image, flanked symmetrically on both sides by a series of alternate dark and bright bands of decreasing intensity, is obtained on the screen.

According to Huygen's-Fresnel principle, each point of the plane wavefront incident on the slit AB. Each point in AB sends out secondary wavelets in all directions. The rays (waves) proceeding in same direction as the incident waves are focussed at P_0 by lens L. While through an $\theta°$ angle is focussed at P_1 by L. Let find the resultant intensity at P_1, now draw a AC perpendicular to BC. As paths from plane AC to P_1 are equal.

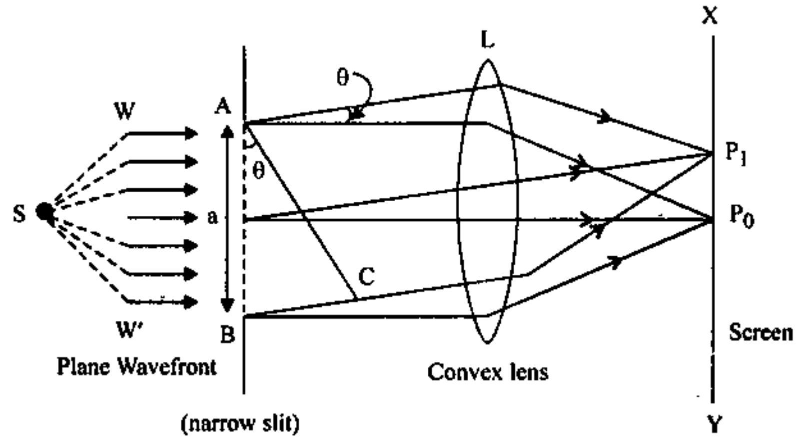


Fig. 11.

The path difference between the wavelengths A and B at $\theta°$ is

$$BC = AB \sin\theta = a \sin\theta$$

and corresponding phase difference as

$$\delta = \frac{2\pi}{\lambda} \times BC = \frac{2\pi a}{\lambda} \sin\theta$$

Let the AB of the slit is divided into n equal parts, the phase difference between the waves from any two consecutive parts is

$$\delta = \frac{2\pi}{\lambda} a \sin\theta \left(\frac{1}{n}\right) = d$$

Hence, the resultant amplitude at P_1 is given by

$$R = \frac{a \sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{nd}{2}\right)}$$

$$R = \frac{a \sin\left(\frac{na\pi \sin\theta}{\lambda n}\right)}{\sin\left(\frac{\pi}{n\lambda} a \sin\theta\right)}$$

$$R = \frac{a \sin\left(\frac{\pi a}{\lambda} \sin\theta\right)}{\sin\left(\frac{\pi}{n\lambda} a \sin\theta\right)} \quad \text{... (1)}$$

Let $\beta = \pi a \sin\theta / \lambda$, then equation (1) becomes as

$$R = \frac{a \sin\beta}{\left(\frac{\beta}{n}\right)} \qquad \left[\because \quad \frac{\beta}{n} \text{ is small}\right]$$

$$R = \frac{na \sin\beta}{\beta} = \frac{M \sin\beta}{\beta},$$

where $M = na$.

Therefore, the resultant amplitude at P_1 is

$$I = R^2 = M^2 \frac{\sin^2\beta}{\beta^2} \quad \text{... (2)}$$

Position of Maxima and Minima :

(i) *For Minima* : From equation (2), then

the condition is $\dfrac{\sin\beta}{\beta} = 0$

or $\sin\beta = 0$, But $\beta \neq 0$

or $\beta = \pm\, m\pi.$

Here, $m = 1, 2, 3, \ldots$ etc.

But $$\beta = \frac{\pi a \sin\theta}{\lambda}$$

$$\therefore \quad \frac{\pi a \sin\theta}{\lambda} = \pm\, m\pi$$

or $$a\sin\theta = \pm\, m\lambda \quad \ldots(3)$$

which is required condition.

(ii) *For Maxima* : The intensity of light is maximum when

$$\frac{dI}{d\beta} = 0$$

From equation (2),

$$M^2\left(\frac{2\sin\beta}{\beta}\right)\left(\frac{\beta\cos\beta - \sin\beta}{\beta^2}\right) = 0$$

or $$\frac{\beta\cos\beta - \sin\beta}{\beta^2} = 0 \qquad \left(\because \quad \frac{\sin\beta}{\beta} \to 1\right)$$

or $$\beta\cos\beta - \sin\beta = 0$$

or $$\beta = \tan\beta \quad \ldots(4)$$

It is solved by graphically method and plotting the curves

$$y = \beta \quad \ldots(5)$$

$$y = \tan\beta \quad \ldots(6)$$

The equation (5) is a straight line through origin 0, making an angle of $\frac{\pi}{4}$. While equation (6) is a discontinuous curve having a number of branches as shown in fig. (12).

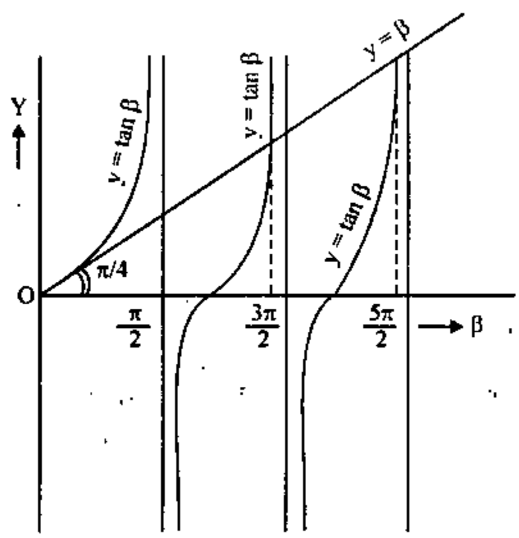


Fig. 12.

From fig. (12),

$$\beta = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

or $$\beta = 0, 1.430\pi, 2.462\pi, \ldots$$

Putting these values in equation (2), we get the intensities of various maxima.

Thus, the intensity of the central maxima is

$$I_0 = M^2 \left(\frac{\sin\theta}{\theta}\right)^2 = M^2 \qquad \left[\text{when} \quad \theta = \frac{\pi}{2}\right]$$

(a) Maximum intensity of Ist subsidiary is given by

$$I_1 = M^2 \left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{M^2}{22} = \frac{I_0}{22}$$

(b) Intensity of second subsidiary is given by

$$I_2 = M^2 \left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = \frac{M^2}{61} = \frac{I_0}{61}$$

Thus, ratio of the successive maximum intensities is

$$\frac{1}{1} : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} \ldots \text{etc.}$$

Clearly most of the incident light is concentrated in central maximum which occurs in the direction.

$$\therefore \qquad \beta = 0$$

$$\frac{a\pi \sin\theta}{\lambda} = 0$$

or $$\sin\theta = 0$$

or $$\theta = 0°$$

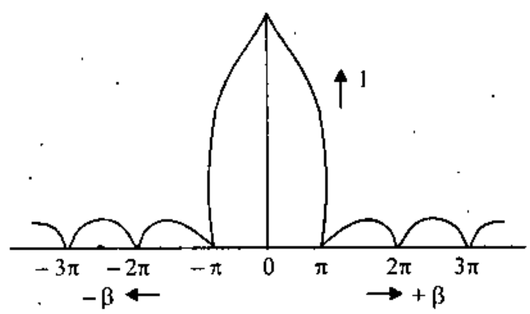


Fig. 13.

Thus, the same direction as the incident light.

Now plot a graph between I and β as shown in fig. (13) we get, a below graph (13) is known as intensity division curve. Clearly from graph, diffraction pattern consists of a bright principle maximum in the direction of incident light having alternatively minima and maxima of rapidly decreasing intensity on either side of it. The minima lies at $\beta = \pm\, m\pi (m = 1, 2, 3, \ldots \text{etc.})$.

• 4.6. FRAUNHOFER DIFFRACTION AT TWO SLITS

Construction : Let a parallel beam of monochromatic light of wavelength be incident normally upon two rectangulars AB and CD, placed closed and parallel to each other shown in fig. (14). Let the width of each slit be 'a' and the distance between the slits be b so that the distance between corresponding points of two slits is equal to $(a + b)$. Let the diffracted rays be focussed by a convex lens L on a screen XY placed in the focal plane of the lens L.

Explanation : According to Huygen's principle, every point in the slits AB and CD sends out secondary wavelets into all directions. All the secondary waves travelling in a direction parallel to OP_0. Thus, P_0 corresponds to the position of the central bright maximum, as all the wavelets traverse equal paths and are therefore in same phase. The resultant amplitudes due to all wavelets diffracted from each slit at P_1 is

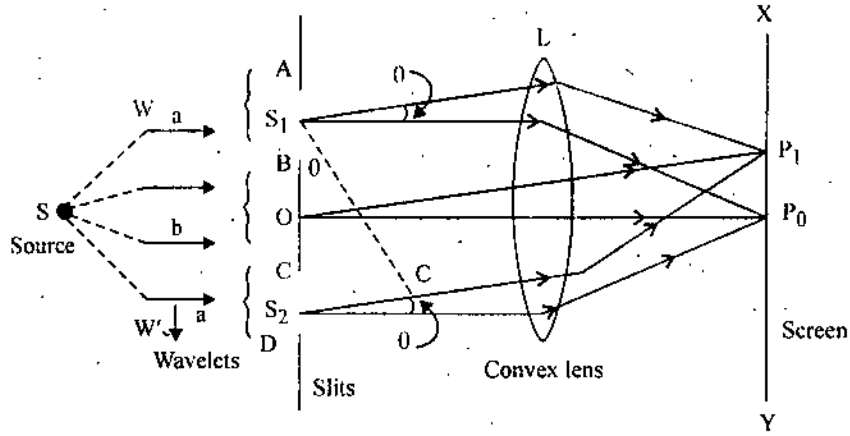


Fig. 14.

$$M \sin\left(\frac{\beta}{\beta}\right)$$

and the resultant phase in this direction is

$$\beta = \frac{\pi a \sin\theta}{\lambda}.$$

Thus, two slits are equivalent to coherent sources placed at their middle points S_1 and S_2, each sending a wave of amplitude in a direction θ to normal. Consequently the resultant amplitudes at P_1 on XY, is due to the interference of these two waves originally in same phase β, but having certain phase difference δ on reached P_1 depending upon δ (phase difference).

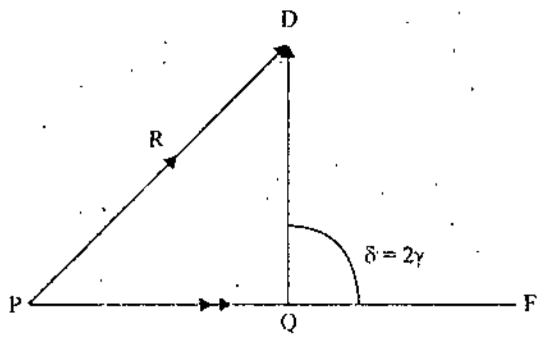


Fig. 15.

Let us drop perpendicular S_1H from S_1 on S_2H. The δ between two waves at P_1 is given by

$$S_2H = S_1 S_2 \sin\theta = (a+b)\sin\theta$$

or $$\delta = \frac{2\pi}{\lambda} \times S_2H = \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\gamma \text{ (say)} \qquad \dots (1)$$

The resultant amplitude R at P_1 can be obtained by vector addition method.

From fig. (15),

$$\therefore \quad PQ = QD = \frac{M\sin\beta}{\beta} \qquad \because \angle DQF = \delta$$

$$\therefore \quad PD^2 = PQ^2 + QD^2 + 2PQ \,.\, QD\cos DQF$$

or $$R^2 = 2\left(\frac{M\sin\beta}{\beta}\right)^2 + 2\left(\frac{M\sin\beta}{\beta}\right)^2(\cos\delta)$$

$$\left[\because PQ = QD = \frac{M\sin\beta}{\beta}\right]$$

$$R^2 = 2\left(\frac{M\sin\beta}{\beta}\right)^2(1+\cos\delta)$$

$$R^2 = 2\left(\frac{M\sin\beta}{\beta}\right)^2 .\, 2\,.\cos^2\frac{\delta}{2}$$

$$R^2 = 4\,.\,\frac{M^2\sin^2\beta}{\beta^2}\cos^2\frac{\delta}{2}$$

$$R^2 = \frac{4M^2 \sin^2 \beta}{\beta^2} \cos^2 \gamma \qquad (\because \ \delta = 2\gamma) \ \ldots (2)$$

[by using the equation (1)]

Hence, resultant intensity I at P_1 is given by :

or $$I \propto R^2$$

$$I \propto \frac{4M^2 \sin^2 \beta}{\beta^2} \cos^2 \gamma$$

$$I = 4R_0^2 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma \qquad (\text{where, } R_0^2 = M^2). \ \ldots (3)$$

Thus, the resultant intensity in the pattern depends on two factors below.

(i) $\frac{R_0^2 \sin^2 \beta}{\beta^2}$ which gives diffraction pattern of single slit.

(ii) $\cos^2 \gamma$ which gives the interference pattern due to light waves of the same amplitudes from two slits.

Dependence of intensity on $R_0^2 \sin^2 (\beta/\beta^2)$: The diffraction term $R_0^2 \sin^2 \frac{\beta}{\beta^2}$ gives a principal maxima if $\beta = 0$, at P_0 on screen. The maximum has on either side alternate minima and secondary maxima of diminishing intensity. The minima are obtained in the directions given by

$$\sin \beta = 0$$

$$\beta = \pm m\pi$$

or $$\frac{\pi a \sin \beta}{\lambda} = \pm m\pi \qquad \left[\because \ \beta = \frac{\pi a \sin \theta}{\lambda}\right]$$

$$a \sin \theta = \pm m\lambda \qquad \ldots (4)$$

where $m = 1, 2, 3, \ldots$ etc.

but $m \neq 0$.

The position of secondary maxima is given by

$$\beta = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2} \text{ etc.} \qquad \ldots (5)$$

Dependence of intensity on $\cos^2 \gamma$: For minima condition,

$\therefore$ $$\cos^2 \gamma = 0$$

or $$\gamma = \pm (2n + 1) \frac{\pi}{2}$$

or $$\sin \theta = \frac{(2n + 1)\lambda}{2(a + b)}$$

Here, $n = 0, 1, 2, 3, \ldots$(by using equation (2)) $\left[\because \ \gamma = \frac{\pi}{\lambda} (a + b) \sin \theta\right]$

or $$\theta = \frac{\lambda}{2(a + b)}, \frac{3\lambda}{2(a + b)}, \frac{5\lambda}{2(a + b)}, \ldots \qquad \ldots (6)$$

Similarly maxima occurs for these values of θ for which

$$\cos^2 \gamma = 1$$

$$\gamma = \pm n\pi$$

or $$(a + b) \sin \theta = \pm n\lambda \qquad \left[\because \ \gamma = \frac{\pi(a + b) \sin \theta}{\lambda}\right]$$

or $$\theta = \frac{\lambda}{(a + b)}, \frac{2\lambda}{(a + b)}, \frac{3\lambda}{(a + b)}, \ldots \qquad \ldots (7)$$

Thus, the separation $\Delta\theta$ between any two consecutive minima/maxima is $\frac{\lambda}{(a+b)}$.

or $$\boxed{\Delta\theta \propto \frac{1}{(a+b)}} \quad \dots (8)$$

In this way interference term $\cos^2\gamma$ gives a set of equidistant dark and bright bands.

(A) Intensity Distribution Curve : We can plot the intensity distribution curve in double slit Fraunhofer diffraction as shown in fig. (16).

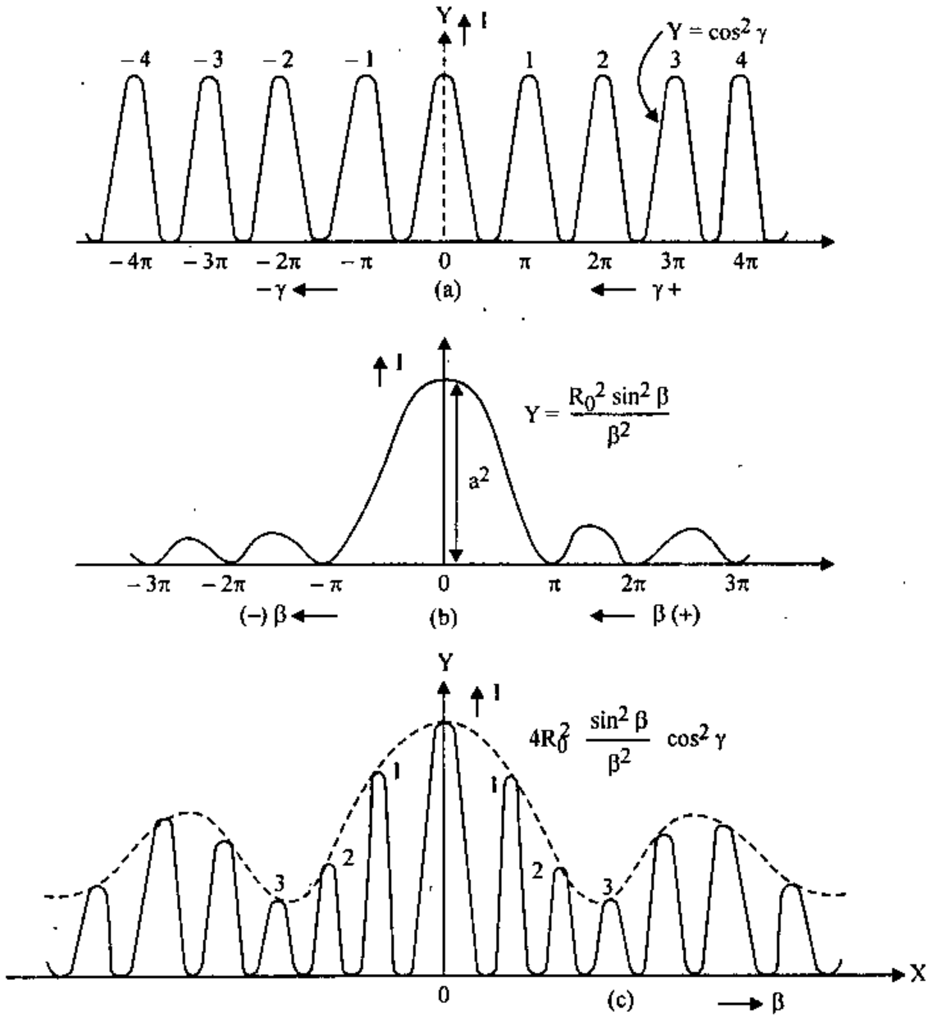


Fig. 16.

Fig. (16(a)) is a graph of interference term $\cos^2\gamma$ against γ. The maxima occurs at $\gamma = \pi, 2\pi, 3\pi$ etc. on each side and the values orders for various maxima marked. Fig. (16(b)) is a graph of diffraction term $\frac{\sin^2\beta}{\beta^2}$ against β, maxima occurs at $\beta = \pi, 2\pi, 3\pi$ etc.

If consider a case $2a = b$, then interference maxima will be absent in the diffraction pattern. Hence, resultant intensity distribution curve is shown in fig. (16(c)). It represents the complete double slit diffraction pattern.

(B) Effect of increasing width of slit and distance between slits : If we increase the width of slit (a), the envelope of pattern (fringes) are changed. So, the central peak is sharp. The fringes spaces does not change. Hence, less interference maxima now fall within central diffraction maximum if the separation of slits are increased then fringes close together, the envelope of the pattern remains unchanged. Hence, more interference maxima fall within the central envelope.

(C) Effect of increasing wavelength : On increasing wavelength of light, the envelope becomes broader, and the fringes move further apart.

• 4.7. PLANE DIFFRACTION GRATING

An arrangement consisting of an equidistant parallel rectangular N slits of equal width separated by equal opaque portion is called as a **diffraction grating**. The diamond points cut a large number of grooves in the glass surface or on speculum metal. These grooves act as an opaque obstacle while the space between them acts as a narrow slit. When a wavefront is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such a grating is called a transmission grating.

Action of a diffraction grating (Diffraction at N slits) : Let a parallel beam of monochromatic light of wavelength λ be incident normally on N parallel slits each of width 'a' and separated by a distance 'b'. Such an all the rays issuing from the spaces reach P_0 on the screen (XY) in phase shown in fig. (17).

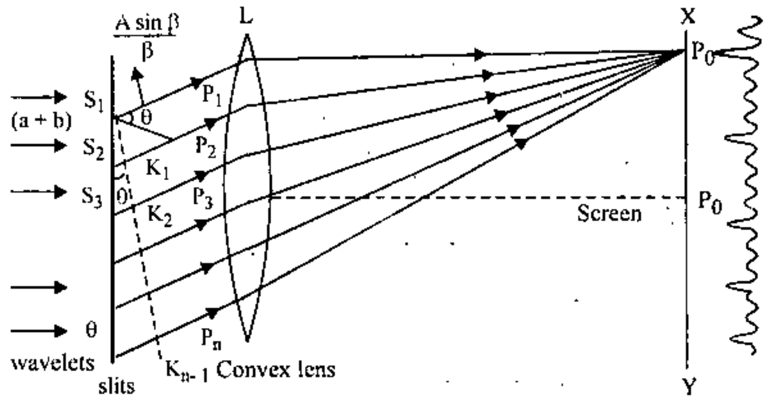


Fig. 17.

A part of light gets diffracted in various directions, those diffracted at θ an angle with the initial direction reach P_1 on passing through a convex lens L in different phases. As a result dark and bright bands on both sides of the central maximum are obtained.

According to the Huygen's principle, as soon as the plane wavefront is incident on the slits, all points in each slit become the sources of secondary disturbances in all directions. By the theory of Fraunhofer diffraction at a single slit, the secondary waves from all points in a slit diffracted in a direction θ are equivalent to a single wave of amplitude.

$$A_0 = \frac{A \sin \beta}{\beta}$$

where A is the amplitude due to a single slit in normal direction and β is the phase difference between the wavelets from the end and middle points of slit.

$$\beta = \frac{\pi}{\lambda} a \sin \theta \qquad \text{... (1)}$$

We have N diffracted parallel rays, one each from the middle points of $S_1, S_2, S_3, \ldots S_N$ the slits. Draw S_1K_1 perpendicular to S_2K_1 then the path difference between the rays from the slits S_1 and S_2 is given by

$$S_2K_1 = S_1S_2 \sin \theta = (a + b) \sin \theta$$

across phase difference

$$\delta = \frac{2\pi}{\lambda} \times S_2K_1$$

$$\delta = \frac{2\pi}{\lambda} (a + b) \sin \theta = 2\gamma \qquad \text{... (2)}$$

From fig. (18),

$$MP_1 = 2OM \sin\gamma$$

and $$MP_N = 2OM \sin N\gamma$$

Eliminating OM, we get

$$MP_N = 2 \cdot \frac{MP_1}{2\sin\gamma} \cdot \sin N\gamma$$

$$MP_N = A_0\left(\frac{\sin N\gamma}{\sin\gamma}\right)$$

or $$R = \left(\frac{A\sin\beta}{\beta}\right)\left(\frac{\sin N\gamma}{\sin\gamma}\right)$$

Hence, the resultant intensity is given by

$$I = \left(\frac{A\sin\beta}{\beta}\right)^2\left(\frac{\sin N\gamma}{\sin\gamma}\right)^2 \quad ...(3)$$

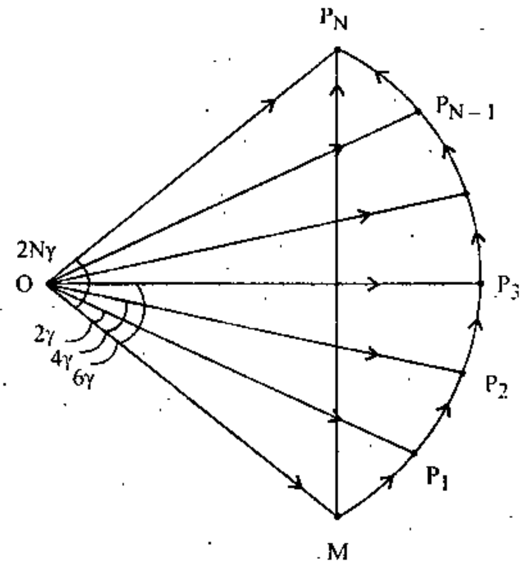


Fig. 18.

The Ist factor $\left(\frac{A\sin\beta}{\beta}\right)^2$ express intensity distribution due to diffraction at a single slit while that factor $\left(\frac{\sin N\gamma}{\sin\gamma}\right)^2$ may be said to arise from interference between the N diffracted waves from the N slits.

Principal Maxima :

If $\sin\gamma = 0$ or $\gamma = \pm n\pi$

Here, $n = 0, 1, 2, 3, ...$ etc

then $\sin N\gamma = 0$

and $\frac{\sin N\gamma}{\sin\gamma} = \%$ = Indeterminate form.

It is solved by limiting method. Thus,

$$\underset{\gamma \to \pm n\pi}{limit}\left(\frac{\sin N\gamma}{\sin\gamma}\right) = \pm N$$

Therefore from equation (3), the intensity is then given by

$$I = \left(\frac{A\sin\beta}{\beta} \cdot N\right)^2$$

which is maximum. These maxima are most intense and hence they are called **principal maxima** as obtained if

$$\gamma = \pm n\pi$$

or $$\frac{\pi}{\lambda}(a + b)\sin\theta = \pm n\pi$$

or $$(a + b)\sin\theta = \pm n\lambda \quad ...(4)$$

where $n = 0, 1, 2, ...$ etc. shows the order of the interference maximum. For $n = 0$, we get zero order maxima. For $n = 1, 2, 3, ...$ etc we obtain the Ist, IInd, third, ... etc. Principal maxima. Equation (4) does not contain N so that the position of the principal maxima are independent of number of slits for a given separation.

Minima : As series of minima occur when

$$\sin N\gamma = 0$$

$$\frac{\sin N\gamma}{\sin\gamma} = 0 \text{ but } \gamma \neq 0$$

and from (3), we have $I = 0$.

Which is minimum, thus for minima, we have

$$\sin N\gamma = 0$$

$$N\gamma = \pm\, m\pi$$

or $$N \cdot \frac{\pi}{\lambda}(a+b)\sin\theta = \pm\, m\pi \qquad \left[\because \quad \gamma = \frac{\pi}{a}(a+b)\sin\theta\right]$$

or $$N(a+b)\sin\theta = \pm\,\frac{m\pi}{\lambda} \qquad \text{... (5)}$$

where $m = 0, 2N, 4N, \ldots, nN$.

When $m = 0, N$ gives principal maxima, for $m = 1, 2, 3, \ldots, N-1$ gives minima. Hence, there are $(N-1)$ equally spaced minima between two adjacent maxima.

Secondary Maxima : As there are $(N-2)$ maxima between two adjacent principal maxima, to find the secondary maxima from equation (3),

$$\therefore \qquad \frac{dI}{d\gamma} = 0$$

$$\therefore \qquad \left(\frac{A\sin\beta}{\beta}\right)^2 2\left[\frac{\sin N\gamma}{\sin\gamma}\right]\left[\frac{N\sin N\gamma\sin\gamma - \sin N\gamma\cos\gamma}{\sin^2\gamma}\right] = 0$$

or $$N\tan\gamma = \tan N\gamma \qquad \text{... (6)}$$

To find the value of $\frac{\sin^2 N\gamma}{\sin^2\gamma}$ from equation (6), we get from fig. (19) say.

$$\sin N\gamma = \frac{N\tan\gamma}{\sqrt{1+N^2\tan^2\gamma}}$$

$$\therefore \qquad \left(\frac{\sin N\gamma}{\sin\gamma}\right)^2 = \frac{N^2\tan^2\gamma}{(1+N^2\tan^2\gamma)\sin^2\gamma}$$

$$= \frac{N^2}{(\cos^2\gamma + N^2\sin^2\gamma)}$$

$$= \frac{N^2}{[1+(N^2-1)\sin^2\gamma]}$$

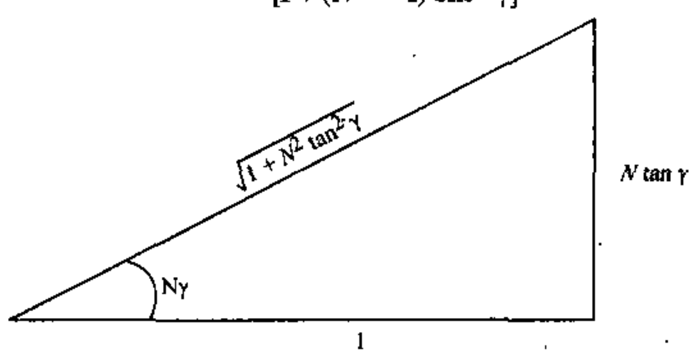


Fig. 19.

which shows the intensity of secondary maxima varies as $\frac{N^2}{1+(N^2-1)\sin^2\gamma}$ while that *of principal maxima varies as* N^2 *therefore intensity ratio of secondary maxima and* intensity of principal maxima is

$$\frac{1}{1+(N^2-1)\sin^2\gamma}$$

so that larger the value of N, weaker are secondary maxima.

Formation of Spectray : When a beam of light of wavelength of λ falls normally on a grating, the principle maxima are formed in the direction given by

$$(a+b)\sin\theta = \pm\,\lambda n$$

Here, $n = 0, 1, 2, \ldots$ etc.

This equation shows that a given order of n, the θ angle of diffraction varies with the wavelength. The longer λ the greater is the angle of diffraction. Hence, if incident light

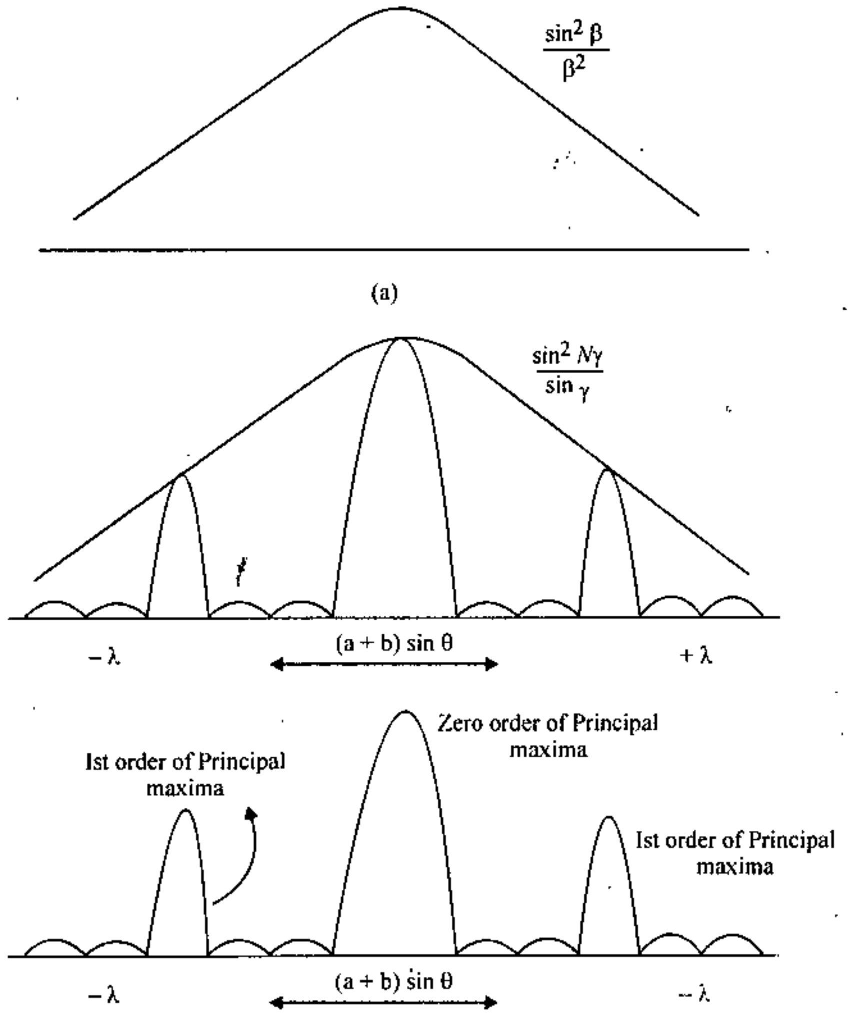


Fig. 20.

is white then each order will contain principal maxima of different λ in different directions. The principal maxima of all λ for $n = 1$ will form the Ist order of spectra and so on. The principal maxima for all values of λ to $n = 0$, *i.e.*, $\theta = 0$. Hence, zero order maxima will be white having on either side of it Ist order spectrum, IInd order spectra, ... etc.

There spectra shown in fig. (20).

• 4.8. THEORY OF PLANE TRANSMISSION GRATING

Let in fig. (21), $A'B'$ be the section of a plane transmission grating placed perpendicular to the plane paper. Let 'a' be the width of slits each and separated by a distance 'b'. Then $(a + b)$ is called the **grating element**.

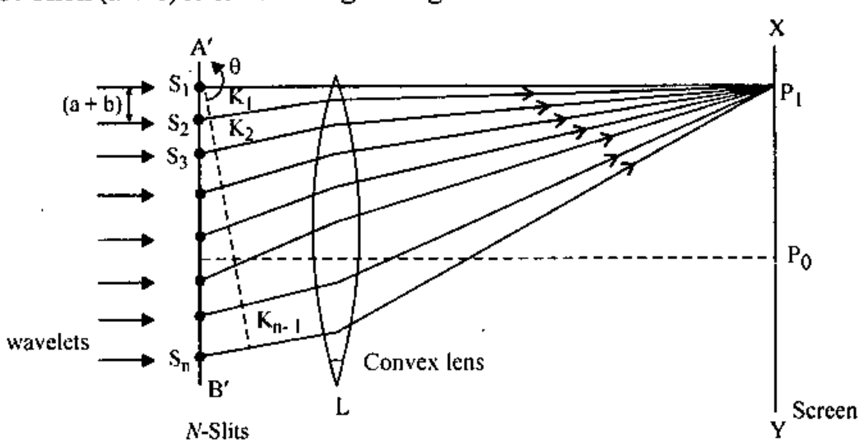


Fig. 21.

Let a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. According to Huygen's principle all the points in each slits send out secondary wavelets in all directions. And reach to point P_1 by L.

Let us consider the wavelets starting from the middle points $S_1, S_2, \ldots, S_N$ of slits diffracted at θ angles. And draw a perpendicular S_1K_1 to S_2K_1 from S_1. Here the path difference between wavelets S_1S_2 then

$$S_2K_1 = S_1S_2 \sin\theta = (a+b)\sin\theta$$

The two waves will reinforce each other when brought to focus at P_1 by L, if the path difference is multiple of whole number of λ. Then by reinforcement we have

$$(a+b)\sin\theta = n\lambda$$

where $n = 0, \pm 1, \pm 2, \ldots$, *etc.*

For $n = 0$ gives a maxima and $n = \pm 1, \pm 2, \ldots$, etc. gives maxima's on both sides of zero order spectra.

Determination of λ : For determination of $(a+b)$, the grating element is determined from the number of rulings per inch on the grating if this member is N', then

$$N'(a+b) = 1 \text{ inch} = 2.54 \text{ cm}$$

or
$$(a+b) = \frac{2 \cdot 54}{N'} \text{ cm.}$$

Procedure : The whole experiment is performed in the following two parts :

(1) Adjustments

(2) Measurements

1. Adjustments

(A) Adjustments of spectrometer : Before using the spectrometer, the following adjustments should be made :

(i) Focus the eye-piece of the telescope on the crosswires.

(ii) Adjust the collimator and telescope for emitting and receiving parallel rays respectively. This should be done by Schuster's method.

(iii) Level the prism table optically with a spirit level.

(B) Adjustments of the grating for normal incidence : To place the grating perpendicular to the light coming from the collimator, proceed as follows :

(i) Bring the telescope in line with the collimator and focus the direct image of the slit on the vertical cross-wires. Note the readings of both the verniers for this position of the telescope.

(ii) Rotate the telescope through 90° and then clamp it.

(iii) Mount the grating in the grating holder on the prism table in such a way that the light from the collimator falls on the back of the grating (*i.e.*, on the side onot having the rulings).

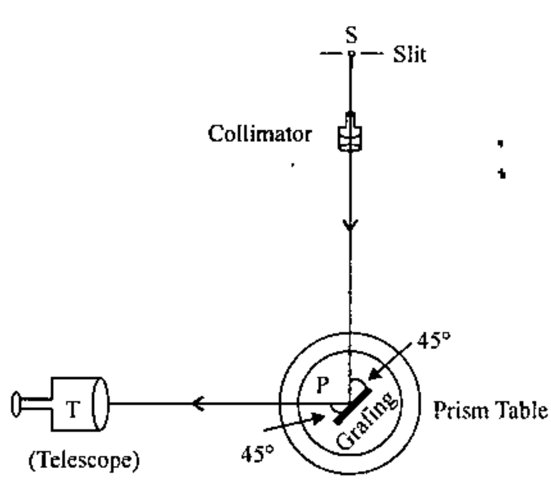


Fig. 22.

(iv) Now, rotate the prism table so that the image of slit (due to reflection from the grating surface) is formed exactly on the vertical cross-wire. It is shown in fig. (22).

Adjust the screws P and Q, if necessary to get the reflected image in the centre of the field of view of the telescope.

(v) Note the readings of both the verniers and rotate the prism table through 45° or 135° so that the ruled surface of the grating be normal to the incident rays and faces the telescope. Now clamp the prism table and release the telescope.

(C) Adjustment of rulings of the grating parallel to the main axis of the instrument : Rotate the telescope about the main axis of the instrument. When the axis of the telescope is parallel to that of the collimator, an image of the slit will be seen lying with its centre on the intersection of the cross wires of the telescope. When telescope is moved on either side of this position, the diffracted images of the slit or spectral lines will be visible in the field of view. If necessary, adjust the levelling screw P so as to get the middle of all the diffracted image on the intersection of the cross-wires of the telescope. When this adjustment is completed, the rulings of the grating become parallel to the main axis of the instrument.

(D) Adjustment of the slit parallel to the rulings of the grating : For this setting, rotate the slit in its own plane till the diffracted images (observed above) or the spectral lines become well defined (sharp) and bright. Take care that the distance between the slit and the collimating lens should remain unaltered as the slit is rotated.

2. Measurement of the Angle of Diffraction

The spectrum obtained by the grating is shown in fig. (23).

(i) Rotate the telescope to one side (say left) of the direct image (zero order) of the slit and receive the first order spectrum in the field of view of the telescope.

(ii) Set the vertical cross-wire on different spectral lines (Violet, Green and Red) of first order spectrum. *(For this, clamp the telescope and then move it slowly by tangent screw till the vertical cross-wire turn by turn, coincides with different spectral lines).* Note down the readings of both the verniers in each setting.

(iii) Now unclamp the telescope and rotate it to the other side (say right) of the direct image till the first order spectrum is again visible in the field of view. Clamp the telescope and adjust the vertical crosswire on various spectral lines by finer movement of the telescope. Note down the readings of both the verniers in each setting.

(iv) If second order spectrum is also visible, then repeat the above procedure (steps (i), (ii) and (iii)) for the visible colours of second order spectrum on both sides of direct image and then note the readings.

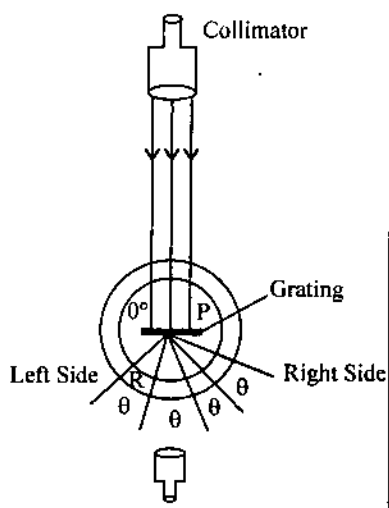


Fig. 23.

(v) Find out the difference in the readings of the same verniers (V_1 from V_1 and V_2 from V_2) for the same spectral lines of first order spectrum obtained in the two side settings. It gives 2θ, which is twice the angle of diffraction for that particular colour. Half it to get θ, the angle of diffraction. Similarly, find θ for other spectral lines.

(vi) Now calculate the angles of diffraction for all the spectral lines of second order.

(vii) Note down the number of lines ruled per inch (N) on the grating surface *(usually written on the grating)*.

• 4.9. CONCAVE REFLECTION GRATING

This grating is invented by **Henry Rowland in 1882.** An consists of a polished spherical concave surface of a metal *(i.e.,* speculum). The surface is ruled with fine parallel lines equally spaced along the chord of the arc joining the extreme rulings, when light is incident on such a grating. It is diffracted and automatically focussed without the uses of lenses and shown in fig. (24). This type of grating is known as **concave reflection grating.**

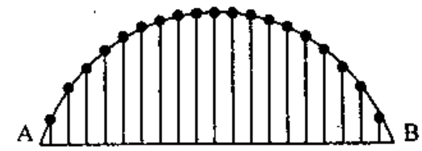


Fig. 24.

Theory : In fig. (25), let GG' be the surface of the Concave Grating in which the rulings are perpendicular to the plane of the paper. Let C be the centre of curvature and R be the radius, of the curvature. With R as diameter draw a

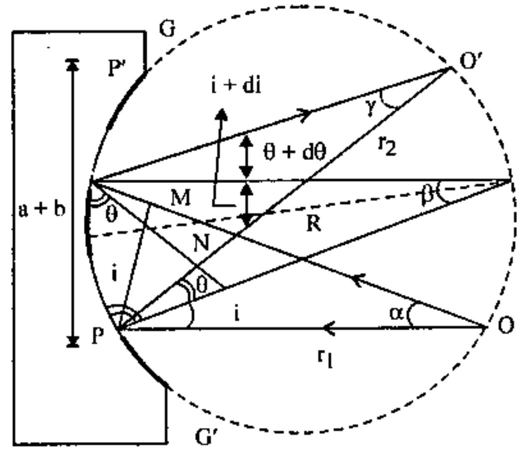


Fig. 25.

dotted circle which touches the grating surface at P and P'. Let P and P' be the corresponding points in consecutive polished space of the grating so that $PP' = (a + b)$ is the grating element. Let θ be a vertically slit and which illuminated by a monochromatic light λ. Light rays and OP and OP' incident at i and $i + di$ angles respectively are diffracted at θ and $\theta + d\theta$ angles. If to diffracted rays are brought to a focus at O', the path difference between the rays OPO' and $OP'O'$

$$= (OP' + O'P') - (OP + PO')$$
$$= (OP' - OP) - (PO' - P'O')$$
$$= P'M - PN$$
$$= PP' \sin i - PP' \sin\theta$$
$$= PP' (\sin i - \sin\theta)$$
$$= (a + b)(\sin i - \sin\theta)$$

The rays will reinforce each other and produce a maxima at O'. If the difference in their paths is an integral multiple of λ, *i.e.*,

$$(a + b)(\sin i - \sin\theta) = n\lambda \quad \text{... (1)}$$

if the point O' is taken on the same side at O, the condition for maxima at O' will be

$$(a + b)(\sin i - \sin\theta) = n\lambda \quad \text{... (2)}$$

or $$(a + b)(\sin i - \sin\theta) = \text{constant.}$$

Thus, the concave grating obeys just the same equation as the plane grating for oblique incident.

Now take the differentiate equation (2) we get

$$\cos i\, di - \cos\theta\, d\theta = 0 \quad \text{... (3)}$$

If α, β and γ be the angles as shown in fig. (25), then from geometry of fig.

$$\alpha + i = \beta + i + di$$

and $$\beta + \theta = \gamma + \theta + d\theta$$

whence $$di = \alpha - \beta$$

and $$d\theta = \beta - \gamma$$

Hence, equation (3) becomes

$$\cos i(\alpha - \beta) - \cos\theta(\beta - \gamma) = 0 \quad \text{... (4)}$$

if $$PO = r_1,\ PO' = r_2, \text{ then}$$

$$r_1 \cdot \alpha = PM = (a + b)\cos i$$
$$R \cdot \beta = PP' = (a + b)$$

$$r_2 \cdot \gamma = P' N = (a + b) \cos \theta$$

$$\therefore \qquad \alpha = \frac{(a+b)\cos i}{r_1}, \beta = \frac{a+b}{R}, \gamma = \frac{(a+b)\cos\theta}{r_2}$$

Putting these values in equation (4), we get

$$\cos i \left[\frac{\cos i}{r_1} - \frac{1}{R}\right] - \cos\theta \left[\frac{1}{R} - \frac{\cos\theta}{r_2}\right] = 0$$

This is the general equation, which relates the position of diffracted image with position of source of light. It is satisfied all values of θ, i, its possible if

$$\left.\begin{aligned} r_1 &= R \cos i \\ r_2 &= R \cos \theta \end{aligned}\right\}$$

Hence, the grating, slit and spectral lines lie on the circumference of the same circle. This circle is called **the Rowland Circle.**

Rowland Mounting : It consists of two strong iron rails SQ and SQ' shown in fig. (26). Lying at right angles to each other in a horizontal plane. The grating G and photographic plate P are mounted on two carriages B and B' which can move along the rails and are joined by rigid beam of length R. A slit 'S' is placed at the junction of two rails. Since, $Q'SQ$ is a $\pi/2$ angles and length of the beam $\beta\beta'$ is equal to diameter of Rowland Circles. When white light is passed through the slits and falls on the grating, it is diffracted and the spectra of various order are focussed on the circle. However, the only portion of spectrum is used which is incident on the plate P. Which is always at the incident on grating. When the beam BB' is moved then G and P are moved to new positions such as G'' and P'' and spectra of different orders are obtained.

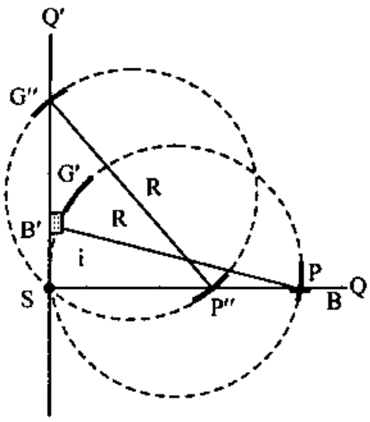


Fig. 26.

Determination of λ (wavelength) : Since, the angles of diffraction along the diffraction GP is zero. The condition for a maximum at P given by equation (2), becomes

$$(a + b) \sin i = n\lambda$$

But $$\sin i = \sin SGP = \frac{SP}{R}$$

$$\therefore \qquad \frac{(a+b)\,SP}{R} = n\lambda$$

which is the required formula for obtaining the value of λ. If $(a + b)$ and R are constants then

or $$SP \propto n.$$

• 4.10. VARIOUS MOUNTINGS

The concave reflection grating can be used to obtain spectra in a number of ways, called **mountings.** In all the mountings grating is normal to Rowland Circle and slit, eye-piece are placed on the same circle.

(1) Rowland Mounting

To answer see the question number (3), Fig. 26.

Merits (Advantages) : The spectrum is normal and the rail OP can be graduated to read wavelength directly.

Demerits (Disadvantages) : (i) It shows some astigmatism.

(ii) It needs large space.

(iii) It is expensive and involves a considerable amount of mechanism.

(iv) It is difficult to control temperature.

(2) Paschen Mounting

In fig. (27), the mounted grating 'G'' and slit 'S'' is fixed on the Rowland Circle on a rigid support at a proper position which give a desired angle of incidence. The source S' is placed behind the slit. The photographic plate holder 'P'' is clamped to a circular steel frame along the circle. The spectra of different orders are focussed simultaneously on the plate.

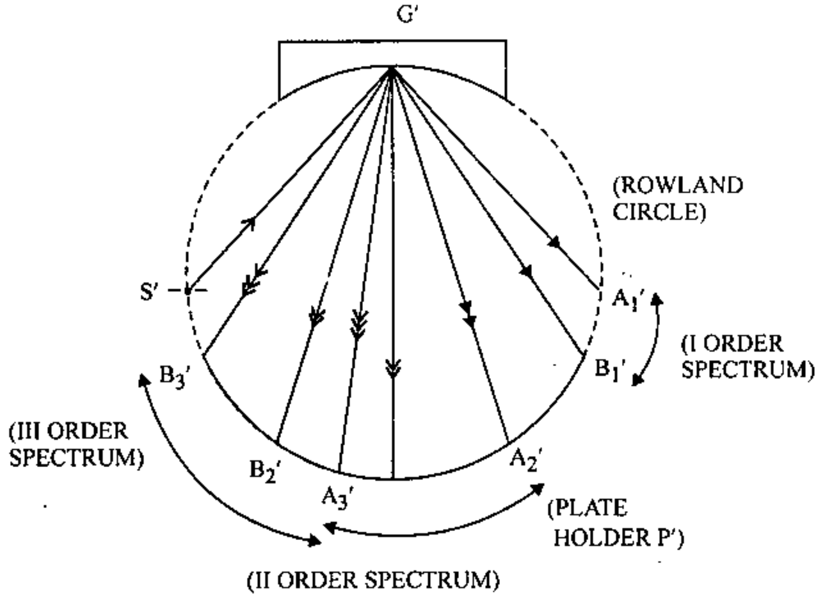


Fig. 27.

Merits : (1) The arrangement is very rigid.

(2) Two or more slits can be placed at different points on the circle to work at different incident angles.

Demerits : (1) It shows strong astigmatism.

(2) The spectrum is normal only in the region which falls near the grating normal.

(3) It is difficult to avoid temperature changes.

• 4.11. DIFFERENCE BETWEEN THE GRATING AND PRISM SPECTRA

S.N.	Prism Spectra	Grating Spectra
1.	It is formed by dispersion.	It is formed by diffraction.
2.	It is formed only by one spectrum.	The grating form a number of spectra of different orders on each side of central image.
3.	The prism is bright.	It is much fainter.
4.	It depends upon the prism material.	It is independent of grating material.
5.	The dispersion produced by prism if $\frac{d\delta}{d\lambda} \propto \frac{1}{\lambda^3}$.	The dispersion produced by grating if $\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$.

• 4.12. DISPERSIVE POWER OF GRATING

It is defined as the rate of change of the angle of diffraction with wavelength of light.

If $d\lambda$ is the rate of change of wavelength and $d\theta$ is the rate of change of angle then it is expressed

$$\frac{d\theta}{d\lambda}.$$

For grating maxima is given by

$$(a + b) \sin \theta = n\lambda$$

It is differentiated w.r. to λ

$$\therefore \qquad (a + b) \cos \theta \frac{d\theta}{d\lambda} = n$$

or

$$\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a + b) \cos \theta}} \qquad \ldots (1)$$

This is the expression for the dispersive power.

This equation (1) shows that the dispersive power is :

(1) directly proportional to order n.

(2) inversely proportional to $(a + b)$.

(3) inversely proportional to $\cos \theta$. If larger value of θ then smaller value of $\cos \theta$ and higher dispersive power.

• 4.13. RESOLVING POWER OF AN OPTICAL INSTRUMENT

The ability of an optical instrument to form distinctly separate images of two objects vary close together is called its **resolving power.**

(a) Rayleigh's criterion of resolution (or Rayleigh Limit Resolution)

Lord Rayleigh proposed an arbitrary criterion which has been universally adopted. According to it,

Two spectral lines of equal intensity should be regarded as separate, i.e., just resolved, if the central maximum of the diffraction pattern due to one coincides with the first secondary minimum of other and vice versea.

This is known as ***Rayleigh's criterion of* resolution** and can be conveniently applied to calculate the resolving power of telescope, grating, microscope etc.

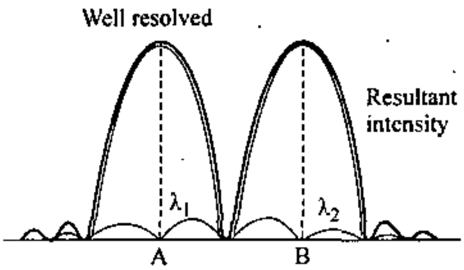


Fig. 28.

In order to illustrate the criterion, consider the diffraction pattern due to two wavelengths λ_1 and λ_2 shown in fig. (28). The difference in wavelength is such that the central maxima of λ_1 and λ_2 are quite separate and distinct at zero point intensity in the middle of resultant intensity curve. Thus, the two spectral lines appear well resolved. The difference in wavelength is smaller, then the central maximum of one coincide with the Ist minimum of the other shown in fig. (29). The resultant intensity curve shows a distinct **dip** in the middle of two central maxima. This is easily understood if we remember that the intensity distribution in grating is of the form

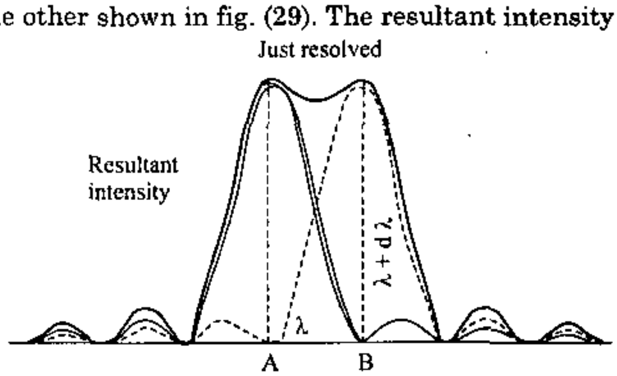


Fig. 29.

$$I = I_{max} \frac{\sin^2 \beta}{\beta^2}$$

If $\beta = \pi$, therefore the midway between the two central maxima shown in fig. (29), if $\beta = \frac{\pi}{2}$.

$$I_{mid} = 2I_{max} = \frac{\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)^2} = \frac{8}{\pi^2} I_{max}$$

$$\frac{I_{mid}}{I_{max}} = \frac{8}{\pi^2} = 0.810$$

If the difference in wavelengths of two spectral lines is so small, then central maxima corresponding to the two wavelengths come still closer. The two intensity curves (shown in fig. (30)) show a sufficient overlapping and the two images cannot be distinguished as separate. Hence, two spectral lines are not resolved.

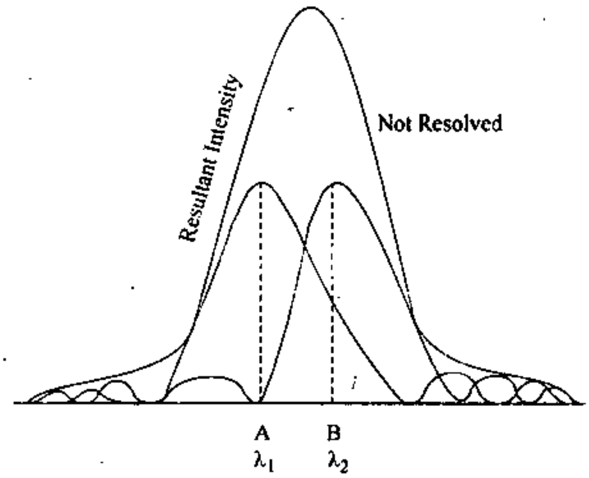


Fig. 30.

Rayleigh's criterion provides us a simple means of determining the resolving power of optical instruments because when the two spectral lines are just resolved, the principal maximum of one coincides with the first minimum of the other, *i.e., the angular separation between the principal maxima of the two spectral lines is equal to half the angular width of either principal maximum.* If an optical instrument just resolves two spectral lines of wavelength λ and $\lambda + d\lambda$, then the resolving power of chromatic resolving power of the instrument is given by $\frac{\lambda}{d\lambda}$.

(b) Resolving Power of a Plane Transmission Grating

The resolving power of a grating represents its ability to form separate spectral lines for wavelength very close together, it is expressed as $\frac{\lambda}{d\lambda}$.

In fig. (31), let a parallel beam of light whose wavelength is λ and $(\lambda + d\lambda)$ incident normally on the grating. If the principal of nth, λ is formed in the direction θ_n.

$\therefore \qquad (a + b) \sin \theta_n = n\lambda \qquad \text{... (1)}$

For minima, the Ist minimum adjacent to nth maximum at $(\theta_n + d\theta_n)$ then

$N(a + b) \sin \theta = mA \qquad \text{... (2)}$

where N is total number of rulings on the gratings and m takes all integral values except

$O, N, 2N, \ldots, nN.$

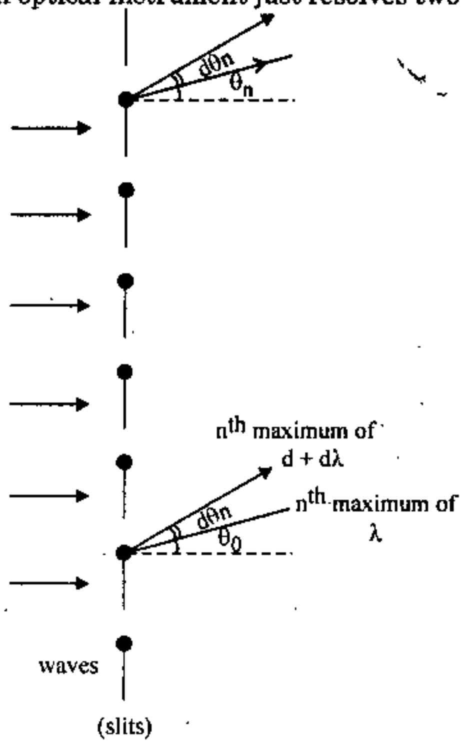


Fig. 31.

Clearly Ist minima adjacent to nth maxima at $(\theta_n + d\theta_n)$ for $m = (nN + 1)$. From equation (2),

$$N(a + b) \sin (\theta_n + d\theta_n) = (nN + 1)\lambda$$

or $$(a + b) \sin (\theta_n + d\theta_n) = \frac{(nN + 1)}{N} \lambda \qquad \text{... (3)}$$

By Rayleigh's criterion equation (1) becomes (for $\lambda + d\lambda$)

$$(a + b) \sin (\theta_n + d\theta_n) = n(\lambda + d\lambda) \qquad \text{... (4)}$$

Comparing equations (3) and (4) we have

$$\left(\frac{nN + 1}{N}\right) \lambda = n(\lambda + d\lambda)$$

or $$\lambda = Nnd\lambda$$

or $$\lambda / d\lambda = Nn = R \text{ (Resolving power of grating)} \qquad \text{... (5)}$$

if $n = 0$, equation (5) becomes,

Hence, all wavelengths being independent of order (n)

Thus equation (5) can be written as

$$R = Nn = \frac{N(a+b)\sin\theta_n}{\lambda}$$

or $$\boxed{R \propto N(a+b)} \quad \text{... (6)}$$

where $N(a + b)$ is the total width of the grating.

(c) Experimental Determination

The resolving power of a grating may be experimentally determined by finding the minimum width of the grating for just resolution of two close lines, say D_1 and D_2 lines of sodium.

The grating is mounted on the prism-table of an adjusted spectrometer. An adjustable slit is placed between the telescope and grating with its edges parallel to the rulings of the grating shown in fig. (32). The slit of the spectrometer is illuminated by sodium light, and the D_1 and D_2 lines in the first order are observed through the telescope. The adjustable slit is now closed gradually until the D_1 and D_2 'just' merge together. The slit-width is then measured by a travelling microscope. Let it be w. Then w is the minimum width of the grating for resolving D_1 and D_2 lines.

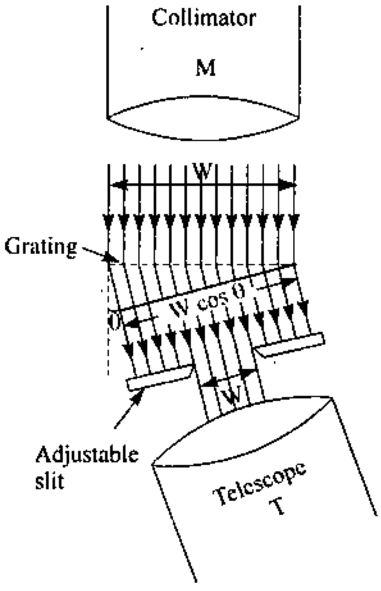


Fig. 32.

Grating-width w measured above corresponds to a resolving power of 982. Since, the resolving power of the grating is proportional to its width, its value R for the whole grating will be given by

$$R = 982 \times \frac{W\cos\theta}{w},$$

where W is the width of the total ruled surface of the grating and θ the mean angle of diffraction of D_1 and D_2.

• 4.14. LIMIT OF RESOLUTION OF A TELESCOPE

Its limit of resolution is defined as the angle subtended at the objective by the two point objects which are just resolved when seen through the telescope. The telescope used to see distant objects which subtend a small angle at its objective.

In the following fig. (33), suppose a distant point source of light S, (like planets, stars etc), lying on the axis of the telescope objective. Its boundary like a circular aperture, the image is formed by the objective. The radius of angular disc is given by

$$\theta = \frac{1\cdot 22\lambda}{d} \text{ radian.}$$

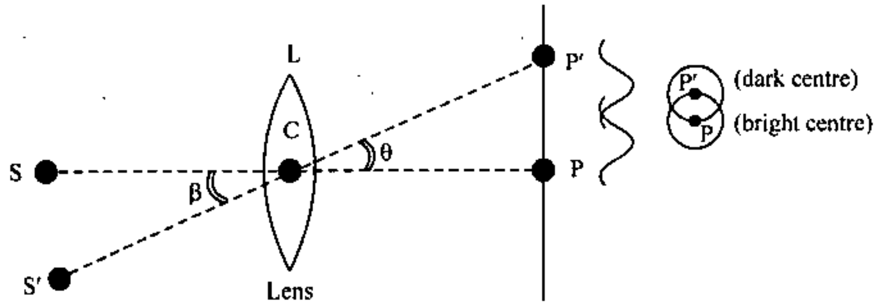


Fig. 33.

where d is the diameter of the objective.

It S' be second source close to source S. Its image will be a differentiation pattern having centre P' on the line $S'C$. Where C is the centre of lens L.

According Rayleigh's criterion, the image of the object just resolved if centre P of the central bright disc due to S falls on the Ist dark rings due to S'.

$$\therefore \qquad \angle PCP' = \theta$$

if β be the angle subtended by SS', at the objective in the limit resolution from figure (33) we have

$$\beta = \theta = \frac{1 \cdot 22\lambda}{d} \text{ radian}$$

where β **is known as limit resolution of the telescope.**

Experimental Determination : To adjustable slit is fitted just infront the objective of the telescope where resolving power is required. The pair of slit is prepared by cutting with a razor blade two fine parallel lines on a glass plate coated with tinfoil they are illuminated by light.

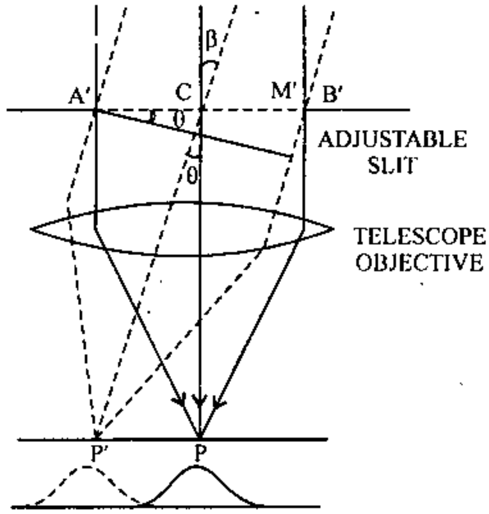


Fig. 34.

The telescope is placed at a sufficient distance from the illuminated slits and directed towards them. The adjustable slit is kept widely opened. The eye-piece is focussed on the cross-wires. Then the distance between the cross-wires and the objective is varied until well-defined images of the two illuminated slits are seen. Now, the width of the adjustable slit is gradually decreased until the limit of resolution is reached at this stage the width a of the slit is measured by means of a microscope. The resolving power is then calculated by means of the following relation.

$$R = \frac{\lambda}{a}$$

where λ is wavelength of light.

Consider [in fig. (34)] a light falling normally on $A'B$ slit each point of $A'B$ is send out secondary waves in all direction by Huygen's principle. These waves meet at P of the objective in the same phase, therefore P is the central maxima of the diffraction pattern. Now again consider the waves diffracted at θ and normal to $A'B$. These are focussed at P' of objective. Let draw a line $A'M$ normal to the rays. Then the path difference between waves of starting from $A'B'$ is $B'M$ if $B'M = \lambda$ then the waves from $A'C$ shall reach P' with path difference of $\lambda/2$. Therefore each point of $A'C$ in CB. Therefore point P' will be Ist minima of pattern which is

$$B'M = a \sin\theta = \lambda$$

or
$$\sin\theta = \frac{\lambda}{a}$$

$\therefore \qquad a >> \lambda$, $\sin\theta$ is very small then put $\sin\theta = \theta$

$$\therefore \qquad \theta = \frac{\lambda}{a}$$

(Separation between central maxima and Ist minima)

at $\theta = 0$, the central maxima is obtained.

Consider it object 'S'' is near to 'S' object. According to Rayleigh criterion, S and S' will be just resolved when central maxima of S' falls at P', the Ist minima of S. Hence, limit resolution the angular separation between central maxima of S and S' is

$$\theta = \frac{\lambda}{a} \approx \beta$$

which is the minimum angle of resolution. Then $\frac{\lambda}{a}$ is measured of the resolving power of telescope. If d be the linear separation between S and S' and D is the distance from the T (telescope) then

$$\beta = \frac{d}{D}$$

The $\frac{d}{D}$ is called the practical resolving power, and $\frac{\lambda}{a}$ is called the theoretical resolving power.

• 4.15. RELATION BETWEEN MAGNIFYING POWER AND THE RESOLVING POWER OF TELESCOPE

From the theory of telescope, we know that the magnifying power of a telescope is given by

$$M = \frac{\text{Diameter of objective (entrance pupil)}}{\text{Diameter of exit pupil (eye ring)}} = \frac{D}{d} \qquad \text{... (1)}$$

the magnifying power is said to be normal if diameter d of the eye ring is equal to the diameter d_e of the pupil of the eye. Hence,

Normal magnifying power

$$= \frac{\text{Diameter of the objective}}{\text{Diameter of the eye}} = \frac{D}{d_e}$$

Further, the limit of resolution of the telescope objective of diameter D is given by

$$d\theta = \frac{1 \cdot 22\lambda}{D} \qquad \text{... (2)}$$

and the limit of resolution of the unaided eye is given by

$$d\theta' = \frac{1 \cdot 22\lambda}{d_e} \qquad \text{... (3)}$$

$$\therefore \quad \frac{\text{Limit of resolution of the eye}}{\text{Limit of resolution of the telescope}} = \frac{d\theta'}{d\theta}$$

$$= \frac{\left(\frac{1 \cdot 22\lambda}{d_e}\right)}{\left(\frac{1 \cdot 22\lambda}{D}\right)}$$

$$= \frac{D}{d_e} = \text{Normal magnifying power of the telescope.} \qquad \text{... (4)}$$

Thus, *the limit of resolution of telescope, multiplied by its normal magnifying power is equal to the limit of resolution of the unaided eye.*

• SUMMARY

- The zones for calculating the resultant intensity are called Fresnel's half period zones.
- An arrangement consisting of an equidistant parallel rectangular N slits of equal width separated by equal opaque portion is called as a diffraction grating.
- The grating, slit and spectral lines lie on the circumference of the same circle. This circle is called Rowland circle.
- The telescope used to see distant objects which subtend a small angle at its objective.
- The $\frac{d}{D}$ is called the practicle resolving power, and $\frac{\lambda}{a}$ is called the theoretical resolving power.

• STUDENT ACTIVITY

1. What do you mean by diffraction of light?

2. Write the difference between Fresnel's and Fraunhofer's classes of diffraction.

3. Distinguish between interference and diffraction.

4. What are Fresnel's half period zones?

5. What will be the effect in diffraction pattern formed by a narrow wire, if the thickness of wire is increased?.

6. Explain the effect of slit width in Fraunhofer's diffraction.

7. What will be the effect on Fraúnhofer's diffraction, if the wavelength of light is changed?

8. Give the merits and demerits of Rowland mounting.

9. Explain Paschen mounting and give its merits and demerits.

10. In what respect does a prism spectrum differ from grating spectrum?

11. Define dispersive power of grating.

12. What do you mean by resolving power of an optical instrument?

13. What is the relation between magnifying power and resolving power of telescope?

• TEST YOURSELF

1. Describe the method of dividing a cylindrical wavefront into half period strips and find its effect at an external point.
2. Describe with necessary theory, the Fresnel's type of diffraction due to a straight edge. Show the intensity distribution in the diffraction.
3. Explain the diffraction pattern formed by a narrow wire, illuminated by monochromatic light from a narrow slit parallel to the wire.
4. Describe Fraunhofer diffraction due to a single slit and deduce the positions of maxima and minima and find their relative intensities.
5. Explain Fraunhofer's diffraction due to double slit. How does its intensity distribution curve differ from the curve obtained due to a single slit?
6. What do you understand by the plane diffraction grating? Give the construction and theory of a plane diffraction grating.
7. Determine the wavelength of monochromatic light with the help of transmission diffraction grating.
8. Give the theory of concave reflection grating and deduce the conditions of focussing the spectra. Describe the working of Rowland's mounting.
9. Describe the Rayleigh limit of resolution. Deduce an expression for resolving power of a plane transmission grating.
10. Give a method for experimental determination of resolving power of prism.
11. Diffracted light is collected by a lens as in a telescope in which classes ?
    (a) Fresnels (b) Fraunhofer
    (c) Both of (a) and (b) (d) None of these
12. The continuous locus of the either particles in the same phase of vibrations is known as the :
    (a) Wavefront (b) Waves
    (c) Zones (d) None of these
13. In an experiment for observing diffraction pattern due to a straight edge, the distance between the slit source and straight edge is 6 m and that between the straight edge and eye-piece is 4 m. What is the position of Ist maxima when $\lambda = 6000Å$?
    (a) 0.2 cm (b) 0.3 cm
    (c) 0.4 cm (d) None of these
14. Light of wavelength 6000Å through a narrow circular aperture of radius 0.09 cm. At what distance along the axis will the first maximum intensity be observed?
    (a) 132 cm (b) 135 cm
    (c) 138 cm (d) None of these
15. A fine aperture is placed at a distance of 12 cm from a sharp razor blade edge held vertically at a distance of 25 cm from the screen. If the aperture be illuminated by light ($\lambda = 5890$ Å), what is the height of the 6th bright band above the line joining the edge to the aperture ?
    (a) 0.243 cm. (b) 0.240 cm
    (c) 0.343 cm (d) None of these
16. A screen is placed at a distance of 60 cm. from a circular aperture of 0.6 cm radius. What is the radius of the Ist half period zone when $\lambda = 6000Å$?
    (a) 0.06 cm (b) 0.05 cm
    (c) 0.04 cm (d) None of these
17. In above problem number 16, how many zones are contained in the circular aperture?
    (a) 100 (b) 200
    (c) 300 (d) None of these

18. If a be the distance between the slit and the straight edge, and P the distance between the straight edge and the eye-piece, the position of the nth maximum (bright band) is :
   (a) $x_n = \sqrt{\frac{P(a+P)(2n+1)\lambda}{a}}$
   (b) $x_n = \sqrt{\frac{P(a-P)(2n+\lambda)}{a}}$
   (c) $x_n = \sqrt{\frac{P(a+P)(2n-1)\lambda}{a}}$
   (d) None of above
19. How the centre of the illuminated region in diffraction of a circular aperture?
   (a) Dark (b) Bright
   (c) Dark or bright (d) None of these
20. In a Fraunhofer class, the incident wavefront is :
   (a) Spherical (b) Elliptical
   (c) Plane (d) Cylindrical
21. What is the width of the central maximum in the Fraunhofer diffraction pattern due to a narrow slit?
   (a) $\theta = \pm \frac{\lambda}{a}$ (b) $\theta = \frac{a}{\lambda}$
   (c) $\theta = \pm \frac{a}{\lambda}$ (d) None of these
22. What is the value of $\underset{\theta \to 0}{Lim} \frac{\sin\theta}{\theta}$?
   (a) ∞ (b) 1
   (c) 0° (d) None of these
23. The formula of dispersive power of Grating is –
   (a) $\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$ (b) $\frac{d\theta}{d\lambda} = \frac{(a+b)\cos\theta}{n}$
   (c) $\frac{d\lambda}{d\theta} = \frac{n}{(a+b)\cos\theta}$ (d) None of these
24. The spectral lines in grating spectra are almost :
   (a) Circular (b) Straight
   (c) Elliptical (d) None of these
25. The resolving power of a grating is :
   (a) Large (b) Small
   (c) Zero (d) None of these
26. How many orders will be visible if the wavelength of the light falling normally on a grating having 2000 lines/cm is 5000 Å?
   (a) 20 (b) 10
   (c) 5 (d) None of these
27. For a grating with grating element is 18000Å, what is dispersions in the Ist order spectra around $\lambda = 5000$Å with assuming normal incidence ?
   (a) 5.78×10^{-5} rad/Å (b) 5.70×10^{-5} rad/Å
   (c) 5700 rad/Å (d) None of these
28. In a grating spectrum, which spectral line in the IIIrd order will overlap, with 3rd order line of 5461Å
   (a) 5460Å (b) 5461Å
   (c) 5400Å (d) None of these

29. A plane transmission grating having 5500 lines/cm is used to produce a spectrum of Hg light. What will be the angular separation of the two yellow lines 5770 and 5790Å in the second order?
   (a) 9′ of arc (b) 10′ of arc
   (c) 8′ of arc (d) None of these
30. How many orders will be visible if the wavelength of light falling normally on a grating having 2540 lines/inch of 5000 Å.
   (a) 2 (b) 3
   (c) 4 (d) None of these
31. The ratio of $\frac{\lambda}{d\lambda}$ is known as :
   (a) Resolving power of device (b) Chromatic resolving power of device
   (c) Both of (a) and (b) (d) None of these
32. Resolving power of telescope is equal to
   (a) $\frac{1.22\lambda}{d}$ (b) $\frac{1.20\lambda}{d}$
   (c) $\frac{1.22\,d}{\lambda}$ (d) None of these
33. What is the smallest separation between two point-stars which a telescope of 10 cm diameter objective can resolve? (when $\lambda = 6000$ Å)
   (a) 1.5° (seconds) (b) 2.0° (seconds)
   (c) 2.5° (seconds) (d) None of these
34. The Na yellow doublet has wavelength 5890Å and 5896 Å. What should be the resolving power of a grating to resolve them ?
   (a) 980 (b) 981
   (c) 982 (d) None of these
35. What should be the minimum number of lines in a plane grating required to resolve the sodium yellow doublet for IInd order?
   (a) 491 (b) 490
   (c) 591 (d) None of these
36. In problem number 35, if order is Ist then minimum number of lines is
   (a) 982 (b) 980
   (c) 882 (d) None of these
37. What should be the least width of a plane diffraction grating having 500 lines/cm to resolve the Na yellow doublet in IInd order?
   (a) 2.0 cm (b) 3.0 cm
   (c) 1.0 cm (d) None of these

ANSWERS

11. (b)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (c)	
20. (c)	**21.** (a)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (a)	**26.** (b)	**27.** (a)	**28.** (b)	
29. (b)	**30.** (a)	**31.** (c)	**32.** (a)	**33.** (a)	**34.** (c)	**35.** (a)	**36.** (a)	**37.** (c)	

UNIT 5

POLARIZATION

STRUCTURE

- Polarization of Light
- Brewster's Angle
- Doubly Refracting Crystal
- Double Refraction
- Huygen's Postulates and lheory of Double Refraction
- Nicol Prism
- $\lambda / 4$ Plates (Quarter Wave Plates)
- Detection of Different Polarized Light
- Production of Light (Plane Polarised)
- Distinction between Unpolarised Light and Partially Plane-Polarised Light
- Optical Rotation
- Specific Rotation
- Biquartz Polarimeter
  - Summary
  - Student Activity
  - Exercise

LEARNING OBJECTIVES

After going this unit you will learn :

- Pictorial representation of light like ordinary light, unpolarised light, plane polarised vibrations.
- Brewster's law.
- Uniaxial and biaxial crystals; optic axes of crystal.
- Principal section of crystal.
- Nicol prism as a analyser and half wave plates.
- Production of circularly polarised light and elliptically polarised light.
- Fresnel's explanationof optical rotation.
- Laurent's half shade polarimeter.

5.1 POLARIZATION OF LIGHT

In fig. (1), let light (ordinary) from a source S falls on tourmaline crystals M and M'. Which both cuts parallel to its crystallogrpahic axis shown in dotted. The emergent

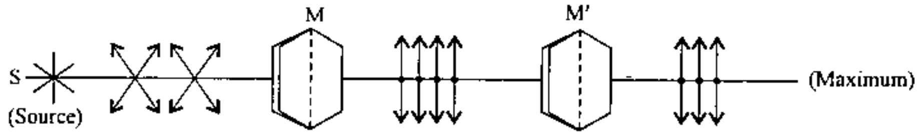


Fig. 1.

beam will be slightly coloured then intensity is maximum. If now keeping crystal M fixed, crystal M' is rotated about the beam as axis, the intensity of the emergent light decreases and no light comes out of the crystal M' when M' is at right angles of M (in fig. 2).

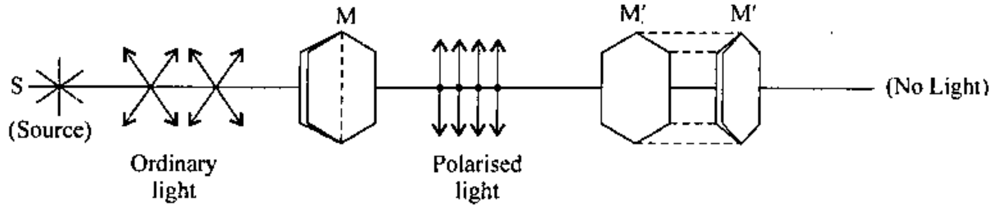


Fig. 2.

If crystal M' is further rotated, light reappears and becomes maximum again when the two crystals are parallel. It can be easily inferred that light is in transverse wave motion. This shows that light passing by crystal M is not symmetrical about the direction of propagation but its vibrations are confined only to a single line in a plane perpendicular to the direction of propagation which has acquired the property of one sideness **is called polarized light.** Thus, polarization of light means departure from complete symmetry about the direction of propagation.

According to Maxwell's, E.M. theory of light is nothing but the propagation of mutually perpendicular vibrating electric and magnetic vectors. Both being perpendicular to the direction of propagation. The electric vector function as the light vector. Hence-plane polarised light may be defined as the light in which the light vector vibrates along a fixed straight line in a plane perpendicular to the direction of propagation.

Difference between ordinary and polarised light : When ordinary light is passed through a *single* rotating tourmaline crystal, there is no variation in the intensity of the emergent light. This means that ordinary light is symmetrical about its direction of propagation, *i.e.*, the light vector vibrates along all possible straight lines in a plane perpendicular to the direction of propagation.

In the case of polarised light obtained by passing it through tourmaline plate, there is a lack of symmetry about the direction of propagation of the light. If this light is again viewed through another rotating tourmaline plate, there will be a change in the intensity of light and it will be completely cut off when the axis of the two crystals are perpendicular to each other.

Pictorial Representation of Light

In an unpolarised beam of light all directions of vibrations at right angles to that of propagation of light are possible, hence it is represented by fig. 3 (i). In a plane polarised beam of light, the vibrations are along a single straight line if the vibrations are parallel to the plane of paper. It is shown in fig. 3 (ii) if they are along a straight line perpendicular to the plane of paper. They are shown in fig. 3 (iv).

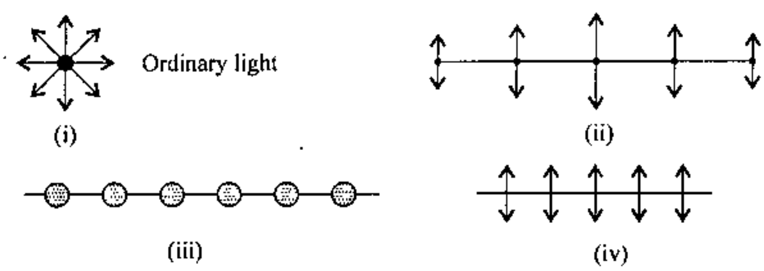


Fig. (3). (i) Ordinary light; (ii) Unpolarised light; (iii) Plane polarised vibrations parallel to plane of paper, (iv) Plane polarised vibrations are to plane of paper.

• 5.2 BREWSTER'S ANGLE

In 1811, Sir David Brewster's study the polarisation of light by reflection, using the different reflecting surface. He observed that for a particular angle of incidence **known as the angle of polarisation**, the reflected light is completely plane polarised with the plane of vibration perpendicular to the plane of incidence.

Brewster's law : According to the law, **"when light is incident at the polarizing angle, this reflected and refracted rays are perpendicular to each other."**

$\therefore \qquad \mu = \tan i_p$

Suppose, a beam PO of unpolarised light is incident on the glass surface at polarizing angle i_p, it is reflected along OP' and refracted along OO' as shwon in fig. (4). Then from Brewster's law,

$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p}$$

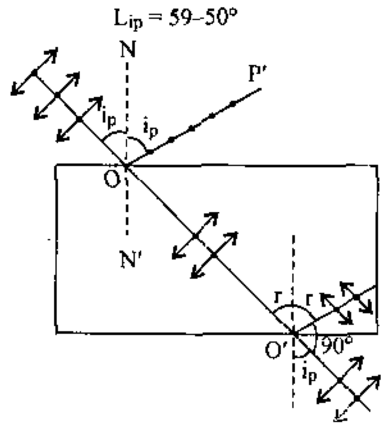


Fig. 4.

Also from Snell's law,

$$\mu = \frac{\sin i_p}{\sin r}$$

$$\therefore \qquad \frac{\sin i_p}{\sin r} = \frac{\sin i_p}{\cos i_p}$$

or $$\frac{\sin i_p}{\sin\left(\frac{\pi}{2} - i_p\right)} = \frac{\sin i_p}{\sin r}$$

or $$\frac{\pi}{2} - i_p = r \quad \text{or} \quad i_p + r = \frac{\pi}{2}$$

We know that, $\angle NON' = \pi$ (from fig.)

$$i_p + \angle POO' + r = \pi$$

$$\therefore \qquad \angle POO' = \pi - (i_p + r) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

that is an polarising angle, the reflected rays is at right angles to the refracted angles.

Brewester's law applies, when reflection takes place at the surface of a rarer medium. In fig. (4), the angle of incidence and angle of refraction at lower surface are r and i_p respectively. Hence, refractive index of second medium (air) with respect to glass (Ist).

$$\frac{1}{\mu} = \frac{\sin r}{\sin i_p}$$

and also

$$\tan r = \frac{\sin r}{\cos r} = \frac{\sin r}{\cos r} \cdot \frac{\sin i_p}{\sin i_p}$$

$$\tan r = \frac{\sin r}{\sin i_p} \cdot \frac{\sin i_p}{\sin i_p} \qquad (\because \sin i_p = \cos r)$$

$$\therefore \qquad \tan r = \frac{\sin r}{\sin i_p} = \frac{1}{\mu}$$

Hence the light reflected at the lower face is completely plane polarised while that refracted into the air is partially polarised plane.

• 5.3 DOUBLY REFRACTING CRYSTAL

In 1669, Bartholinous performed that, if crystals split a rays of light incident upon them into two refracted rays, such **crystals are called double refracting crystals,** such like–quartz, calcite, etc.

They are of two types :

(i) Uniaxial crystals, (ii) Biaxial crystals.

(a) Uniaxial Crystals : These crystals of are in one direction, called the optic axis which two refracted rays travel with same velocity—(calcite).

(b) Biaxial Crystals : There are two optic axes along which refracted rays travel with same velocity—(aragonite).

Examples—

Calcite Crystal : Its a colourless transparent substance found in nature in different forms. Chemically its hydrated calcium carbonate ($CaCO_3$) and can be reduced into rhombhedron by breakage as shown in fig. (5). Each of the faces of rhomobohedron are parallelogram having angles of 78°, 102° nearly. At the two diameterically opposite C and F, the three angles of faces meeting there all obtuse. These corners are **called blunt corners.** And all remaining 4 corners, in two are obtuse angle and two are acute.

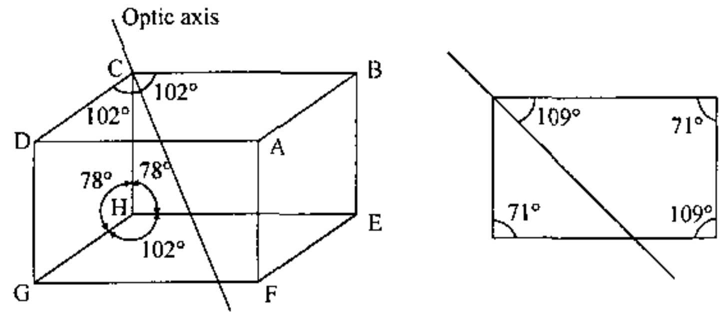


Fig. 5.

Optic axes of Crystal : A line passing through any of the blunt corners and equally, inclined with three faces which meet there, it is **known as the optic axes of crystals.**

Principal Section of Crystal : A plane containing the optic axis and perpendicular to the two opposite faces of the crystal is **called the principal section of the crystal.** As a crystal has 6 faces, for every point inside the crystal there are 3 principal sections one for each pair of opposite crystal faces. A principal section of crystal is a parallelogram having angles of 109° and 71°.

• 5.4 DOUBLE REFRACTION

When a beam of unpolarised light is allowed to pass through a calcite crystal (uniaxial), it splits upto into two refracted beams in place of the usual one a single glass. This phenomenon **is called as double refraction.** And its Ist discovered by **Dutch Philosopher Bartholinus in year 1669.**

The phenomenon of double refraction can be illustrated in a simple manner. If an ink dot is made on a sheet of paper and is viewed through a calcite crystal, two images are observed. If now the crystal is rotated slowly, as shown in fig. (6), one of the two images remains stationary while the IInd image rotated around the first. The image which remains stationary is **called as ordinary image** and the refracted rays which produce this image are **known as the ordinary rays (*O*-rays)** as it obeys the ordinary law of refraction. The second image which rotates around the Ist is **called as extraordinary image** and the refracted rays which produce this image are **called as the extra ordinary rays (*E*-rays)** it does not obey the law of refraction.

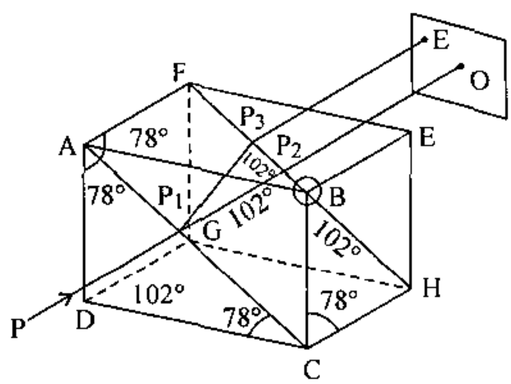


Fig. 6.

If a narrow beam of light AB from a point source is incident on a crystal of calcite making an angle i (angle of incidence), it is spilt-up into two rays (O and E) inside the crystal and the two images are obtained on the screen as shown in fig. (6). The O-ray travelling along inside the crystal makes an angle of refraction r_1 while the E-ray travelling along makes an angle of refraction r_2 and the refractive indices for O-ray and E-rays are respectively.

$$\mu_O = \frac{\sin i}{\sin r_1} \quad \text{and} \quad \mu_E = \frac{\sin i}{\sin r_2}$$

In the case of calcite $r_2 > r_1$ and $\mu_E < \mu_O$, or $V_O < V_E$. It means inside the crystal, the velocity of light for the O-rays is less than that for E-rays. It is found that μ_0 of O-rays is the same for all values of incidence angles while it varies with the angle of incidence in the E-rays case. It is obvious that the O-rays travel with the same speed in all the directions within the crystal while the E-rays have got different speed in different directions. The both rays, ordinary and extraordinary obtained by double refraction, are plane polarised. And their planes of polarisation are at the 90° angles. The vibrations of O-rays are perpendicular to plane of the paper while those of the E-rays takes place in the plane of paper.

• 5.5 HUYGEN'S POSTULATES AND THEORY OF DOUBLE REFRACTION

In order to explain the phenomenon of double refraction in uniaxial crystals. Huygen extended his theory of secondary wavelets. The postulates of the theory are given below :

(i) When light waves are incident on a doubly refracting crystal, every point of it becomes a source of secondary wavelets and sends out not one but two wavefronts—one for the O-ray and the other for E-ray.

(ii) For the O-ray, the crystal is isotropic and homogeneous. Hence, O-ray travels with the same velocity in all directions and the wavefront (or wave-surface) corresponding to it is *spherical*.

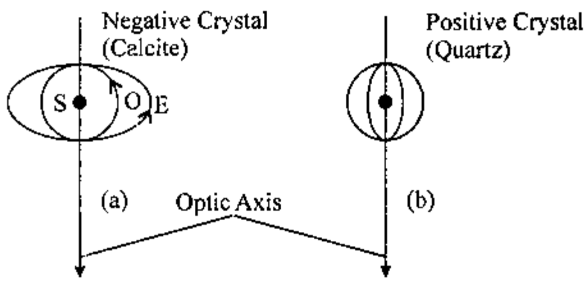


Fig. 7.

(iii) For the E-ray, the crystal is anisotropic (not having identical properties in different directions). Hence, its velocity varies with the directions and the extra-ordinary wave surface can not be a sphere; it is a spheroid or *ellipsoid of revolution.*

(iv) The spherical wavefront corresponding to the O-ray and the ellipsoid of revolution corresponding to the E-ray touch each other at two points. The direction of the line joining these two points is the *optic axis.*

Let us now consider a point source of light S within the crystal. Fig. (7) represents the O and E wavefronts in two dimensions at any instant surrounding the point sources S. If the figure is rotated about the optic axis, the circle gives rise to a sphere (the wave surface of the O-ray) and the ellipse to an ellipsoid (the wave-surface for the E-ray).

In fig. (7b), the ordinary wave surface lies within the extra ordinary wave surface. The diameter of the sphere is equal to the minor axis of the ellipsoid. Such crystals are **called as negative crystals** when ordinary wave surface lies outside the extra ordinary wave surface. The diameter of the sphere is equal to the major axis of the ellipsoid such crystals are **called as positive crystals.**

Case–I : Optic axis in the plane of incidence and parallel to the crystal surface.

(a) Oblique Incidence : In fig. (8), consider a plane wavefront AB incident on the plane surface MM'. At the wavefront touches at the point A it becomes the source of O (spherical) and E (ellipsoidal) wavelets. These show by semi circle and semi ellipse with in the crystal touch each other along MM'. By the time reach C, the O-rays travelling a distance AF' and E-rays a distance AG'.

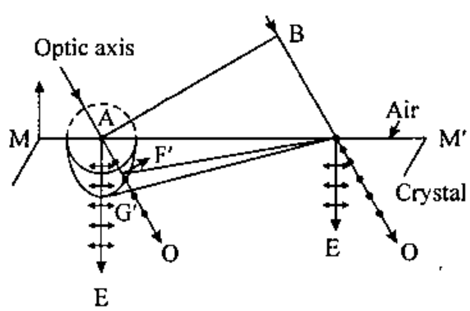


Fig. 8.

$$\therefore \quad \frac{BC}{V_a} = \frac{AF'}{V_0} = \frac{AG'}{V_e}$$

Here, $V_\alpha \rightarrow$ Velocity in the air,

$V_O \rightarrow$ Velocity of O-rays,

$V_e \rightarrow$ Velocity of E-rays,

$$\therefore \qquad AF' = \frac{BCV_0}{V_a} = \frac{BC}{\mu_0}$$

$$AG' = \frac{BCV_e}{V_a} = \frac{BC}{\mu_e}$$

If μ_E is the principal refractive index along E-rays. A tangent CF' from C to θ-wave surfaces and CG' from C to E-wave surfaces determine the E wavefronts inside the crystal.

(b) Normal Incidence : In fig. (9), a plane wave front is incident on surfaces MM', $C\,D'$ and $F\,G'$ are the positions of the ordinary and extra ordinary refracted wavefronts at the same instant. AO and AE (also BO and BE) are the ordinary and extraordinary refracted rays. Although, the rays are travelling along the same directions yet there is double refraction because the rays are travelling with different velocities.

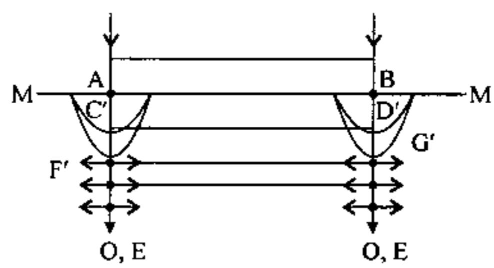


Fig. 9.

Case II : Optic axis in the plane of incidence and perpendicular to the crystal surface.

(a) Oblique Incidence : Fig. (10) represents a plane wavefront AB falling obliquely on the calcite crystal surface MM'. The optic axis is in the plane of incidence and perpendicular to surface. The spherical O-wave surface and ellipsoidal E-wave surface originating from A touch other at Q in the direction of the optic axis. CF and CG show the refracted wavefront at instant when the wavelets from B reaches C. AF and AG are the ordinary and extraordinary rays.

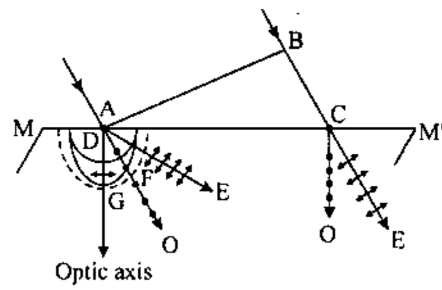


Fig. 10.

(b) Normal Incidence : In fig. (11), depicts this case of normal incidence. AB is the normally incident plane wavefront. The two refracted wavefronts CD and FG coincide at all instants. There is no separation between the ordinary and extraordinary rays. Both travel in the same direction (along the optic axis) with the same velocity. The refractive index for the E-ray is the same as that for the O-ray. Thus, the phenomenon of double refraction does not occur in this case.

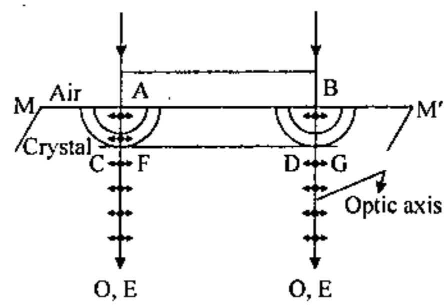


Fig. 11.

Case III : Optic axis in the plane of the incidence and inclined to the crystal surface.

(a) Oblique Incidence : Let AB be the incident plane wavefront falling obliquely on the crystal surface MM'. The crystal is so cut that the optic axis lies in the plane of incidence and is in the direction AD' shown dotted in fig. (12).

The point A of the wavefronts AB, where it strikes the crystal surfaces MM'. Ist becomes the centre of two wavelets inside the crystal—one the ordinary and other the extra ordinary by principle of Huygen's. The circular shape of the ordinary and elliptical shape of extraordinary waves calcite being a negative crystal, sphere lies inside

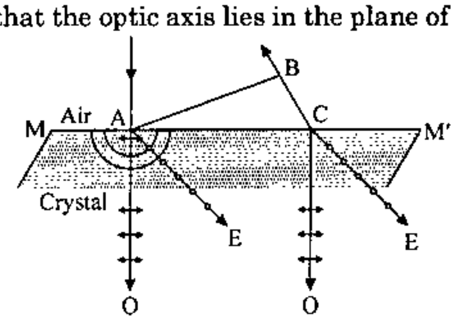


Fig. 12.

the ellipsoid and touch each other at points which lie on the optic axis. G' is the point of intersection of AE and ellipsoid if V_a is the velocity of light in air and V_0 and V_e is respectively the velocities of O-rays along AF' and E-rays along AE. Then

$$\frac{BC}{V_a} = \frac{AF'}{V_0} = \frac{AG'}{V_e}$$

Hence, $$AF' = V_0 \frac{BC}{V_a} = \frac{BC}{\mu_0}$$

and $$AG' = \frac{V_e BC}{V_a} = \frac{BC}{\mu_e}$$

Hence $$\mu_e < \mu_a < \mu_0$$

or $$V_a > V_e > V_0$$

The E-rays surface is an ellipsoid touching the θ-wave surface at D. AD then gives the semi-minor axis of ellipse and equal to BC/μ_0. At $\pi/2$ to the axis, the μ_e is minimum and is **called principal refractive index denoted by μ_e.**

(b) Normal Incidence : In fig. (13) illustrates in this case. AB is the plane wavefront incident normally. CD' and $F'G'$ are the positions of the ordinary and extraordinary refracted wavefronts at the same instant. AO and AE are ordinary and BO and BE extra ordinary refracted rays. Similar constructions can be drawn for different positions of the optic axis w.r. to plane of the incidence and crystal surface.

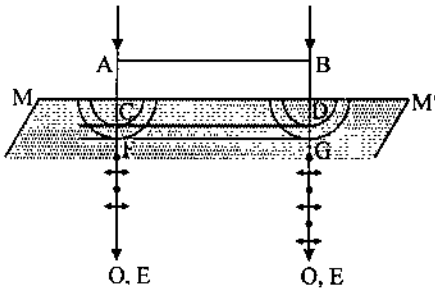


Fig. 13.

Case IV : Optic axis perpendicular to the plane of incidence and parallel to the crystal surface.

(a) Oblique Incidence : Let a plane wavefront AB incident obliquely on the surface MM'. The point 'A' becomes the sources of ***O* and *E*** wave surface and both wave surfaces (O and E) are figures of revolution about the axis. Their section in the plane of incidence are circular. In fig. (14) CF and CG are respectively ordinary and extra-ordinary refracted wavefronts at instant. AF and AG shows the O and E rays.

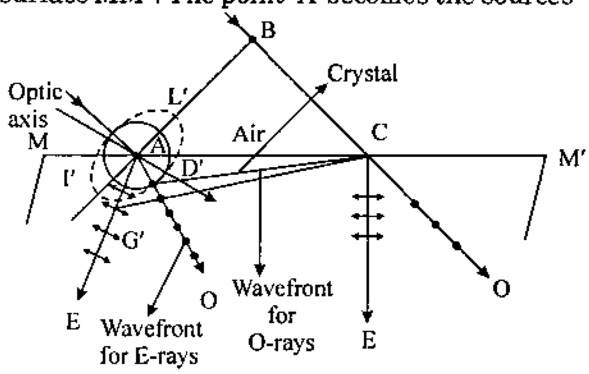


Fig. 14.

(b) Normal Incidence : In fig. (15), a plane wave-front AB is incident normally on the surface MM'. CD and FG are the ordinary and extra-ordinary refracted waves at same instant. AO and AE with BO and BE are the ordinary and extra-ordinary rays, O and E lie along same direction but different velocities.

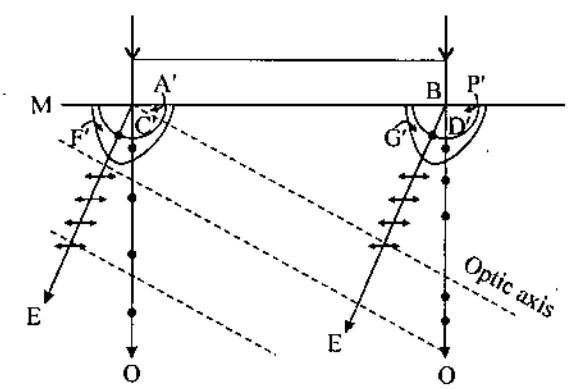


Fig. 15.

• 5.6 NICOL PRISM

Nicol prism is based for its action on the phenomenon of double refraction. It is an optical device made from a calcite crystal for producing and analysing plane polarised light. **It is invented by William Nicol in 1828.**

Construction : A calcite crystal $PQRS$ (fig. 16) about three times as long as it is wide is taken. Its end faces PQ and RS are ground such that the angles in the principal section becomes 68° and 112° instead of 71° and 109°. The crystal is then cut apart along the plane $P'S$ perpendicular to both principal section and the end faces $P'Q$ and RS'. The two cut surfaces are ground and polished optically flat.

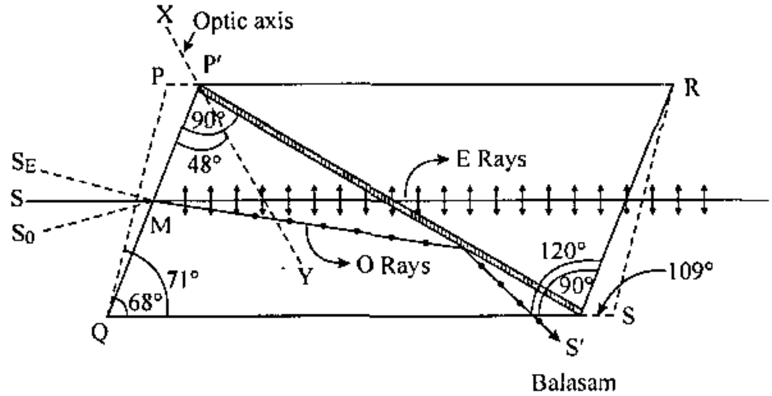


Fig. 16.

Working : When a ray SM of unpolarised light nearly parallel to QS' is incident on the face $P'Q$. It is split up into two refracted rays, the O-ray and the E-ray. Both the rays are plane-polarised. The O-ray has vibrations perpendicular to the principal section of the crystal while the E-ray has vibrations in the principal section. When the O-ray reaches the layer of the canada balsam, it is passing from an optically denser to a rarer medium. Since, the length of the crystal is large, the angle of incidence of the O-ray at the calcite-balsam surface becomes greater than the critical angle (69°) for the O-ray., Hence, the O-ray is totally reflected at the calcite-balsam surface and is absorbed by the tube containing the crystal. The E-ray, however, on reaching the calcite-balsam surface passes from a rarer to a denser medium and is transmitted. Since the E-ray is plane-polarised, the light emerging from the Nicol is plane-polarised with vibrations parallel to the principal section. These vibrations are parallel to the shorter diagonal of the end face of the crystal.

Limitations : The Nicol prism works only when the incident beam is slightly convergent or slightly divergent. If the incident ray makes angle much smaller than SMQ with the face $P'Q$, the O-ray will strike the calcite-balsam surface at an angle less than the crictical angle (69°). Therefore, the O-ray will also be transmitted and the light emerging from the Nicol prism will not be plane-polarised.

If the incident ray makes an angle much greater than SMQ, the E-ray will become more and moie parallel to the optic axis xy so that its refractive index will increase and become greater than that of the balsam. Then the E-ray will also be totally reflected from the calcite-balsam surface and no light will emerge from the Nicol prism. Hence, to obtain plane-polarised light, the incident beam should not to be wide. With the dimensions chosen; the semivertical angle of lens of incident light $S_E MS_0$ should not exceed 140, its make order to the ray SM lies within the angle range 140 that the end faces are *grounded to modify the angles.*

Uses : Its uses as the analyser and polariser.

Nicol Prism as an Analyser : (1) When an unpolarised ray of light is incident on a Nicol prism N (shown in fig. 17 (i), the ray energing from N is plane polarised with vibrations in the principal section of N. If waves incident on a prism N' (parallel to N), its vibrations will be in the principal section of N'. Hence, waves like as a E-rays in N'. And will be completely transmitted with maximum intensity.

(2) Now if N' be rotated such that its principal section becomes normally to N. (shown in fig. (17) (ii)). The vibration in the plane polarised ray incident on N' will be perpendicular to N'. Then waves like behaves as ordinary ray.

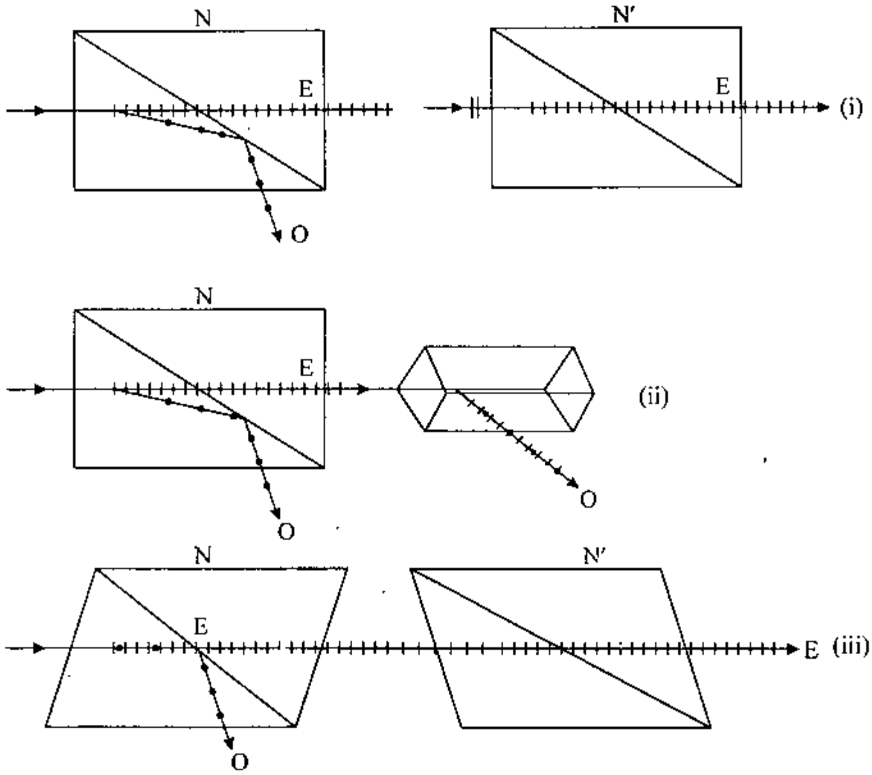


Fig. 17.

Inside in N' and will lost by total reflection at Balsam surface. Then no light will occur from N'. In this position, N and N' said to be crossed, if N' again rotated (parallel to N) shown in fig. 17 (iii). The intensity of emergent light will be maximum.

The prism N is called polariser and N' is called analyser.

If the given light (ordinary) on viewing through a rotating Nicol prism shows vibrations in minimum (zero) intensity. Hence given light is plane polarised.

• 5.7 λ/4 PLATE (QUARTER WAVE PLATE)

It is a double reflected crystal's plate having thickness such as to produce a path difference of $\frac{\lambda}{4}$ between O and E waves, **is called as $\frac{\lambda}{4}$ plate.**

Let us consider a AB wavefront incident on a plate's surface. It is broken upto O and E waves. By Huygen's wave theory, both waves travelling with different velocities and path perpendicular if thickness of plate is 'd'. Then velocities of O and E waves in, inside the plate are μ_{od} and μ_{ed} respectively. Also path difference between them is

$$\delta = (\mu_0 - \mu_0)\, d$$

if $\quad \delta = \frac{\lambda}{4}$

then $\quad \frac{\lambda}{4} = (\mu_0 - \mu_e)\, d$

$\therefore \quad \boxed{d = \frac{\lambda}{(\mu_0 - \mu_e)\, 4}}$

But in a +ve crystal axis

$$\mu_e > \mu_0$$

$\therefore \quad d = \frac{\lambda}{4}(\mu_e - \mu_0)$

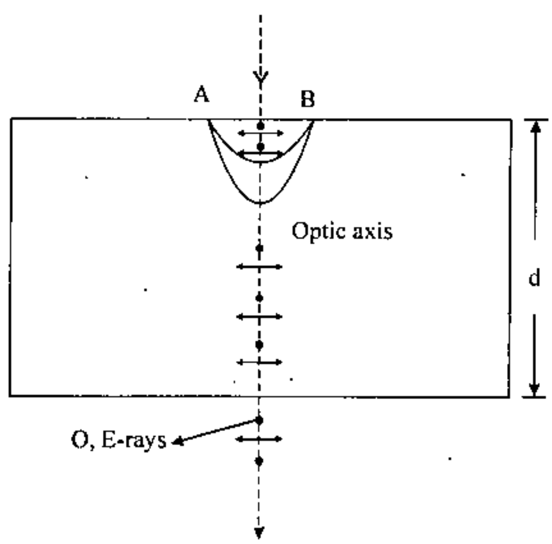


Fig. 18.

It is used to producing circularly and elliptically polarised light.

Half wave plate ($\lambda / 2$ plate) : A double refracting crystal plate having a thickness d and produce a path difference $\frac{\lambda}{2}$ between O and E waves. **Both of them, then it is called a $\frac{\lambda}{2}$ plate.**

If d be the thickness of such a plate, then for a negative crystal, such as calcite, we have

$$(\mu_0 - \mu_e)\, d = \frac{\lambda}{2}$$

or
$$d = \frac{\lambda}{2(\mu_0 - \mu_e)}.$$

If linarly-polarised light is passed through $a\,\lambda/2$ plate, the emergent light is also linearly-polarised but its direction of vibration is inclined at 2θ angle to that in the incident light, where θ is the angle between the incident vibration and the principal section of the plate. Hence, such a plate is used in polarimeter as half-shade devices to divide the field of view into two halves presented side by side.

• 5.8 DETECTION OF DIFFERENT POLARIZED LIGHT

(a) Let A be the amplitude of vibration of the incident light wave and θ be the angle between the direction of the incident vibration and the optic axis of the $\frac{\lambda}{4}$-plate. On entering the plate, the incident wave is splitted into two plane-polarised components. One component has vibrations parallel to the optic axis (E-wave) and the other has vibrations perpendicular to the optic axis (O-wave); the amplitudes of vibration being $A \cos\theta$ on emerging from the plates it have a $\frac{\lambda}{4}$ path differences.

Let $\qquad a = A\cos\theta$ and $b = A\sin\theta$

if X and Y are the axes of cartesian coordinates.

Then waves are vibrates and can be written as

$$\therefore \qquad X = a\sin\left(\omega t + \frac{\pi}{2}\right) = \alpha\cos\omega t \qquad \text{... (1)}$$

and
$$Y = b\sin\omega t \qquad \text{... (2)}$$

Now eliminating 't' between equations (1) and (2) we get

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$$

which represents the ellipse equation whose minor and major axis are normally to optical axis.

Now there is arises some cases.

(i) If $\theta = 0$, then $A = a\cos\theta = \alpha\cos 0° = a$ $\qquad (\because \cos 0° = 1)$

and $\qquad b = A\sin\theta = A\sin 0° = 0$ $\qquad (\because \sin 0° = 0)$

$\therefore$ Equations (1) and (2) can be written as :

$$\left.\begin{array}{l} x = A\cos\omega t \\ y = 0. \end{array}\right]$$

Thus, the emergent light is plane polarised having vibrations parallel to axis.

(ii) If $\theta = 90°$, then $a = 0$, $b = A$

Hence, energent light is plane polarised having vibrations perpendicular to optically axis just as incident light.

(iii) If $\theta = 45°$, then $a = b$ and equation (3) becomes

$$x^2 + y^2 = a^2 = \frac{1}{2}A^2$$

This shows that emergent light is circularly polarised.

If the light incident on the $\frac{\lambda}{4}$ plate, it is circularly polarized, the incident wave can be resolved into the plane polarised O and E component having path difference $\frac{\lambda}{4}$. The plate removes this path difference so that plane polarised light emerges.

• 5.9 PRODUCTION OF LIGHT (PLANE POLARISED)

To produce plane-polarised light, a beam of ordinary light is sent through a Nicol prism in a direction almost parallel to the long edge of the prism. Inside the prism, the beam is broken up into two components, O and E. The O-component is totally reflected at the Canada Balsam layer and is absorbed. The E-component emerges out. It is plane-polarised with its vibration parallel to the shorter diagonal of the end face of the Nicol.

Detection : To detect plane-polarised light, it is examined through another Nicol prism rotating about the direction of propagation of light. If the intensity of the emerging light varies with zero minimum, the light is plane-polarised.

Production of Circularly Polarised Light : The circularly polarised light can be produced by allowing plane polarised light obtained from a Nicol prism to fall normally on a quarter-wave plate such that the direction of vibration in the incident plane-polarised light makes an angle of 95° with the optic axis of the plate.

Inside the plate, the incident wave of amplitude A (say) is divided into an E component $A \cos 45°$ perpendicular to the optic axis. And an O-component $A \sin 45°$ perpendicular to the optic axis. These components emerge from the plate with a phase difference of $\frac{\pi}{2}$.

Let $A \cos 45° = A \sin 45° = a$. If the axis of x and y be taken along and perpendicular to the optic axis, then the emerging components can be written as

$$x = a \sin\left(\omega t + \frac{\pi}{2}\right) = a \cos \omega t \qquad \text{... (i)}$$

and $$y = a \sin \omega t.$$

Eliminating t between eq. (i) and (ii), the resultant vibration is

$$x^2 + y^2 = a^2,$$

which represents a circle. Hence, the light emerging from the $\frac{\lambda}{4}$-plate is circularly-polarised.

Detection : The circularly polarised light, when seen through a rotating Nicol prism, shows no variation in intensity. It thus resembles unpolarised light. Hence to confirm that the given light is circularly polarised it is first passed through a $\frac{\lambda}{4}$-plate (which converts it into plane-polarised light) and then through the rotaing Nicol prism. The light now shows a variation in intensity with zero minimum.

Production of Elliptically Polarised light : The elliptically polarised light can be produced by allowing plane-polarised light obtained from a Nicol prism to fall normally on a quarter-wave plate such that the direction of vibration in the incident plane-polarised light makes an angle other then 0°, 45° and 90° with the optic axis of the plate.

In this case, incident wave is divided inside the plate into E and O components of unequal amplitudes $A \cos \theta$ and $A \sin \theta$. The path difference of both of them is $\frac{\lambda}{4}$.

Let if $$a = A \cos \theta,\ b = B \sin \theta$$

from emerges components of waves.

$$\left.\begin{aligned} x &= a \cos \omega t \\ y &= b \sin \omega t \end{aligned}\right\}$$

or $$\frac{x^2}{a^2}+\frac{y^2}{b^2}$$ (which is elliptical equation)

Hence, the light emerging from the $\frac{\lambda}{4}$ plate is elliptically polarised.

Detection : When the elliptical light is incident on the rotating Nicol prism. And prism is adjusted for maximum intensity. The principal section of the Nicol prism is now parallel to the major axis for the elliptically vibration.

Another $\frac{\lambda}{4}$-plate is now introduced between the first plate (which produces the elliptically polarised light) and the Nicol prism such that the optic axis of the second plate is parallel to the principal section of the Nicol prism (which has been adjusted for maximum intensity). The optic axis of the plate thus becomes parallel to the major axis of the elliptic vibration. The light after passing through the second $\frac{\lambda}{4}$-plate becomes plane-polarised. If the Nicol prism be now rotated, the intensity will vary with zero minimum.

• 5.10 DISTINCTION BETWEEN UNPOLARISED LIGHT AND PARTIALLY PLANE-POLARISED LIGHT

The distinction can be made by passing the light through a rotating Nicol prism. The unpolarised light will not show any variation in intensity; it will appear of same intensity for all orientations of the Nicol. The partially plane-polarised light on passing through the rotating Nicol will, however, show an intensity variation but will never be completely cut off.

Distinction between Circularly polarised and Unpolarised Light : When circularly polarised light passed through a rotating Nicol prism, no variation in the intensity of the emergent light is observed. This is because inside the Nicol prism the circularly polarised light is broken up into two plane-polarised components of equal amplitudes, having vibrations parallel and perpendicular to the principal section of the Nicol prism. The perpendicular component is lost by total reflection, while the parallel component is transmitted. Since, the same happens in all positions of the Nicol prism, the intensity of the emergent light is always the same. For exactly the same reason an unpolarised light shows no variation in intensity on rotation of the Nicol prims. Thus, a rotating Nicol prism fails to distinguish between circularly polarised light and unpolarised light.

To distinguish between the two, the light is first made to pass normally through a quarter-wave plate, and then examined through the rotating Nicol prism. If the given light is circularly polarised, then on entering the quarter-wave plate it will be broken up into two components of *equal* amplitudes having a phase difference of $\pi/2$. The quarter-wave plate will introduce a further phase change of $\lambda.\ \pi/2$ so that the phase difference between the two components on emergence will be either 0 or π. In either case the emergent light will now be *plane-polarised*. Hence if this be examined through a rotating Nicol prism, the intensity will be a maximum when the principal section of the Nicol is parallel to the vibrations in the emergent plane-polarised light. Light will be completely cut off when the principal section of the Nicol prism is perpendicular to these vibrations. Hence, if the rotation of the Nicol prism shows variation in intensity with **zero minimum**, the given light is circularly-polarised.

If the given light be unpolarised, then on passing through the quarter-wave plate, it will remain unchanged. (This is because the unpolarised light has vibrations in all directions, never making a constant angle with the optic axis of the plate). Therefore when examined through a rotating Nicol prism, it shows no variation in interfere.

Distinction between Circulary polarised Light and a Mixture of Circularly-polarised and Unpolarised Light : The given light is first passed through a $\frac{\lambda}{4}$ plate and then through a rotating Nicol prism. If the light emerging from the Nicol prism varies in intensity with zero minima then the given light is circularly polarised. If it varies in intensity but with *non-zero* minima, then the given light is a mixture of circularly polarised light and unpolarised light.

If the given light is circularly polarised, it is converted into plane-polarised light by the $\frac{\lambda}{4}$ plate. Hence, it suffers complete extinction twice in each rotation of the Nicol prism. If the given light is a mixture of circularly polarised and unpolarised light, then on passing through the $\frac{\lambda}{4}$-plate, the circularly polarised light is converted into plane-polarised light while the unpolarised light still remains unpolarised. Thus, we get a mixture of plane-polarised and unpolarised light. This mixture, when examined through a rotating Nicol prism, shows variation in intensity but is never completely extinguished.

Distinguish between the Elliptical and Mixture of Plane and Unpolarised Light : To distinguish between the two, the light is first made to pass normally through a quarter-wave plate and then examined through the rotating Nicol. The quarter-wave plate is so placed that its optic axis is parallel to the principal section of the Nicol prism adjusted for maximum intensity, so that the optic axis is also parallel to the major axis of the elliptic vibration. If the given light is elliptically polarised, then on entering the quarter-wave plate it will be broken up into two *unequal* components vibrating perpendicular to each other and having a phase difference of $\frac{\pi}{2}$. The $\frac{\lambda}{4}$-plate introduces a further phase difference of $\frac{\pi}{2}$. Hence, the phase difference on emergence from the plate is either 0 or π. In either case, the light emerging from the plate is plane-polarised. Hence on being examined through the rotating Nicol prism, intensity will very *with zero minimum*. If the given light is a mixture of unpolarised light and plane-polarised light, it will remain as such when passed through the quarter-wave plate. Therefore, on being examined through the rotating Nicol prism, the intensity, will very with a *non-zero minimum*.

Distinction between Circularly Polarised Light and Eliptically Polarised Light : The given light is passed through a rotating Nicol prism. If the intensity of the transmitted light remains constant, the given light is circularly polarised. If, however, the intensity varies but never falls to zero, then the given light is elliptically polarised.

• 5.11 OPTICAL ROTATION

The property of rotating the plane of vibration of plane polarised light about its axis (direction) of travel by some crystal is called **as optical activity** and the phenomenon is called **optical rotation.** The angle through which plane of perpendicular is rotated is called as **angle of rotation.**

Fresnel's Explanation of Optical Rotation : It is the resultant of two opposite circular motions of the same frequency. Fresnel made the following assumptions :

(i) The incident polarised light on entering a substance is broken up into two circularly polarised waves, one clockwise and the other anti-clockwise.

(ii) In an optically-inactive substance, the two waves travel with the same velocity, but in an optically-active substance, they travel with different velocities. (In a dextro-rotatory substance the clock-wise wave travels faster, while in the leavo-rotatory substance the anti clockwise travels faster). Hence, a phase difference is developed between them as they traverse the substance.

(iii) On emergence, the two circular components recombine to form plane-polarised light whose plane of polarisation is rotated with respect to that of the incident light by an angle depending on the phase difference between them.

Suppose that plane-polarised light is incident normally on a quartz plate cut perpendicular to the optic axis. Let the first face of the plate be in the $x-y$ plane. Let the vibrations in the incident light be represented by

$$y = a \cos \omega t. \qquad \text{... (i)}$$

These vibrations just on entering the crystal, are broken up into two equal and opposite circular motions (fig. 19) which are represented by

$$\left.\begin{aligned} x_1 &= \frac{a}{2}\sin \omega t \quad \text{clockwise} \\ y_2 &= \frac{a}{2}\cos \omega t \quad \text{circular - motion} \end{aligned}\right\} \quad \text{... (ii)}$$

$$\left.\begin{aligned} x_2 &= -\frac{a}{2}\sin \omega t \quad \text{anti - clockwise} \\ y_2 &= \frac{a}{2}\cos \omega t \quad \text{circular - motion} \end{aligned}\right\} \quad \text{... (iii)}$$

These are circular components which are propagated by plates with different velocities, then phase differences between them.

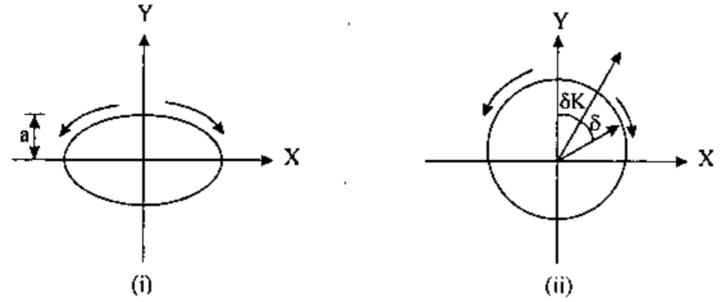


Fig. 19.

The emergent circular component is given by

$$\left.\begin{aligned} x_1 &= \frac{a}{2}\sin(\omega t + \delta) \\ y_1 &= \frac{a}{2}\cos(\omega t + \delta) \end{aligned}\right\} \quad \text{... (iv)}$$

The resultant displacement is given by

$$\left.\begin{aligned} x = x_1 + x_2 &= a\cos\left(\omega t + \frac{\delta}{2}\right)\sin\frac{\delta}{2} \\ y = y_1 + y_2 &= a\cos\left(\omega t + \frac{\delta}{2}\right)\cos\frac{\delta}{2} \end{aligned}\right\}$$

or $\quad \frac{x}{y} = \tan\frac{\delta}{2}$ (straight line equation)

if μ_1 and μ_2 are refractive indices of crystal in direction of axis anticlock and clockwise, circularly polarised light is the thickness of crystal plate, then phase difference is given by

$$\delta = \frac{2\pi}{\lambda}(\mu_1 - \mu_2)\,D$$

Hence, for rotation,

$$\theta = \frac{\delta}{2} = \frac{\pi}{2}(\mu_1 - \mu_2)\,D$$

$\therefore \quad \mu_1 = \frac{v}{v_1}, \mu_2 = \frac{v}{v_2}$

$\therefore \quad \theta = \left(\frac{\pi}{\lambda}\right) Dv \left(\frac{1}{v_1} - \frac{1}{v_2}\right)$

if $\quad v_1 > v_2$

$\therefore \quad \delta = -ve$

then substance is anticlockwise rotationary

if $\quad v_1 < v_2$

$\therefore \quad \delta = +ve$

Hence, clockwise rotationary

if $\quad v_1 = v_2,$

$\therefore \quad \delta = 0$

Hence, substance is optically inactive.

• 5.12 SPECIFIC ROTATION

The specific rotation of substance at a given temperature and for given a wavelength of light, is defined as the rotation (in degrees) produced by 1 decimetre length of substance when its concentration is 1 gm/cm^3. That is for a solution.

$$\text{Specific rotation} = \frac{\theta}{lC}$$

where θ is the angle of rotation in degrees, l is the length of the solution in decimetre and C is the concentration of the solution in gm/cm^3.

Laurent's half shade polarimeter : It is shown in fig. (20). Which consists of a source of light, a polariser and an analyser provided with a graduated circular scale C. A glass tube D, containing the optically active solution and closed on both side, by plane glass plates with metal caps, is mounted between the polariser and the analyser on a rigid iron base.

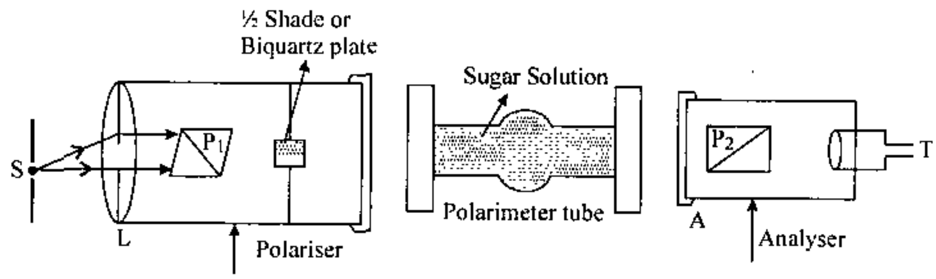


Fig. 20.

Monochromatic light from the source S (usually a sodium lamp) after passing through a slit is rendered parallel by a convex lens L and falls on a polarising Nicol prism P. After passing through P the polarised light passes through a half shade device H (or biquartz plate) and then through the tube D and then falls on the analysing Nicol prism (P). The emergent light is viewed through a telescope T. The analysing Nicol prism A can be rotated about the light as axis and its rotation can be measured by vernier moving over the fixed circular degree scale.

Working : Let us suppose for a moment that the half-shade device H is not present. The position of the Nicol prism P_2 is adjusted so that the field of view is completely dark when the tube D is empty. In this position, P_2 is crossed with respect to P_1. The reading of P_2 is taken. Now the tube D is filled with the experimental solution. This rotates the plane of polarisation of the light coming from P_1 through some angle, say θ, so that some light is transmitted by P_2. Now P_2 is rotated until the field of view again becomes dark and its reading is noted. This will happen when P_2 has been rotated through the angle θ in the direction of the optical rotation produced. Hence, the difference of the two readings of P_2 gives the rotation of the plane of polarisation.

Action of the 1/2 shade device : All the difficulty of Nicol to prism's removed by $\frac{1}{2}$ shade device H if immediately after the Nicol P_2, in fig. (21).

It is a combination of two half circles, plates ACA' and ADA'. The plate ACA' is of quartz and cuts it parallel to optic axis. When the ADA' is glass both are connected along AA'. The thickness of the quartz introduced a phase difference π of OE.

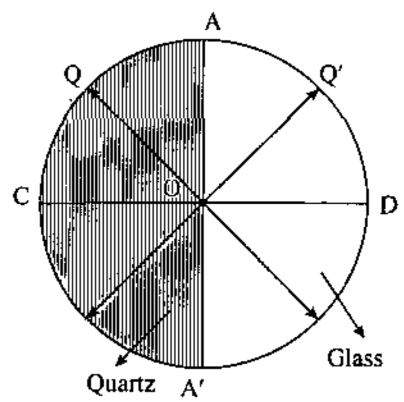


Fig. 21.

The plane-polarised light from P_1 falls normally on the half-shade plate. Supose this light has vibrations along OQ'. It is transmitted through glass-half as such and emerges with vibrations still along OQ'. Inside the quartz-half, however, the light is divided into two components, one E-component parallel to the optic axis (OA) of the quartz and the other O-component perpendicular to the optic axis, *i.e.*, along OD. The O-component travels faster in quartz. Therefore, on emergence, it gains a phase of π over the E-component. Hence, now the O-component has vibrations along OC, the E-component still having vibrations along OA. Therefore, the light emerging from quartz has resultant vibrations along OQ where $\angle AOQ = \angle AOQ'$.

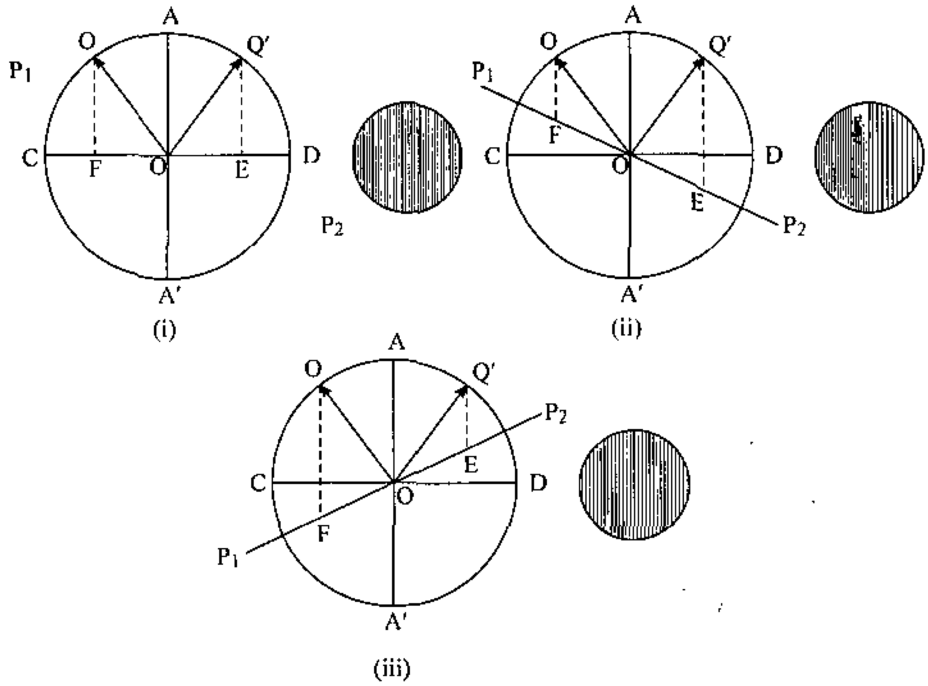


Fig. 22.

Now, if the principal section P_2OP_2 of the analysing Nicol P_2 is parallel to COD, both halves of the field appear slightly but *equally* illuminated. This is because the components OE and OF of the emergent vibrations OQ' and OQ along the principal section of P_2 are small but *equal*. If P_2 is slightly rotated from this position in the clockwise direction, the component OE appreciable decreases while OE appreciably increases. Hence, the right-half of the field becomes darker and the left-half brighter. Similarly, a slight rotation of P_2 in the anti-clockwise direction makes the right-half brighter and the left-half darker.

Determination of specific rotation : The Nicol P_2 is first adjusted to two halves of the field are slightly illuminated the sugar solution is then placed between the $\frac{1}{2}$ shade and P_2 and P_2 is again adjusted so that two halves of the field are equally illuminated. In IInd order, the solution rotates the plane of vibration of OQ' and OQ at [In fig. (23)] in same direction. When P_2 must be rotated further at θ in rotated direction. Hence, difference between positions of P_2 gives θ.

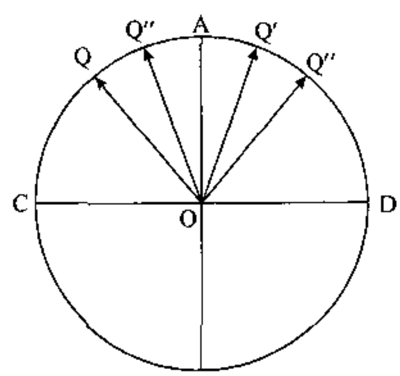


Fig. 23.

The length l of the tube containing the solution measured in d.m. the specific rotation of sugar solution is obtained

$$S = \frac{\theta}{l \times C}.$$

• 5.13 BIQUARTZ POLARIMETER

Principle : It works on the principle of rotatory dispersion. According to Biot's law, the angle of rotation $\theta \propto 1/\lambda^2$ approximately. When a beam of composite polarised light is incident normally on biquartz plate, the plane of polarisation for different wavelengths will be rotated through different extents, resulting the phenomenon of dispersion.

Construction : It consists of two semicircular plates of quartz cemented together to form a disc, each of the plates cut perpendicular to the optic axis, one piece is made from left handed quartz and the other from right handed.

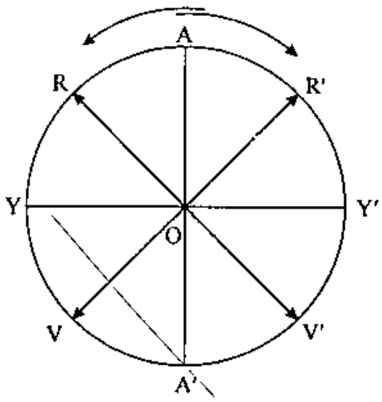


Fig. 24.

Working : It is a combination of two semicircular plates one of left-handed quartz and the other of right-handed quartz; both cut perpendicular to the optic axis and cemented together so as to form a complete circular plate. Each rotates the plane of polarisation of the yellow light through 90°, one anti-clockwise and the other clockwise. When *white* light, rendered plane-polarised by the polarising Nicol, travels through the biquartz normally, it is travelling along the optic axis. Therefore, rotatory dispersion occurs in each half plate of the biquartz. If the vibrations in the incident light are along *AM'* then, after passing through the biquartz, the vibrations of yellow light are along a direction perpendicular to *AA'*. The vibrations of red and blue lights are along different directions as shown in the figure. If the principal section of the analysing Nicol prism be parallel to *M'*, the yellow light will be quenched in both halves of the field, while the red and blue lights will be present in the same proportion in each half. Therefore, the two halves of the field will be equally-illuminated with a reddish-violet tint, **called the 'sensitive tint' or 'tint of the passage'**. Biquartz permits the use of white light and also determines the rotation for the yellow light, moreover, contrast in colour can be judged more accurately then in intensities, hence it is a more sensitive device.

• SUMMARY

- Polarization of light means departure from complete symmetry about the direction of propagation.
- A plane containing the optic axis and perpendicular to the two opposite faces of the crystal is called the principal section of the crystal.
- The diameter of the sphere is equal to the major axis of the ellipsoid such crystals are called positive crystals.
- The angle through which plane of perpendicular is rotated is called as angle of rotation.
- A principal section of crystal is a parallelogram having angles of 109° and 71°.

• STUDENT ACTIVITY

1. What do you mean by polarization of light ?

2. Distinguish between polarized and unpolarized light.

3. Define Brewster's angle.

4. What do you mean by doubly refracting crystal ?

5. Mention the uses of Nicol's prism.

6. What are quarter wave plate and half wave plate ?

7. Distinguish between polarized light and unpolarized light.

8. Distinguish between cricularly polarized and unpolarized light.

• TEST YOURSELF

1. What do you understand by double refraction ? What are ordinary and extraordinary rays in a uniaxial crystal ? How can you show that they are plane polarized ?
2. Give Huygen's postulates for double refraction of a uniaxial crystal. Discuss how you will locate the position in ordinary and extraordinary wavefronts in different cases.
3. Describe the construction and action of Nicol's prism.
4. Plane polarized light is normally incident on a quarter wave plate. Discuss the conditions under which elliptically polarized light, plane polarized light and circularly polarized light can be obtained.
5. Describe how, with the help of Nicol prism and $\frac{\pi}{4}$ plates polarized light are produced and detected.
6. What do you mean by optical rotation ? Give an outline of Fresnel's theory of optical rotation.
7. Define specific rotation. Describe the construction and working of a Laurent half shade polarimeter explaining fully the action of the half shade device and obtain the specific rotation of sugar solution.
8. Describe the working and principle of biquartz polarimeter. Give the advantages of biquartz over half shade polarimeter.
9. The principle of doubly refracting crystal is enunciated by :
   (a) Bartholinous (b) Huygen
   (c) Newton (c) None of these
10. The example of Biaxial crystal as :
   (a) Aragonite (b) Cacite
   (c) Both of (a) and (b) (d) None of these
11. The principle of Nicol prism is based upon :
   (a) Double refraction (b) Double interference
   (c) Both of (a) and (b) (d) None of these
12. What should be the thickness of a $\frac{\lambda}{4}$ plate, when the wavelength of light is equal to 5890 Å ?
   (If $\mu_0 = 1 \cdot 55, \mu_E = 1 \cdot 54$)
   (a) $1 \cdot 47 \times 10^{-3}$ cm (b) $1 \cdot 47 \times 10^{3}$ cm
   (c) $1 \cdot 47 \times 10^{-5}$ cm (d) None of these
13. What is the wavelength of light when the thickness of quarter wave plate is $1 \cdot 47 \times 10^{-3}$ cm ?
   (given that $\mu_0 = 1 \cdot 55, \mu_E = 1 \cdot 54$)
   (a) 5890 Å (b) 5896Å
   (c) 5899 Å (d) None of these

14. What is the refractive index of O-rays when the thickness of the $\frac{\lambda}{4}$ plate is $0 \cdot 0016$ cm ?
(given that $\mu_E = 1 \cdot 5533$, $\lambda = 5893$ Å)
(a) $1 \cdot 523$ (b) $1 \cdot 5442$
(c) $1 \cdot 5042$ (d) None of these

15. What is the thickness of doubly-refracting crystal plate required to introduced a path difference of $\frac{\lambda}{2}$ between the **O and E** rays when $\lambda = 5000$ Å, $\mu_0 = 1 \cdot 55$ and $\mu_E = 1 \cdot 54$?
(a) $2 \cdot 5$ Å (b) $2 \cdot 0$ Å
(c) $1 \cdot 5$ Å (d) None of these

16. What is the refractive index of $\frac{\lambda}{2}$ calcite crystal plate, when the thickness of the $\frac{\lambda}{2}$ plate is $0 \cdot 003$ cm ?
(If $\mu_0 = 1 \cdot 55$, $\lambda = 6000$ Å)
(a) $1 \cdot 50$ (b) $1 \cdot 54$
(c) $1 \cdot 58$ (d) None of these

17. The example of positive crystal is :
(a) Calcite (b) Quartz
(c) Aragonite (d) None of these

18. When a given polarised light is passed through a rotating Nicol, if the intensity of the transmitted light remains constant, the given polarised light as :
(a) Plane (b) Circular
(c) Unpolarised (d) Elliptical

19. When a given polarised light is passed through a rotating Nicol prism and a $\frac{\lambda}{4}$-plate. if the intensity of the transmitted light is varied with zero minima, then the given polarised light as :
(a) Plane (b) Circular
(c) Unpolarised (d) None of these

20. In question no. 19, if the intensity of the transmitted light is not varied then the given light as :
(a) Plane Polarised (b) Circular polarised
(c) Unpolarised (d) None of these

21. In above question 18th, if the intensity of transmitted light varies but never falls to zero, then given polarised light is :
(a) Plane (b) Circular
(c) Unpolarised (d) Elliptical

22. The optical rotation θ of any solution is given by :
(a) $\theta = SCl$ (b) $S = \theta, Cl$
(c) $\theta = \frac{S}{Cl}$ (d) None of these

23. A sugar solution in a tube of length 20 cm produces optical rotation of 13°. The solution is then diluted to $\frac{1}{3}$ of its previous concentration. What should be the optical rotation produced by 30 cm long tube containing the dilute solution ?
(a) $5 \cdot 5°$ (b) $6 \cdot 5°$
(c) $7 \cdot 5°$ (d) None of these

24. A sugar solution in a tube of length 10 cm produces optical rotation of 15°. The solution is then diluted to $\frac{1}{3}$ of its previous concentration, what should be the optical rotation produced by 20 cm long tube, containing the dilute solution ?

(a) 10° (b) 20°
(c) 30° (d) None of these

25. The specific rotation of quartz at 6000 Å is 27 deg/mm. What is the difference in the refractive indices ?
(a) 9×10^{-5} (b) 8×10^{-5}
(c) 7×10^{-5} (d) None of these

26. The rotation of the plane of polarisation of light ($\lambda = 5893$ Å) in a material of 10°/cm. What is the phase differences in the refractive indices of the material for clockwise and anticlockwise circularly-polarised light ?
(a) $3 \cdot 274 \times 10^{-7}$ (b) $3 \cdot 20 \times 10^{-7}$
(c) $2 \cdot 274 \times 10^{-7}$ (d) $3 \cdot 274 \times 10^{7}$

27. If the difference in the refractive indices is $8 \cdot 4 \times 10^{-5}$ and specific roration of quartz is $29 \cdot 73$ deg/mm, what is the value of λ of light ?
(a) 5086 Å (b) 5076 Å
(c) 6086 Å (d) 5068 Å

28. What should be value of the specific rotation of quartz at 6000 Å ? While given that difference between the refractive indices is $9 \cdot 0 \times 10^{-5}$?
(a) 27°/mm (b) 26°/mm
(c) 25°/mm (d) None of these

29. The optical rotation is measured of certain substances by :
(a) Laurent 1/2 shade polarimeter (b) Laurent full-shade polarimeter
(c) Both of (a) and (b) (d) None of these

30. The rotation of plane of vibration is defined in mathematical form by :
(a) $\theta = \frac{\pi}{\lambda}(\mu_A - \mu_B)$ (b) $\theta = \frac{\lambda d}{\lambda}(\mu_B - \mu_A)$
(c) $\theta = \frac{\pi d}{\lambda}(\mu_A - \mu_B)$ (d) None of these

ANSWERS

9. (a) 10. (a) 11. (a) 12. (a) 13. (a)S 14. (b) 15. (a) 16. (b) 17. (b)
18. (a) 19. (b) 20. (c) 21. (d) 22. (b) 23. (b) 24. (a) 25. (a) 26. (a)
27. (a) 28. (a) 29. (a) 30. (c)