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SYLLABUS

ELECTRICITY AND MAGNETISM

SC-104

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UNIT 1

ELECTROSTATICS

STRUCTURE

- Coulomb's Law
- Continuous Charge Distribution
- Principle of Superposition
- Electric Intensity
  - Student Activity
- Electric Flux
- Electric Flux Through a Cylinder
- Gauss's Law
- Electric Field of a Uniformaly Charged Sphere
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  - Student Activity
- Electric Polarisation of Matter
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LEARNING OBJECTIVES

After going this unit you will learn :

- Explanation of Coulomb's law in vector form and its importance.
- Linear, surface and volume charge distributions.
- Electric field due to different things.
- Polar Molecules, non-polar molecules and ferro electrics.
- Electric polarisation, displacement and susceptibility.
- Electric flux and Gauss's law.
- Molecular polarisability and Langevin-Debye relation.
- Electronic, ionic and orientation polarisability.

1.1. COULOMB'S LAW

According to this law, *"Two point charges attract or repel each other with a force which is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them"*.

The force is replusive, if the charges are like and attractive in case of unlike charges and this electrostatic force between two charges is central in nature. Coulomb's law in electrostatic holds for stationary charges and the two charges should be points in size.

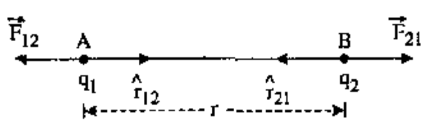


Fig. 1

Coulomb's Law in Vector Form

Let us consider two like charges q_1 and q_2 at points A and B in vacuum at a distance r [Fig. (1)]. The two charges will exert equal repulsive forces on each other.

Let $\vec{F}_{12}$ be the force on charges q_1 due to charge q_2 and $\vec{F}_{21}$ be the force on charge q_2 due to charge q_1 then according to Coulomb's law, the magnitude of force on charge q_1 due to q_2 (or on charge q_2 due to q_1) is given by

$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \quad \ldots (1)$$

Let $\hat{r}_{12}$ be a unit vector pointing from charge q_1 to q_2 and $\hat{r}_{21}$ is a unit vector pointing from charge q_2 to q_1.

As the force $\vec{F}_{21}$ is along the direction of unit vector $\hat{r}_{12}$ then force on q_2 due to q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} \quad \ldots (2)$$

Again as the force vector $\vec{F}_{21}$ is along the direction of unit vector $\hat{r}_{12}$, then force on q_1 due to q_2

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{21} \quad \ldots (3)$$

The equations (2) and (3) are required form of Coulomb's law in vector form.

Importance of Coulomb's Law in Vector Form

Coulomb's law in vector form is more informative than in its scalar form for the following reasons :

(1) In vector form, Coulomb's law shows that the force $\vec{F}_{21}$ and $\vec{F}_{12}$ are equal and opposite. Since $\hat{r}_{12}$ and $\hat{r}_{21}$ are the unit vectors pointing in opposite directions so, we have

$$\hat{r}_{21} = -\hat{r}_{12}$$

Put this value in equation (2), then we get

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}(-\hat{r}_{12}) = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{r}_{12} \quad \ldots (4)$$

From equations (3) and (4), we get

$$\vec{F}_{12} = -\vec{F}_{21} \quad \ldots (5)$$

(2) From Coulomb's law in vector form, it is clear that the electrostatic force between two charges is a **central force.** It act along the line joining two charges.

From equations (3) and (4), it is clear that the force experienced by one charge due to the other acts in the direction of unit vector $\hat{r}_{12}$ or in the direction opposite to that of $\hat{r}_{12}$. Since $\hat{r}_{12}$ is a unit vector along the line joining the two charges, the electrostatic force between two charges is a central force.

• 1.2. CONTINUOUS CHARGE DISTRIBUTION

Any charge which covers a space with dimensions much less than its distance from an observation point is assumed to be a point charge.

A system of closely spaced charges is said to form a continuous charge distribution. It does not mean that electric charge is continuous or charge is no longer discrete. It only means that distribution of discrete charges is continuous with small space between the charges.

Continuous charge distribution are of three types :

(i) Linear charge distribution : When the charge is distributed uniformly along a line *e.g.*, a straight line or circumference of a circle.

(ii) Surface charge distribution : When the charge is distributed continuously over same area (*e.g.*, a membrane).

(iii) Volume charge distribution : When the charge is continuously distributed over a volume (*e.g.*, a sphere or a cube). It is represented in terms of ρ: where $\rho = \frac{dq}{dV}$. It is measured in cm^{-3}.

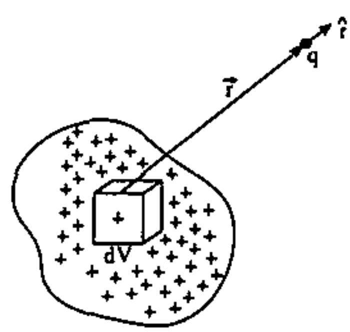


Fig. 2

Now we will obtain the expression for the total force F_q on a test charge q due to continuous volume charge distribution.

A volume charge density ρ is defined as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} \quad \text{... (1)}$$

Similarly a surface charge density σ may be defined as "the limit of charge per unit surface area as the area becomes infinitesimal" *i.e.*,

$$\rho = \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} \quad \text{... (2)}$$

where ρ and σ are net charge densities.

Now the force acting on a point charge q due to a continuous charge distribution

$$\vec{F}_q = \frac{q}{4\pi\varepsilon_0}\int \frac{dq}{r^3}\cdot\vec{r} = \frac{q}{4\pi\varepsilon_0}\int \frac{\vec{r}}{r^3}\,dq \quad \text{... (3)}$$

where dq = charge on a small element of charge distribution at a distance $\vec{r}$ from q.

In above expression integration sign means that all volume and surface charge distribution.

If a charge is distributed through a volume V with a density ρ on the surface S which bounds V with a density then we may write

$$\int dq = \int_V \rho\, dV + \int_S \sigma\, dS$$

so by equation (3), we get

$$\vec{F}_q = \frac{q}{4\pi\varepsilon_0}\int_V \frac{\vec{r}}{r^3}\rho(\vec{r})dV + \frac{q}{4\pi\varepsilon_0}\int_S \frac{\vec{r}}{r^3}\sigma(\vec{r})dS \quad \text{... (4)}$$

This is the required expression for the force between a point charge and a continuous charge distribution.

If the point charge q is located a point $\vec{r}$ instead of origin the equation (4) reduce to

$$\vec{F}_q = \frac{q}{4\pi\varepsilon_0}\int_V \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}\rho(\vec{r}')\,dV' + \frac{q}{4\pi\varepsilon_0}\int_S \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\sigma(\vec{r}')\,dS' \quad \text{... (5)}$$

Here the variable $\vec{r}'$ is used to locate a point with in the charge distribution *i.e.*, it plays the role of source point $\vec{r}_i$.

• 1.3. PRINCIPLE OF SUPERPOSITION

We know Coulomb's law gives the force between two charges. But when there are more than two charges then the principle of superposition is used to determine the force on a charge due to other charges.

According to this principle *"when a number of charges are interacting then the total force on a given charge is the vector sum of the individual forces exerted on the given charge by all the other charges".*

The force between two charges is not affected by the presence of other charges.

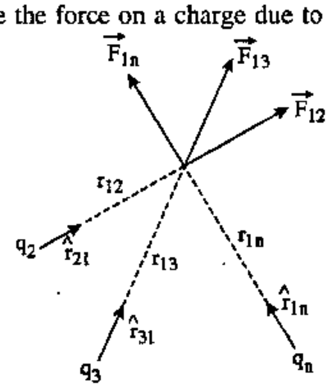


Fig. 3

Let us consider n point charges $q_1, q_2, q_3, \ldots\ldots q_n$ are distributed in space in a discrete manner and the charges are interacting with each other. We have to calculate the total force on charge q_1 due to other remaining charges.

Let the charges $q_1, q_2, \ldots\ldots q_n$ exert forces $\vec{F}_{12}, \vec{F}_{13}, \ldots\ldots, \vec{F}_{1n}$ on charge q_1 then according to the principle of superposition the total force on q_1 is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1n} \quad \text{... (1)}$$

Let $\vec{r}_{12}$ be the distance between q_1 and q_2 then

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}^2}\hat{r}_{21}$$

where $\hat{r}_{21}$ = unit vector from charge q_2 to q_1.

Similarly, the force on charge q_1 due to other charges is

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{r_{13}^2} \hat{r}_{31}$$

....

....

$$\vec{F}_{1n} = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{q_1 q_n}{r_{1n}^2} \hat{r}_{n1}$$

Put these values in equation (1), then we get

$$\vec{F}_1 = \frac{1}{4\pi\,\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{31} + \ldots\ldots \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{n1}\right] \quad \ldots (2)$$

in the same way we can find force on charge q_2 due to all other charges, *i.e.*,

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \ldots\ldots + \vec{F}_{2n}$$

so

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2 q_1}{r_{21}^2}\hat{r}_{12} + \frac{q_2 q_3}{r_{23}^2}\hat{r}_{32} + \ldots\ldots + \frac{q_2 q_n}{r_{2n}^2}\hat{r}_{n2}\right]$$

• 1.4. ELECTRIC INTENSITY

The electric field intensity at a point due to a source charge may be defined as "the force experienced by a unit charge placed at that point without disturbing the source charge. The electric field intensity is also called ***strength of electric field.***

Let us consider a positive charge q_0 experiences a force $\vec{F}$, which is placed at a point at which the electric field is to be determined.

The electric field at this point is

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

This is the required expression for the electric field intensity.

In other words, "Electric intensity at a point due to a source charge may be defined as the force experienced per unit +ve charge on a small positive charge placed at that point.

If $\vec{F}$ is a force experienced by positive charge q_0 placed at any point. Then the electric field at that point is

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

The limit $q_0 \to 0$ means that on placing the charge at the observation point, the source charge will not be disturbed.

Electric field is a vector quantity and its unit is **newton coulomb^{-1}** or volt meter^{-1}.

Electric intensity due to a continuous charge distribution : The force on test charge q_0 at position $\vec{r}$ relative to a point charge q is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q q_0}{r^3} \vec{r}$$

so the electric field intensity at point $\vec{r}$ due to q is

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{q}{r^3} \vec{r}$$

then the force on a charge q_0 at point $\vec{r}$ due to n point charges $q_1, q_2 \ldots\ldots q_n$ which are placed at $\vec{r}_1, \vec{r}_2 \ldots\ldots \vec{r}_n$ respectively is

$$\vec{F} = \frac{q_0}{4\pi\,\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

so the electric field intensity is

$$\vec{E} = \lim_{q_0 \to 0} \frac{1}{4\pi\,\varepsilon_0} \sum_{i=1}^{n} q_i \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3}$$

Now let us consider a continuous charge distribution of volume charge density $\rho(\vec{r})$ in volume V and surface charge density $\sigma(\vec{r})$ on the surface S which bounds V.

If a test charge q_0 is placed at point $\vec{r}$ then the force acting on it is

$$\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\,\rho(\vec{r'})\,dV' = \frac{q_0}{4\pi\varepsilon_0}\int_S \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\,\sigma(\vec{r'})\,dS'$$

since $\vec{E} = \frac{\vec{F}}{q_0}$ so electric field strength at $\vec{r}$ is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=0}^{n} q_i \frac{\vec{r}-\vec{r_i}}{|\vec{r}-\vec{r_i}|^3} + \frac{1}{4\pi\varepsilon_0}\int_V \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\,\rho(\vec{r'})\,dV' + \frac{1}{4\pi\varepsilon_0}\int_S \frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|^3}\,\sigma(\vec{r'})\,dS'$$

This is the general expression for the electric field intensity. In most cases one or more of the terms will not be needed. This formula can be used to determine the electric field intensity at each point in the space in the vicinity of a system of a charge distribution. Thus $\vec{E} = \vec{E}(\vec{r})$ is a vector point function or vector field.

• STUDENT ACTIVITY

1. What is the importance of expressing Coulomb's law in vector form ?

2. State the principle of superposition of charges.

• 1.5. ELECTRIC FLUX

"The electric flux through a surface held inside an electric field represents the total number of electric line of forces passing through the surface in a direction normal to the surface."

Electric flux is a scalar quantity and it is denoted by ϕ.

Suppose that a surface having area S is placed inside an electric field of intensity $\vec{E}$ as shown in the fig. 4.

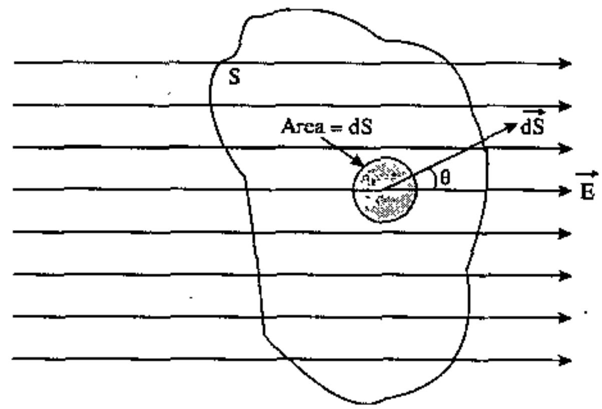


Fig. 4

Here we have to calculate the electric flux through the surface of area S. For this let us consider a small area dS of the surface S. The elementary area dS can be represented by a vector $\vec{dS}$ which is directed along normal to the area element dS. Let electric field $\vec{E}$ makes an angle θ with $\vec{dS}$ then component of electric field along normal to the dS *i.e.*, along area vector dS is

$$E_n = E \cos\theta$$

Hence electric flux passing through the area dS in a direction along normal is

$$d\phi = E_n\, dS = (E\cos\theta)\, dS$$

or

$$d\phi = \vec{E}\cdot\vec{dS}$$

The electric flux through the whole area is

$$\boxed{\phi = \int_S \vec{E}\,.\,\vec{dS} = \int_S E_n\, dS}$$

Thus, electric flux linked with a surface in an electric field may be defined as the surface integral of the electric field over that surface.

The unit of electric flux is $Nm^{-2}\,C^{-1}$.

• 1.6. ELECTRIC FLUX THROUGH A CYLINDER

Let us consider a cylinder of radius R with its axis parallel to a uniform electric field $\vec{E}$.

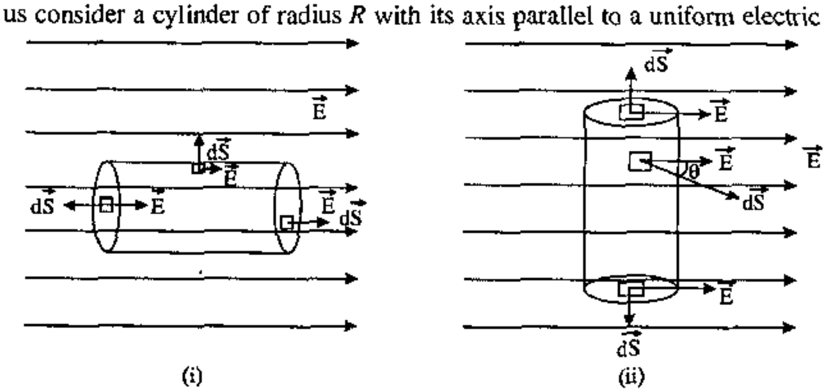


Fig. 5

The electric flux through the whole of the cylinder is the sum of the fluxes passing through the surface through the right plane surface and through the cylindrical surface. That is

$$\Phi = \int_S \vec{E}\,.\,\vec{dS}$$

$$= \underbrace{\int \vec{E}\,.\,\vec{dS}}_{\text{left plane surface}} + \underbrace{\int \vec{E}\,.\,\vec{dS}}_{\text{right plane surface}} + \underbrace{\int \vec{E}\,.\,\vec{dS}}_{\text{cylindrical surface}}$$

Consider one patch on each of these surface, fig. 5 (i). The angle between $\vec{E}$ and $\vec{dS}$ is 180° for all patches on the left surface, 0° for all patches on the right surface and 90° for all patches on the cylindrical surface. Thus

$$\Phi = \int E dS \cos 0° + \int E\, dS \cos 180° + \int E\, dS \cos 90°$$

$$= E \int dS - E \int dS + 0 = 0.$$

The electric flux through the whole cylinder immersed in field $\vec{E}$ is zero.

When the cylinder is placed with its axis perpendicular to the field, fig. 3 (ii), the angle between $\vec{E}$ and $\vec{dS}$ will be 90° for all patches on the plane surfaces so that their contribution to the electric flux is zero. The angle between $\vec{E}$ and $\vec{dS}$ is different for different patches on the cylindrical surface. For a particular patch for which the angle is θ, there is also an opposite patch the angle for which is (180° – θ). Thus the patches on the cylindrical surface mutually cancel and the net electric flux is zero. So, the electric flux through the cylinder placed perpendicular to the field is also zero.

• 1.7. GAUSS'S LAW

According to this law, *the electric flux Φ through any closed hypothetical surface of any shape drawn in an electric field is equal to* $\frac{1}{\varepsilon_0}$ *times the net charge enclosed by the surface,* that is

$$\Phi = \oint \vec{E}\,.\,\hat{n}\, dS = \frac{q}{\varepsilon_0}$$

where $q = q_1 + q_2 + q_3 + \ldots\ldots$ is the total net charge inside the closed surface.

The charges outside the closed surface do not contribute to the total flux. This hypothetical closed surface is called *Gaussian Surface.*

Proof : Consider a charge $+q$ inside an arbitrary closed surface S. Let dS is a small patch of area. It is surrounding a point P on the surface and $OP = r$.

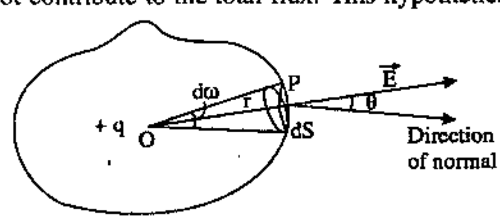


Fig. 6

If $\vec{E}$ is the electric field intensity at P due to charge $+q$ at O (along OP), then the electric flux through patch of area dS is given by

$$d\Phi = \vec{E}\,.\,\vec{dS} = E\, dS \cos\theta$$

where θ is the angle between the vectors $\vec{E}$ and $\vec{dS}$.

But the electric field intensity at P is,

$$E = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q}{r^2}.$$

$$\therefore \qquad d\Phi = \frac{q}{4\pi\,\varepsilon_0}\cdot\frac{dS\cos\theta}{r^2}.$$

But $\frac{dS\cos\theta}{r^2} = d\omega$, the solid angle subtended by the patch dS at O.

$$\therefore \qquad d\Phi = \frac{q}{4\pi\,\varepsilon_0}\, d\omega.$$

Hence, the total outward electric flux Φ through the surface S, is

$$\Phi = \frac{q}{4\pi\,\varepsilon_0}\oint d\omega.$$

As $\oint d\omega = 4\pi$, the solid angle subtended by the entire closed surface S at O, therefore

$$\Phi = \frac{q}{4\pi\,\varepsilon_0}(4\pi) = \frac{q}{\varepsilon_0}.$$

If there are several charges $+q_1, +q_2, +q_3, -q_4, -q_5$ etc. For positive charges, the electric flux is taken to be positive, while for negative charges, the electric flux is taken to be negative. Therefore, the electric flux through the entire surface is

$$\Phi = \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3 - q_4 - q_5 \ldots\ldots) = \frac{1}{\varepsilon_0}\Sigma q.$$

If the charge q is outside the surface, the total electric flux through the surface is zero. It is so because the cone with vertex at q cuts off areas dS_1, dS_2, dS_3, dS_4, at P, Q, R and S respectively. The flux through areas dS_2 and dS_4 is positive (outward drawn normal) while through areas dS_1 and dS_3 the electric flux is negative (inward drawn normal).

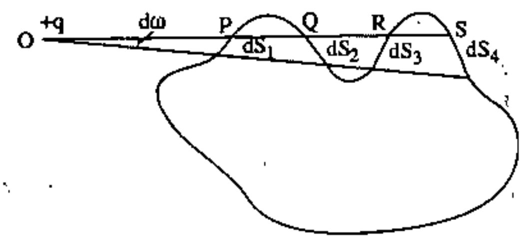


Fig. 7

Electric flux through area $dS_1 = -\frac{q}{4\pi\varepsilon_0} d\omega.$

Electric flux through area $dS_2 = \frac{q}{4\pi\varepsilon_0} d\omega$

Electric flux through area $dS_3 = -\frac{q}{4\pi\varepsilon_0} d\omega.$

Electric flux through area $dS_4 = \frac{q}{4\pi\varepsilon_0} d\omega.$

Hence the total electric flux through the above areas is zero.

Similarly, we can prove that the electric flux will be zero for any of the cone drawn at O through the closed surface. Hence the total electric flux through a closed surface due to external charges is zero.

Coulomb's Law from Gauss's Law

Consider an isolated positive point charge q at O. Now draw a Gaussian sphere S of radius r with q as centre. By symmetry, electric field $\vec{E}$ at any point on the surface of the sphere is along the outward normal at that point. Also it has the same magnitude at every point on the surface.

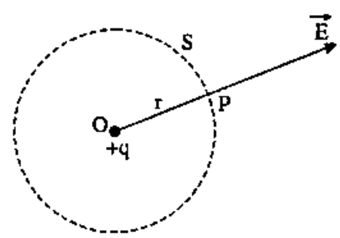


Fig. 8

For any patch on the surface, the vector $\vec{E}$ and $\vec{dS}$ both are along the same direction *i.e.*, angle between them is 0°.

$\therefore \qquad \vec{E}\cdot\vec{dS} = E\,dS\cos\theta = E\,dS\cos 0 = E\,dS$

The electric flux through Gaussian surface is given by

$$\phi = \oint \vec{E}\cdot\vec{dS} = \oint E\,dS$$

But E is constant *i.e.*,

$$\phi = E\oint dS$$

$\oint dS = 4\pi r^2$, Area of the sphere.

$\therefore \qquad \phi = E\,(4\pi r^2)$

According to Gauss's law

$$\phi = E\,(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

It is the magnitude of the electric field strength at a point. If the test charge q_0 is placed, the magnitude of the force experienced by the test charge will be given by

$$F = E\,q_0 = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q q_0}{r^2}$$

Thus Gauss's Law is equivalent to Coulomb's Law.

• 1.8. ELECTRIC FIELD OF A UNIFORMALY CHARGED SPHERE

(i) At external point $(r > R)$: Let us consider a sphere of radius R with centre O. Let a charge q is distributed uniformly on the sphere. Let P be the external point at distance r from O at which the electric field intensity is to be determined.

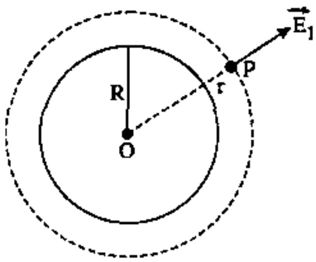


Fig. 9

Now draw a spherical surface through P. This is known as Gaussian spherical surface.

Since $\vec{E_1}$ and vector $\vec{dS}$ are radially outward on the surface. This means that the angle between $\vec{E_1}$ and $\vec{dS}$ is zero.

$$\therefore \qquad \vec{E_1}\cdot\vec{dS} = E_1\,dS\cos 0^\circ = E_1\,dS$$

Hence the electric flux through the Gaussian surface is

$$\phi = \oint \vec{E_1}\,\vec{dS} = \int_S E_1\,dS$$

$$= E_1 \oint_S dS \qquad [\because\ E_1 \text{ is constant}]$$

$$= E_1\,(4\pi\,r^2) \qquad \because \oint_S dS = 4\pi\,r^2$$

But according to Gauss's law, the electric flux is equal to $\frac{1}{\varepsilon_0}$ times the total charge q *i.e.*,

$$\phi = E_1\,(4\pi\,r^2) = \frac{q}{E^0}$$

or

$$\boxed{E_1 = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q}{r^2}}$$

which is same as a point charge q were placed at O. Thus the electric field intensity due to a uniformly charged sphere at an external point is the same as if the entire charge it were concentrated at the centre of the sphere.

(ii) At an internal point $(r < R)$: Let the point inside the sphere at the distance r from O. Now draw a spherical surface through P.

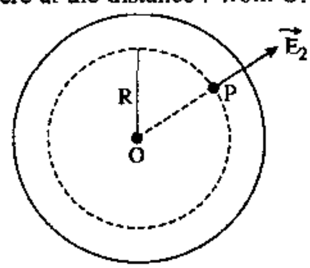


Fig. 10

Electric flux for the Gaussian surface is

$$\phi = E_2\,(4\pi\,r^2)$$

by Gauss law

$$\phi = E_2\,(4\pi\,r^2) = \frac{1}{\varepsilon_0}\,(q)$$

$$\boxed{E_2 = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q}{r^2}}$$

For conducting sphere the charge inside the sphere is zero so

$$E_2 = 0$$

But for nonconducting sphere the charge will be distributed throughout its entire volume.

According to Gauss law the electric flux around the Gaussian surface is $\phi = \frac{q'}{\varepsilon_0}$ where q' is the part of charge q.

$$\therefore \qquad \phi = E_2\,(4\pi\,r^2) = \frac{q'}{\varepsilon_0}$$

$$E_2 = \frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q'}{r^2}$$

As the sphere is uniformly charged, therefore the charge density ρ is constant throughout the sphere.

$$\rho = \frac{q}{\frac{4}{3}\pi R^3} = \frac{q'}{\frac{4}{3}\pi r^3}$$

or
$$q' = q\left(\frac{r}{R}\right)^3$$

∴
$$\boxed{E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qr}{R^3}}$$

Thus the electric field intensity due to uniformly charged sphere at an internal point is proportional to the distance r of the point from the sphere.

(iii) At the surface : $(R = r)$ In this position P lies on the surface of the sphere then $r = R$ so electric field intensity becomes

$$\boxed{E_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}}$$

• 1.9. ELECTRIC FIELD DUE TO AN INFINITELY LONG CHARGED CYLINDER

(i) At an External Point $(r > R)$: Fig. 11 shows a uniformly charged positive cylinder of infinite length. It is having a constant linear charge density λ (charge per unit length). Let P_1 is a point, distant r from the axis of the cylinder at which the electric intensity. $\vec{E_1}$ is to be determined. We draw a coaxial Gaussian cylindrical surface of length l and radius r through P_1. By symmetry, the magnitude $\vec{E_1}$ of the field intensity will be the same at all points on the cylindrical surface, and direction radially outward. Thus, for any path taken on the cylindrical surface, field vector $\vec{E_1}$ and the area vector $\vec{dS}$ both are parallel (radially outward). Therefore

$$\vec{E_1} \cdot \vec{dS} = E_1\, dS \cos 0° = E_1\, dS$$

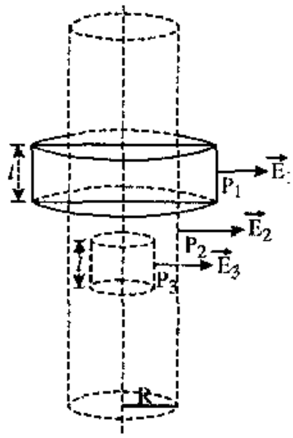


Fig. 11

The electric flux through the cylindrical surface is

$$\Phi = \int_S \vec{E_1} \cdot \vec{dS} = \int_S E_1\, dS$$

$$= \vec{E_1} \int_S dS \quad (\because \vec{E} \text{ is constant})$$

$$= \vec{E}_1\, (2\pi rl).$$

The electric flux through each of the plane surfaces is zero because $\vec{E_1}$ and $\vec{dS}$ are perpendicular to each other everywhere on these surfaces $(\vec{E_1}\, \vec{dS} = E_1\, dS \cos 90° = 0)$.

∴ The total electric flux through the Gaussian surface is

$$\Phi = E_1\, (2\pi rl).$$

According to Gauss's law, this must be equal to $\frac{1}{\varepsilon_0}$ times the charge contained within the Gaussian surface.

Charge contained within the Gaussian surface $= l\lambda$.

∴
$$\Phi = E_1\, (2\pi rl) = \frac{1}{\varepsilon_0}(l\lambda)$$

or
$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda}{r}.$$

The direction of $\mathbf{E}_1$ is radially outward for a positive charge.

(ii) At Surface of Charged Cylinder $(r = R)$: P_2 is such a point at which the field intensity is $\vec{E_2}$ and the electric flux is given as

$$\Phi = E_1\, (2\pi Rl) = \frac{1}{\varepsilon_0}(l\lambda)$$

$$\therefore \qquad \vec{E_2} = \frac{\lambda}{2\pi\,\varepsilon_0 R} = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{2\lambda}{R}.$$

(iii) At an Internal Point ($r > R$) : P_3 is such a point at which the field intensity is $\vec{E_3}$. Consider coaxial cylindrical surface of length l and radius r through P_3.

The electric flux through the Gaussian surface is

$$\Phi = E_3\,(2\pi\,rl)$$

The charge contained within the Gaussian surface is

$$= (\pi r^2 l)\,\rho,$$

where ρ is the charge density of the cylinder and

$$\rho = \frac{\lambda}{\pi R^2}$$

$$\therefore \qquad \Phi = E_3\,(2\pi rl) = \frac{1}{\varepsilon_0}\,(\pi r^2 l\rho)$$

or

$$E_2 = \frac{1}{4\pi\,\varepsilon_0}\,(2\pi\,r\rho) = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{2\lambda r}{R^2}.$$

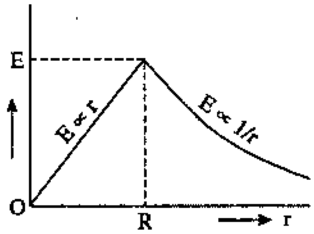


Fig. 12

The variation of electric intensity with distance r is shown in fig. 12.

• 1.10. ELECTRIC FIELD DUE TO AN INFINITE PLANE SHEET OF CHARGE

Fig. 13 shows a thin, non-conducting plane sheet of charge, infinite in extent, and having a surface charge density (charge per unit area) σ. Let A is a point, distant r from the sheet, at which the electric intensity is to be determined.

Let us choose a point B symmetrical with A, on the other side of the sheet. We draw a Gaussian cylinder cutting through the sheet, with its plane ends parallel to the sheet and passing through A and B. Let a is the area of each plane end.

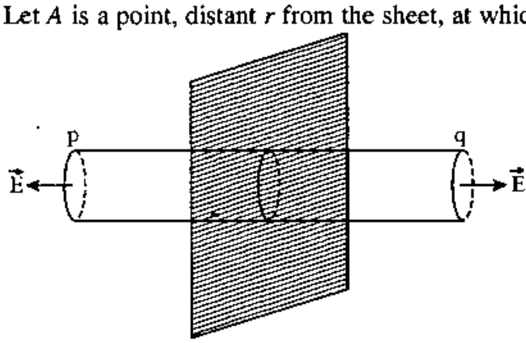


Fig. 13

By symmetry, the electric intensity at all points on either side near the sheet (charged positively) will be perpendicular to the sheet, directed outward. Thus $\vec{E}$ is perpendicular to the plane ends of the cylinder and parallel to the curved surface. Its magnitude E will be the same at A and B and direction outward.

$\therefore$ The electric flux through the two plane ends is

$$\Phi = \oint_S \vec{E}\,.\,\vec{dS} + \oint_S \vec{E}\,.\,\vec{dS}$$

$$= \oint_S E\,dS + \oint_S E\,dS \qquad (\because \cos\theta = 1)$$

$$= E\int_S dS + E\int_S dS$$

$$= EA + EA = 2EA. \qquad \left(\because \int_S dS = A\right)$$

Here electric field through the cylindrical surface is zero because $\vec{E}$ and $\vec{dS}$ are perpendicular to each other. Therefore the total electric flux through the Gaussian cylinder is

$$\phi = 2EA$$

According to Gauss's law

$$\phi = 2EA = \frac{\sigma A}{\varepsilon_0} \quad \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

This is the desired result.

• STUDENT ACTIVITY

1. What is electric flux ?

2. What is Gaussian surface ?

3. What is the importance of Gauss's theorem ?

• 1.11. ELECTRIC POLARISATION OF MATTER

We know that every atom is made of positively charged nuclei and around which negatively charged electrons are revolving in different orbits. In atoms because of their spherical symmetry, the centre of mass of electrons coincide with the nucleus. So atoms do not have permanent electric dipole moments but when the atoms are placed in the electric field then they acquire an induced electric dipole moment in the direction of the field. *This process is known as electric polarisation and the atoms are called polarised.*

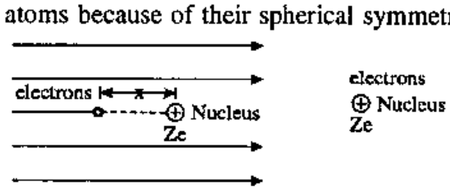


Fig. 14

Polar Molecules

The polar molecules are those which have a permanent electric dipole moment. For example, HCl, CO, H_2O are polar molecules.

In HCl molecules, the electrons of H atom spends more time moving around the Cl atom than around the H atom so the centre of negative charges do not coincide with the centre of positive charges. Thus the molecule has

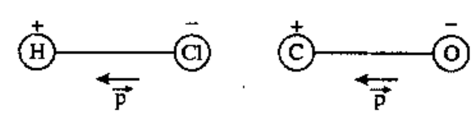


Fig. 15

an electric dipole moment which is directed from Cl atom to the H-atom. In this case the dipole moment of HCl is 3.43×10^{-30} coulomb meter.

Non Polar Molecules

Non polar molecules are those which have resultant dipole moment zero.

For example in CO_2 molecules all the atoms are in straight line so the resultant electric dipole moment is zero as shown in the figure (16).

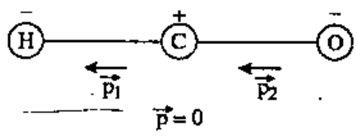


Fig. 16

In absence of any external field the dipole moment of polar molecules are oriented in random manner so there is no dipole moment obtained. But when a static electric field is applied then it tends to orient all the electric dipole in the direction of field.

Ferro electrics

Those substances are said to be ferro electrics which have a permanent polarisation when no external field is applied. For example, Rochelle salt NaK $(C_4H_4O_6).4H_2O$, $BaTiN_3$ etc.

• 1.12. DIELECTRIC

A non conducting medium which can be polarised by an electric field, is known as **dielectric.**

Dielectric in an Electric field : Let us consider a dielectric which is placed in an electric field $\vec{E}_0$ so that it is polarised *i.e.*, its molecules or atoms becomes electric dipole in the direction of external electric field, due to this polarisation a net positive charge on one side of the electric and equal negative charge on the other side which is shown in the figure. Thus the dielectric becomes large dipole.

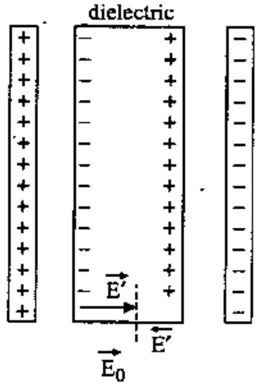


Fig. 17

These charges produce their own electric field $\vec{E}'$ which opposes the external field $\vec{E}_0$. Since $\vec{E}'$ is smaller than $\vec{E}_0$ therefore resultant $\vec{E}$ in the dielectric is weaker than the original field $\vec{E}_0$.

Thus, this phenomenon shows that when a dielectric is placed in an electric field then the field in the dielectric is weaker than the applied field.

Now, we have to prove that the dielectric constant of a conductor is infinite. For this let us consider E and E_0 are the magnitudes of the electric field with dielectric and without dielectric respectively. We have

$$\frac{E_0}{E} = k$$

where k is constant and this constant is known as dielectric constant or permittivity of the medium.

When a conductor is placed in an electric field then the field inside the conductor is reduced to zero *i.e.*, $E = 0$

$$\therefore \qquad \frac{E_0}{0} = k \Rightarrow \boxed{k = \infty}$$

Hence the dielectric constant of a conductor is infinite.

• 1.13. ELECTRIC FIELD STRENGTH

The electric field strength may be defined as "the force per unit charge in an electric field is known as electric field strength".

i.e., $$\vec{E} = \frac{\vec{F}}{q} \qquad \dots (1)$$

Its unit is newton per coulomb.

Electric Polarisation

When a piece of matter is placed in an electric field then it becomes electrically polarised *i.e.*, its molecules or atoms becomes electric dipole moment in the direction of external field.

Thus electric polarisation P of a material may also be defined as the electric dipole moment of the material per unit volume and it is denoted by $\vec{P}$.

The unit of polarisation is coulomb/meter^2.

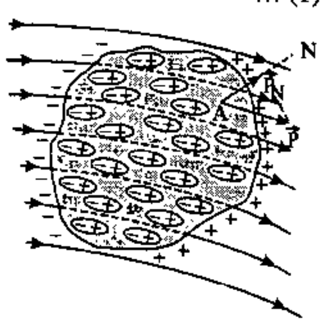


Fig. 18

Let $\vec{P}$ is the dipole moment required in each atom and let n is the number of atoms or molecules per unit volume then electric polarisation is

$$\vec{P} = n\vec{p}$$

Let us consider a slab material of thickness t and surface area S which is placed perpendicular to uniform field $\vec{E_0}$. The polarisation $\vec{P}$ is parallel to $\vec{E_0}$ and perpendicular to S.

$\therefore$ The volume of the slab = tS

$\therefore$ Electric dipole moment = $P\,(tS) = (PS)\,t$

= (charge appeared on each surface S)

and induced charge is

$$q' = PS$$

$$P = \frac{q'}{S} = \sigma_{pol} \qquad \dots (2)$$

Here σ_{pol} is called surface density of the charge due to polarisation.

In the figure (5) the charge density at A is given by

$$\sigma_{pol} = \vec{P}\,.\,\hat{n} = P\cos\theta \qquad \dots (3)$$

Electric Displacement

When a dielectric is placed in an electric field then the polarisation charges on the surface are not free to move through dielectric. These charges are bound to specific atoms. In other materials for example ionised gas, these charges are free to move through material.

Let us consider in figure (19) the surface density of charges on the left hand conducting plate is σ_{free} *i.e.*, $+\frac{q}{S}$ and on the right hand conducting plate is $-\sigma_{free}$ *i.e.*, $-\frac{q}{S}$. These charges produce an electric field to polarise the dielectric slab so that polarisation charges appears on each surface of the slab.

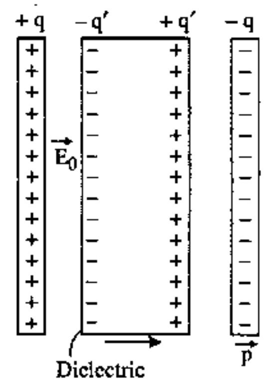


Fig.19

If P is the magnitude of the polarisation in the dielectric slab, then the surface density of charge on the left surface of the dielectric is $\sigma_{pol} = -P$ while on the right surface is $\sigma_{pol} = +P$.

$\therefore$ Net surface density of charge on the left is

$$\sigma = \sigma_{free} - \sigma_{pol}$$

$$\therefore \qquad \sigma = \sigma_{free} - \vec{P} \text{ (on left)} \qquad \dots (4)$$

These net charges give rise to uniform electric field *i.e.*,

$$\vec{E} = \frac{\sigma}{\varepsilon_0}$$

$$\therefore \qquad \vec{E} = \frac{1}{\varepsilon_0}[\sigma_{free} - P]$$

$$\sigma_{free} = \varepsilon_0 \vec{E} + \vec{P}$$

where $\vec{E}$ and $\vec{P}$ are vector quantity in the same direction so a new vector field $\vec{D}$ may be introduced *i.e.*,

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}} \qquad \dots (5)$$

where $\vec{D}$ has the dimension of $\vec{P}$ and it is called electric displacement vector. Its dimension is coulomb/m^2.

Electric Susceptibility

We know when a dielectric is placed in an electric field then the dielectric is polarised. In this position the polarisation, vector $\vec{P}$ is directly proportional to the external field $\vec{E}$ *i.e.*,

$$\vec{P} \propto \vec{E}$$

$$\therefore \qquad \vec{P} = \varepsilon_0 \chi_e \vec{E}$$

where χ_e is called electric susceptibility of the material. It is a pure number *i.e.*, it has no dimension.

Permittivity

Since electric displacement is

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\therefore \qquad \vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E} \qquad [\because \vec{P} = \varepsilon_0 \chi_e \vec{E}]$$

$$\vec{D} = (1 + \chi_e)\, \varepsilon_0 \vec{E} = \varepsilon \vec{E}$$

where $\varepsilon = \varepsilon_0 (1 + \chi_e)$ is called the permittivity of the medium. Its dimension is coulom^2-sec^2/meter^2-kg, *i.e.*, this is unit of ε_0.

Dielectric constant

The dielectric constant may be defined as *"the ratio of permittivity in free space to the permittivity in the medium is known as dielectric constant"* and it is denoted by K, *i.e.*,

$$\boxed{K = \frac{\varepsilon}{\varepsilon_0}}$$

It is a pure number.

• 1.14. GAUSS'S LAW IN DIELECTRIC

According to this law *"the electric flux ϕ passing through any close surface is equal to $\frac{1}{\varepsilon_0}$ lines of the net charge q on the closed surface.*

i.e.,
$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

where $\vec{E}$ = Electric field vector at small part of area $d\vec{S}$ of the surface.

In the fig. (20) two parallel plates charged with charges $+q$ and $-q$. Let the area of each plate is S and vacuum between them.

Let $\vec{E}$ is the uniform electric field between the plates. Let $PQRS$ is a Gaussian surface so the electric flux through the Gaussian surface is

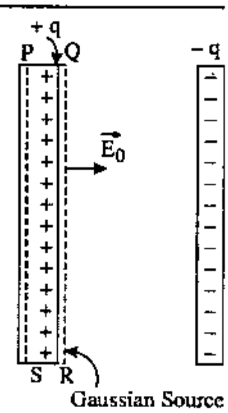


Fig. 20

$$\phi = \oint_S \vec{E_0}\, d\vec{S}$$

$$\therefore \qquad \phi = \int_S \vec{E_0} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

But
$$\int \vec{E_0}\, d\vec{S} = E_0 S \qquad [\because \vec{E_0} \text{ and } d\vec{S} \text{ are parallel}]$$

$$\therefore \qquad E_0 S = \frac{q}{\varepsilon_0}$$

$$E_0 = \frac{q}{\varepsilon_0 S} \qquad \ldots (1)$$

Let us consider a dielectric slab of dielectric constant K is placed between two plates then $-q'$ appears on left surface and $+q$ appears on right surface.

Let $\vec{E_0}$ is the field due to charges $-q'$ and $+q'$ and $\vec{E}$ is the resultant field then electric flux passing through the Gaussian surface $PQRS$ is

$$\phi_k = \int \vec{E} \cdot d\vec{S}$$

net charge contained in the Gaussian surface is $q - q'$ then

$$\phi_k = \int_S \vec{E} \cdot d\vec{S} = \frac{q - q'}{\varepsilon_0} \qquad \ldots (2)$$

$$\therefore \qquad ES = \frac{q - q'}{\varepsilon_0}$$

$$E = \frac{q - q'}{\varepsilon_0 S} \qquad \dots (3)$$

Now divided equation (1) by equation (3), we get

$$\frac{E_0}{E} = \frac{q}{q - q'} = K \qquad \left[\because K = \frac{E_0}{E}\right]$$

$$\therefore \qquad q - q' = \frac{q}{K}$$

Put this value in equation (2), then we get

$$\phi_k = \int \vec{E}.\, d\vec{S} = \frac{q}{K\varepsilon_0}$$

$$\boxed{\int K\vec{E}.\, d\vec{S} = \frac{q}{\varepsilon_0}} \qquad \dots (4)$$

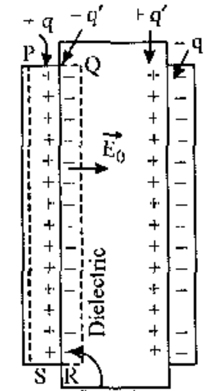


Fig. 21

Equation (4) shows that the change q contained by Gaussian surface is taken to be the free charges only.

• STUDENT ACTIVITY

1. What do you mean by electric polarisation of matter ?

2. Distinguish between polar and non-polar molecules.

3. What are ferro-electrics ?

4. What do you mean by dielectric ?

5. Explain electric field strength.

6. Explain electric polarisation.

7. Explain electric displacement.

8. Explain electric susceptibility.

9. Explain permittivity.

10. Explain dielectric constant.

• 1.15. MOLECULAR POLARISABILITY (α)

When a dielectric is placed in an electric field then it gets polarised but it is electrically neutral on the average, and it produces an electric field both at exterior points and inside the dielectric due to this each atom or molecules of the dielectric acquires an induced dipole moment.

Let $\vec{E}_m$ is the average electric field acting on a molecule and P_m is the average dipole moment acquired by the each molecule of the dielectric. Then it is found that P_m is directly proportional to the electric field $\vec{E}_m$

i.e., $$\vec{P}_m \propto \vec{E}_m$$

or $$\vec{P}_m = \alpha\, \vec{E}_m$$

where α is constant and this constant is known as molecular polarisability or polarisability.

Hence molecular polarisability may also be defined as *"the dipole moment acquired by a molecule per unit polarising field is called polarisability"*.

Polarisability are of three types :

(i) Electronic polarisability : This polarisability may be explained as follows. When a dielectric is placed in a electric field then the negatively charged electrons displace in the direction of applied field relative to the nucleus in the atom. It is present in all dielectric. This polarisability is denoted by α_e.

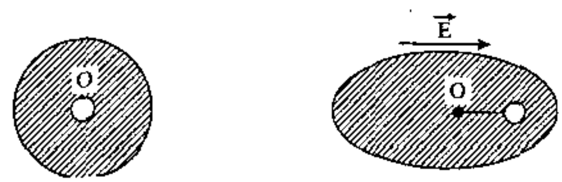


Fig. 22

(ii) Ionic polarisability (α_i) : This polarisability arises due to the displacement of a charged ion with respect to the other ion. In the fig. 23, a molecule contains positive and negative ions. When there is no external field then the ions of molecule are at equal distances from each other but when electric field is applied then such molecule will tend to displace the positive ions relative to the negative ions. Due to this, the dipole moment in the molecules is produced.

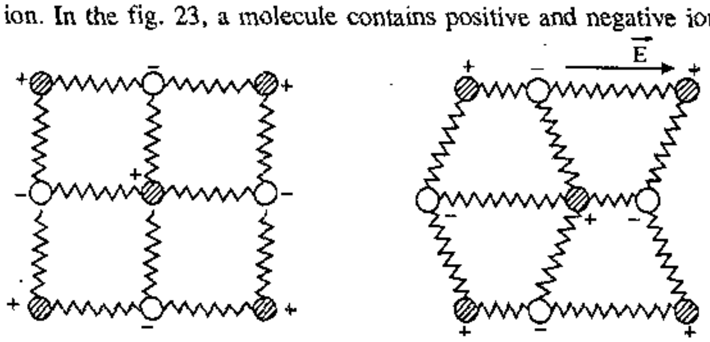


Fig. 23

(iii) Orientation polarisability (α_0) : The orientation polarisability arises due to change in orientation of permanent dipole moment in the presence of the electric field.

From the fig. 24 it is clear that in the absence of an electric field the permanent dipole moment of the molecules are distributed randomly in all directions but when electric field is applied then the dipole moment of the molecules are allign parallel to the field. This orienting tendency is responsible for the orientation polarisability.

From above we concluded that

1. Electronic and ionic polarisability together are refered as deformation or induced polarisability.

2. While both electronic and ionic polarisability depends upon atomic configuration and hence these are independent of temperature while orientation polarisability is inversely proportional to the temperature.

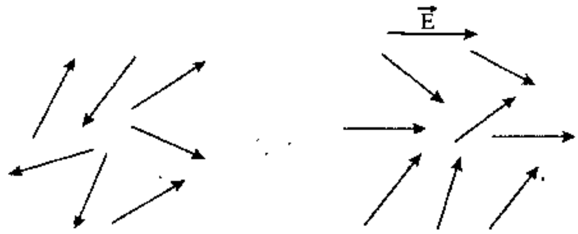


Fig. 24

3. Polarisability depends upon the frequency of the applied field. On increasing the frequency, the orientation and ionic polarisability decreases due to inertia of molecules and ions.

4. The polarisability α which is microscopic quantity is related to dielectric constant in case of non polar molecules while dielectric constant is macroscopic property by the Clausius-Mossotti relation.

$$\frac{N\alpha}{3\varepsilon_0} = \frac{M}{\rho}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right)$$

• 1.16. MOLECULAR FIELD

The electric field which is responsible for polarising a molecule of the dielectric is called the molecular field $\vec{E}_m$. This is the electric field at a molecular position in the dielectric which is produced by all external sources and by all polarised molecules in the dielectric *except the molecule at the point under consideration* (because it will not be polarised by its own field).

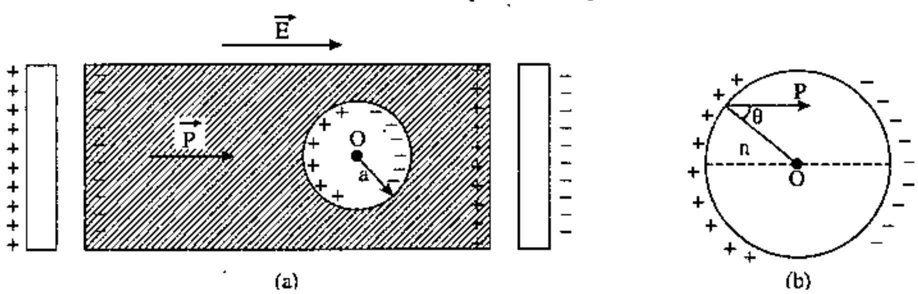


Fig. 25

The molecular field can be calculated in the following way. Suppose that a dielectric sample has been polarised by placing it in the uniform electric field between the parallel plates of a charged capacitor (Fig. 25a). We assume that the polarisation is uniform and that $\vec{P}$ is parallel to the field producing it.

Consider one of the molecules at O, constituting the dielectric. Let us draw a sphere of radius a about this particular molecule. The molecule is thus influenced by the fields :

(1) $\vec{E}_0$ due to the charges on the surfaces of the capacitor plates.

(2) $\vec{E}_1$ due to the charges on the dielectric surfaces facing the capacitor plates.

(3) $\vec{E}_2$ due to the charges on the interior of the spherical cavity of radius a.

(4) $\vec{E}_3$ due to the charges of the individual molecules, other than the molecule under consideration contained within the sphere of radius a.

So the net effective field on a dielectric molecule

$$\vec{E}_m = \vec{E}_0 + \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \quad \dots (1)$$

However,

(1) The field produced by a charges on the plates of the capacitor is

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

or

$$\vec{E}_0 = \frac{\vec{D}}{\varepsilon_0} \quad (\text{as } \vec{D} = \varepsilon_0 \vec{E}_0 = \sigma) \quad \dots (2)$$

(2) The field produced by the charges on the surfaces of dielectric facing the capacitor plates

$$\vec{E}_1 = -\frac{\sigma'}{\varepsilon_0}$$

or $$\vec{E}_1 = -\frac{\vec{P}}{\varepsilon_0} \qquad (\text{as } \sigma' = P) \qquad \ldots (3)$$

(3) The field due to polarisation charge present on the inside of the cavity produces a field

$$E_2 = \frac{1}{4\pi\,\varepsilon_0}\int \frac{P\cos\theta}{a^2}\cos\theta\, ds$$

or $$\vec{E}_2 = \frac{\vec{P}}{4\pi\,\varepsilon_0}\int_0^\pi \frac{\cos^2\theta}{a^2}(2\pi a^2\sin\theta\, d\theta) \qquad (\text{as } ds = 2\pi a\sin\theta\, ad\theta)$$

or $$\vec{E}_2 = \frac{\vec{P}}{2\varepsilon_0}\left[-\frac{\cos^3\theta}{3}\right]_0^\pi$$

i.e., $$\vec{E}_3 = \frac{\vec{P}}{3\varepsilon_0} \qquad \ldots (4)$$

(4) The field due to the individual molecule with in the sphere will be obtained by summing over the fields due to the dipoles with in the sphere. That is

$$E_3 = 0 \qquad \ldots (3)$$

Put the values of E_0, E_1, E_2, E_3, from equations (2), (3), (4), (5) in (1), we get

$$E_m = \frac{D}{\varepsilon_0} - \frac{P}{\varepsilon_0} + \frac{P}{3\varepsilon_0} + 0$$

i.e., $$E_m = E + \frac{P}{3\varepsilon_0} \qquad (\because\ \vec{D} = \varepsilon_0\vec{E} + \vec{P})$$

This is the required expression for molecular field and it is also known as "Lorentz Field Equation". It shows clearly that the field on a molecule is greater than the macroscopic field by a factor $\frac{P}{3\varepsilon_0}$ and is independent of the size of the sphere.

Clausius-Mossotti Relation : According to Lorentz equation the electric field on a single molecule of non-polar isotropic dielectric when in a macroscopic field $\vec{E}$ is given by

$$\vec{E}_m = \vec{E} + \frac{\vec{P}}{3\varepsilon_0} \qquad \ldots (1)$$

Under the action of this field, if the induced dipole moment in the molecule is $\vec{P}_m$, then by definition of polarisability

$$\vec{P}_m = \alpha\,\vec{E}_m = \alpha\left(\vec{E} + \frac{\vec{P}}{3\varepsilon_0}\right) \qquad \ldots (2)$$

If there are n molecule per unit volume of the dielectric, then polarisation will be

$$\vec{P} = n\,\vec{P}_m = n\,\alpha\left(\vec{E} + \frac{\vec{P}}{3\varepsilon_0}\right)$$

Now $P = (K-1)\,\varepsilon_0\,E$. Thus

$$(K-1)\,\varepsilon_0\,E = n\,\alpha\left[E + \frac{(K-1)\,\varepsilon_0\,E}{3\varepsilon_0}\right]$$

or $$(K-1)\,\varepsilon_0 = n\,\alpha\left(1 + \frac{K-1}{3}\right) = n\alpha\left(\frac{K+2}{3}\right)$$

or $$\boxed{\alpha = \frac{3\varepsilon_0\,(K-1)}{n\,(K+2)}} \qquad \ldots (3)$$

This is the Clausius-Mossotti equation.

Putting $K = 1 + \frac{\chi_e}{\varepsilon_0}$

then $$\boxed{\alpha = \frac{3\varepsilon_0}{n}\cdot\frac{\chi_e}{\chi_e + 3\varepsilon_0}}$$

This is the another form of Clausius-Mossotti equation.

Atomic radius : It can be shown that α is proportional to the cube of the radius of the molecule. Hence if K is found and n is known for a gas at a given temperature and pressure, the radius of the atom may be found for monoatomic gases.

• 1.17. LANGEVIN-DEBYE RELATION FOR THE POLARISATION OF POLAR MOLECULES

Langevin modified the Clausius-Mossotti relation so that it may be applied to polar molecules. For this assume that each atom has a permanent dipole moment $\mathbf{p}_0$ and the only force acting on it is that due to the field $\mathbf{E}_m$. The couple acting on the dipole whose axis subtends an angle θ with the field

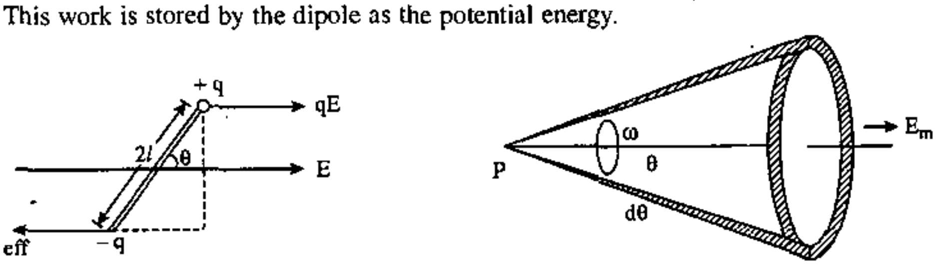
$$C = q\, 2l \sin\theta\, E_m = p_0\, E_m \sin\theta.$$

So the work done on the dipole in a small rotation $d\theta$

$$dW = C\, d\theta = p_0\, E_m \sin\theta\, d\theta$$

i.e.,
$$W = -\, p_0\, E_m \cos\theta \qquad \ldots (1)$$

This work is stored by the dipole as the potential energy.



(a) (b)

Fig. 26

Now on the basis of classical statistical mechanics, the number of molecules per unit volume whose axes make an angle θ with the field is proportional to $e^{(-\Psi/kT)}$, where k is the Boltzmann constant (1.38×10^{-23} J/degree) and T is the temperature in degree Kelvin. Hence the number of dipoles per unit volume whose axes make angle θ and $\theta + d\theta$ *i.e.*, within the solid angle $d\omega$ will be

$$dn = Ae^{(-W/kT)}\, d\omega$$
$$= A\, e^{(-W/kT)}\, 2\pi \sin\theta\, d\theta$$

i.e.,
$$n = 2\pi A \int_0^{\pi} e^{(-W/kT)} \sin\theta\, d\theta \qquad \ldots (2)$$

Further each of dn particles contribute a component of electric moment $p_0 \cos\theta$ parallel to the field while by symmetry the component perpendicular to the field neutralise one another. Hence the polarisation of the atoms,

$$P = 2\pi A \int_0^{\pi} (p_0 \cos\theta)\, e^{(-W/kT)} \sin\theta\, d\theta \qquad \ldots (3)$$

So from equations (2) and (3)

$$\frac{P}{n} = \frac{p_0 \int_0^{\pi} e^{(-W/kT)} \cos\theta \sin\theta\, d\theta}{\int_0^{\pi} e^{(-W/kT)} \sin\theta\, d\theta} \qquad \ldots (4)$$

Now as
$$-\frac{W}{kT} = \frac{-(-p_0\, E_m \cos\theta)}{kT} = \frac{p_0\, E_m}{kT} \cos\theta$$

So if we take

$$\frac{p_0\, E_m}{kT} = u \text{ and } u \cos\theta = t$$

eqn. (4) becomes

$$P = \frac{np_0}{u} \cdot \frac{\int_{-u}^{+u} e^t\, t\, dt}{\int_{-u}^{+u} e^t\, dt} = \frac{np_0}{u} \cdot \frac{\left[t\, e^t - e^t \right]_{-u}^{+u}}{\left[e^t \right]_{-u}^{+u}}$$

or
$$P = \frac{np_0}{u} \cdot \frac{[u\,(e^u + e^{-u}) - (e^u - e^{-u})]}{(e^u - e^{-u})}$$

or
$$P = np_0 \left[\frac{e^u + e^{-u}}{e^u - e^{-u}} - \frac{1}{u} \right]$$

or
$$P = P_s \left[\coth u - \frac{1}{u} \right] = P_s\, L\,(u) \qquad \ldots \text{(A)}$$

where $P_s = np_0$ and is the saturation value of polarisation. Eqn. (A) is known as *'Langevin equation'* and is an expression for the mean effective polarisation of a polar molecule in a field $\vec{E}_m$ at a temperature T.

Agains

$$L\,(u) = \left(\coth u - \frac{1}{u} \right) = \left[\frac{e^u + e^{-u}}{e^u - e^{-u}} - \frac{1}{u} \right]$$
$$= \frac{2\,[1 + (u^2/2)]}{2\,[u + (u^3/6)]} - \frac{1}{u}$$
$$= \frac{1}{u} \left\{ \left[1 + \frac{u^2}{2} \right] \left[1 + \frac{u^2}{6} \right]^{-1} - 1 \right\}$$
$$= \frac{1}{u} \left[\frac{u^2}{3} \right] = \frac{u}{3} = \frac{1}{3} \cdot \frac{p_0\, E_m}{kT}$$

Hence $P = P_s\, L\,(u) = np_0 \left(\frac{1}{3} \cdot \frac{p_0\, E_m}{kT} \right) = \frac{n}{3} \cdot \frac{p_0^2\, E_m}{kT}$ $\qquad \ldots$ (B)

Now as polarisability is defined as dipole moment acquired by a molecule per unit polarising electric field, hence orientation polarisability

$$\alpha_0 = \frac{P}{nE_m} = \frac{p_0^2}{3kT} \qquad \ldots \text{(C)}$$

From equation (B) or (C) it is clear that *polarisation* or *polarisability* of a polar molecule is inversely proportional to the absolute temperature and depends on the nature of molecule (*i.e.*, p_0).

The Debye Relation and Study of Molecular Structure: In case of polar dielectrics along with electronic and ionic polarisibility orientation polarisibility also exists so

$$\alpha = \alpha_e + \alpha_i + \alpha_0$$

or
$$\alpha = \alpha_d + \alpha_0$$
$$\alpha = \alpha_d + \frac{p_0^2}{3kT}$$

This is known as "Langevin-Debye" equation.

where $\boxed{\alpha = \frac{3\varepsilon_0\,(k-1)}{n\,(k+2)}}$

again, we know that electric susceptibility χ_e of the dielectric material is given by

$$\chi_e = \frac{P}{E_m} = \frac{nP_0^2}{3kT}$$

Thus, the electric susceptibility, and hence also the dielectric constant of a polar dielectric is inversely proportional to the absolute temperature.

• SUMMARY

- Electronic force between two charges is called a central force.
- A system of closely spaced charges is said to form a continuous charge distribution.
- The electric field intensity is also called strength of electric field.

- Electric field is a vector quantity and its unit is $N–C^{-1}$ and $V–m^{-1}$.
- Electric flux linked with a surface in an electric field may be defined as the surface integral of the electric field over that surface.
- Hypothetical closed surface is called Gaussian surface.
- $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qr}{R^3} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$
- Molecules which have a permanent electric dipole moment are called polar molecules.
- Substances which have a permanent polarisation when no external field is applied are said to be ferro electrics.
- The dipole moment acquired by a molecule per unit polarising field is called polarisability.
- Clausius-Mossotti relation,

$$\alpha = \frac{3\varepsilon_0}{n} = \frac{\chi_e}{\chi_e + 3\varepsilon_0}$$

- The electric susceptibility and also dielectric constant of a polar dielectric is inversely proportional to the absolute temperature.

• TEST YOURSELF

1. State Coulomb's law in electrostatics. Express it in vector form.
2. What is the meaning of the continuous charge distribution ? Obtain the expression for the force between a point charge and a continuous charge distribution.
3. Define electric intensity and then obtain an expression for the electric field intensity due to a given continuous charge distribution.
4. Prove that for a cylindrical surface immersed in a uniform field E parallel or perpendicular to its axis, the total flux ϕ is zero.
5. State and prove Gauss's law in electrostatics. How Coulomb's law can be derived from this law ?
6. Use Gauss's law to calculate the electric field intensity due to a uniformly charged sphere at (i) an external point (ii) an internal point and (iii) at the surface.
7. Using Gauss's law, find the electric field intensity due to uniformly charged cylinder of infinite length at a distance r from its axis.
8. Use Gauss's law to find the electric intensity at a point near an infinite plane sheet of charge.
9. Show that when a dielectric is placed in an electric field, the field within the dielectric is weaker than the original field. Hence show that the dielectric constant of a conductor is infinite.
10. Explain, how the Gauss's law is modified when a dielectric material is present in the electric field.
11. What is molecular polarisability ? Explain different types of polarisabilities.
12. In SI unit of permittivity is :
    (a) Nm^2C^{-2} (b) $Nm^{-2}C^{-1}$ (c) $C^2N^{-1}m^{-2}$ (d) Am^{-1}
13. When air is replaced by a dielectric medium of dielectric constant K, the maximum force of attraction between two charges separated by a distance :
    (a) decrease K times (b) remains unchanged
    (c) increase k times (d) decrease K^2 times
14. A charge q_1 exerts some force on a second charge q_2. A third charge q_3 is brought near. Then force exerted by q_1 on q_2 will:
    (a) decrease in magnitude (b) increase in magnitude
    (c) remain unchanged
    (d) increase, if q_2 is of the same sign as q_1 and will decrease, if q_2 is of opposite sign
15. Choose the correct relationship among the three electric vectors :
    (a) $\vec{P} = \vec{D} + \varepsilon_0 \vec{E}$ (b). $\vec{P} = \vec{D} - \varepsilon_0 \vec{E}$ (c) $\vec{D} = \vec{E} + \varepsilon_0 \vec{P}$ (d) $\vec{D} = \vec{E} - \varepsilon_0 \vec{E}$
16. Tick the WRONG statement :
    (a) The force between two point charges in a dielectric medium is greater than that in vaccum
    (b) Molecules having symmetrical structure are non-polar
    (c) The dielectric constant of a polar dielectric is higher than that of a non-polar dielectric
    (d) the electric susceptibility of a polar dielectric is inversely proportional to the absolute temperature

17. A rectangular frame of area 10 m^2 is placed in a uniform electric field of 20 NC^{-1}, with normal drawn on the surface of the frame making 60° angle with the direction of field. The electric flux through the surface is :

(a) 100 Vm (b) 200 Vm (c) $50\sqrt{3}$ Vm (d) $100\sqrt{3}$ Vm

18. A closed surface has 'n' electric dipoles located inside it. The net electric flux emerging through the surface is :

(a) $\frac{ne}{\varepsilon_0}$ (b) $\frac{2e}{\varepsilon_0}$ (c) $\frac{2ne}{\varepsilon_0}$ (d) zero

19. A hollow sphere of charge does not produce an electric field at any :

(a) interior point (b) outer point
(c) beyond 2 meter (d) beyond 10 meter

20. An electric dipole of moment P is placed in the positions of stable equilibrium in a uniform electric field of intensity E. The torque required to rotate when the dipole makes an angle θ with the initial position is :

(a) $PE \cos\theta$ (b) $PE \sin\theta$ (c) $P \tan\theta$ (d) $P \cos\theta$

21. When an electric dipole is placed in a uniform electric field, it experiences :

(a) force only (b) torque
(c) both force and torque (d) neither force nor a torque

22. The electric flux through a hemispherical surface of radius R placed in a uniform electric field E parallel to the axis if the circular plane is :

(a) $(2\pi R) E$ (b) $(\pi R^2) E$ (c) $\left(\frac{4}{3}\pi R^3\right)E$ (d) $\left(\frac{2}{3}\pi R^3\right)E$

23. A point charge $+q$ is placed at the mid point of a cube of side L. The electric flux emerging from the cube is :

(a) $\frac{q}{\varepsilon_0}$ (b) $\frac{q}{6L^2\varepsilon_0}$ (c) $\frac{6qL^2}{\varepsilon_0}$ (d) zero

24. Gauss's theorem :

(a) does not hold, if the closed surface encloses discrete distribution of charges.
(b) does not hold, if the closed surface encloses a line, a surface or a volume charge distribution
(c) holds, the surface encloses a point charge only
(d) hold, irrespective of the form in which charges are enclosed by the closed surface

25. Clausius-Mossotti relation for molecular polarizability is :

(a) $\alpha = \frac{\varepsilon_0 (K-1)}{n (K+2)}$ (b) $\alpha = \frac{3\varepsilon_0 (K-1)}{n (K+2)}$
(c) $\alpha = \frac{n (K+2)}{3\varepsilon_0 (K-1)}$ (d) $\alpha = \frac{n (K-2)}{3\varepsilon_0 (K+1)}$

26. The susceptibility for polar dielectrics depends on temperature T is proportional to :

(a) T (b) T^{-1} (c) T^2 (d) T^{-2}

27. In a polar dielectric, centre of positive charges of the atom :

(a) is outside the nucleus
(b) is at the centre of negative charges of the atom
(c) is separated by a small distance from the centre of negative charges of the atom
(d) is separated by a large distance from the centre of negative charges of the atom

28. Force of attraction between two end faces of a polarised dielectric is proportional to :

(a) E (b) E^{-1} (c) E^2 (d) E^{-2}

ANSWERS

12. (c) **13.** (a) **14.** (c) **15.** (b) **16.** (a) **17.** (a) **18.** (d) **19.** (a) **20.** (b)
21. (b) **22.** (b) **23.** (a) **24.** (d) **25.** (b) **26.** (b) **27.** (c) **28,** (c)

UNIT 2

VARYING AND ALTERNATING CURRENTS

STRUCTURE

- Kirchhoff's Laws
- Condition of Balance of Wheatstone's Bridge
- Growth of Current in an Inductive Circuit
- Charging of a Capacitor
- Discharge of a Capacitor through Inductance
- Charging of a Capacitor in LCR Circuit
- Discharge of Capacitor through Inductance and Resistance
- Measurement of High Resistance by Leakage Method
- Energy Stored in an Inductor
  - Student Activity
- Mean Value of Alternating Current
- Lag and Lead of Current
- A.C. Circuit Containing Inductance and Resistance in Series
- A.C. Circuit Containing a Resistance and Capacitance in Series
- A.C. Circuit Containing Inductance and Capacitance in Series
- A.C. Circuit Containing Resistance, Inductance and Capacitance (Series Resonant Circuit)
- Parallel Resonant Circuit
- Quality Factor (Q) of a Series Resonant Circuit
- Sharpness of Resonance
- Choke Coil
  - Summary
  - Student Activity
  - Test Yoruself

LEARNING OBJECTIVES

After going this unit you will learn :

Kirchhoff's laws and its applications in solving the complicated circuit.

Charging and discharging of a capacitor through various circuits and inductances.

- Root mean square value (RMS) and its relation with mean value.
- Different types of A.C. circuit.
- Choke coil and its applications.

2.1. KIRCHHOFF'S LAWS

Kirchhoff given two laws. These laws are used to solve the complicated circuit. These laws are simply the expressions of conservation of electric charge and energy. They are :

(i) Kirchhoff's first law : According to this law *"the algebraic sum of the currents meeting at a junction in a closed circuit is zero"*.

Consider a junction 'O' in the electrical circuit at which the five conductors are meeting. Let I_1, I_2, I_3, I_4 and I_5 the currents as shown in the fig. 1.

Therefore according to Kirchhoff's law

$$\Sigma I = 0$$

or $$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

or $$I_1 + I_2 = I_3 + I_4 + I_5$$

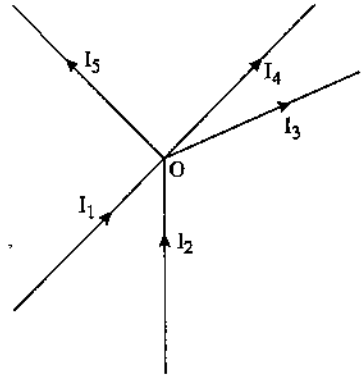


Fig. 1

(ii) Kirchhoff's second law : According to this law "in any closed path of an electrical circuit, the algebraic sum of all the potential differences is zero *i.e.*,

$$\Sigma \Delta V = 0$$

In fact Kirchhoff's second law shows the electrostatic force is a conservative force and the work done by it in any closed path is zero. Consider a closed electrical circuit as shown in the fig. (2). It consists two cells of e.m.f. E_1 and E_2 and three resistances R_1, R_2 and R_3. Apply Kirchhoff's law in closed path $ABEFA$, we get

$$I_3R_2 + I_1R_1 - E_1 = 0$$

$$\therefore \quad E_1 = I_1R_1 + I_3R_2$$

Similarly for closed path $ABCDEFA$

$$E_2 - I_2R_3 + I_1R_1 - E_1 = 0$$

$$E_1 - E_2 = I_1R_1 - I_2R_3$$

$$\Sigma E = \Sigma IR$$

$$\therefore \quad \Sigma E - \Sigma IR = 0$$

i.e., $$\Sigma \Delta V = 0$$

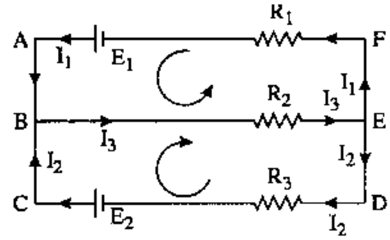


Fig. 2

• 2.2. CONDITION OF BALANCE OF WHEATSTONE'S BRIDGE

According to Wheatstone Bridge principle four resistances P, Q, R and S are arranged to form a bridge as shown in the fig. (3) with a cell E and tapping key K_2 between the points B and D. On closing K_1 first and after K_2 if the galvanometer shows no deflection then bridge is said to be balanced.

Applying Kirchhoff's second law to the closed circuit $ABDA$

$$I_1P + I_gG - (I - I_1)R = 0 \quad \ldots (1)$$

where G is the resistance of galvanometer.

Again applying Kirchoff's second law in $BCDB$

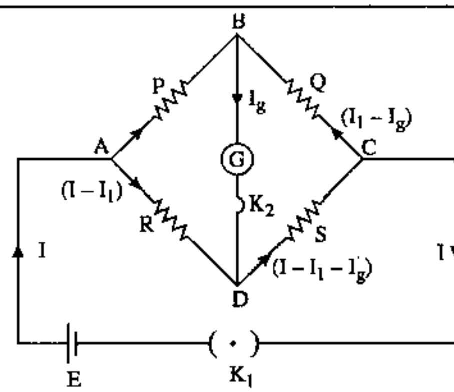


Fig. 3

$$(I_1 - I_g)Q - (I - I_1 + I_g)S - I_gG = 0 \quad \ldots (2)$$

For balancing condition $I_g = 0$ so by eqs. (1) and (2)

$$I_1P - (I - I_1)R = 0$$

$$\therefore \quad I_1P = (I - I_1)R \quad \ldots (3)$$

and $$I_1Q - (I - I_1)S = 0$$

$$I_1Q = (I - I_1)S \quad \ldots (4)$$

Dividing eqn. (3) by (4), we get $\dfrac{P}{Q} = \dfrac{R}{S}$

This is the required condition of balance.

• 2.3. GROWTH OF CURRENT IN AN INDUCTIVE CIRCUIT

Consider a circuit containing a resistance R, a coil of inductance L, a key S with a cell of e.m.f. E as shown in fig. (4). When the key S' is connected to a the circuit may be closed and the current begins to rise in the circuit. The current would have acquired the maximum possible value almost instantaneously. But due to the presence of inductance L in the circuit, when current starts to flow, a back e.m.f. is developed which oppose the growth of current in the circuit and thus reduces rate of growth of current. This rate of growth in such a LR circuit is calculated as follows :

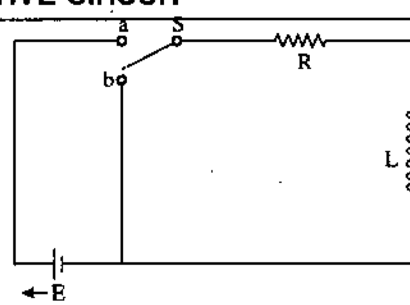


Fig. 4

If during growth of current, i be the current at any instant t, then the back e.m.f. developed in the circuit will be $L\frac{di}{dt}$. The effective e.m.f. in the circuit at that instant will be $\left(E - L\frac{di}{dt}\right)$.

By Ohm's law this should be equal to a resistance × current at that instant *i.e.*, Ri. Thus we may write

$$E - L\frac{di}{dt} = Ri$$

or $$E - Ri = L\frac{di}{dt}$$

or $$dt = \frac{Ldi}{(E - Ri)}$$

which on integration gives

$$t = -\frac{L}{R}\log_e(E - Ri) + A$$

where A is integrating constant. The value of this constant is obtained by condition that current is zero just at start *i.e.*, at $t = 0$, $i = 0$.

Applying this we get

$$0 = -\frac{L}{R}\log_e E + A$$

$$A = \frac{L}{R}\log_e E$$

$$t = -\frac{L}{R}\log_e(E - Ri) + \frac{L}{R}\log_e E$$

or $$-\frac{R}{L}t = \log_e(E - Ri) - \log_e E$$

$$= \log_e\frac{(E - Ri)}{E}$$

or $$e^{-(Rt/L)} = \frac{E - Ri}{E} = 1 - \frac{Ri}{E}$$

or $$\frac{Ri}{E} = 1 - e^{-(Rt/L)}$$

$$i = \frac{E}{R}(1 - e^{-(Rt/L)})$$

$$= i_0(1 - e^{-Rt/L}) \quad \ldots(1)$$

where $i_0 = E/R$ represents the maximum possible current in the circuit and $t = L/R$ is time constant of the circuit. It is called the inductive time constant. A graph between current and time is shown in Fig. (5).

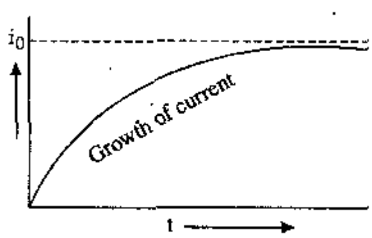


Fig. 5

If we put $t = 1/R$ in eq. (1) we get

$$(i) = i_0\left(1 - \frac{1}{e}\right) = i_0\left(\frac{e-1}{e}\right) = 0.632\, i_0$$

or $$\frac{i}{i_0} = 0.632.$$

Thus the time constant of a circuit may be defined as the time in which the current rises from zero to 0.632 or 63% of its final value.

Decay of Current : Now if circuit is disconnected from battery by connecting switch S to b [Fig. (4)] the current will begin to fall. Again had inductance not been present the current would have fallen from maximum i_0 to zero almost instantaneously. But the inductance L is present in the circuit which opposes the decay of the current and thus reduces the rate of decay of current.

If during decay of current i is the value of current at any instant, t the induced e.m.f. in the circuit will be $\left(-L\frac{di}{dt}\right)$ which by Ohm's law must be equal to resistance × current (= Ri). Thus we write

$$-L\frac{di}{dt} = Ri$$

or $$dt = -\frac{L}{R}\frac{di}{i}$$

which on integration gives $t = -\frac{L}{R}\log_e i + A$

where A is the integration constant which may be obtained by the condition that just at the break of circuit, i has maximum value i_0 *i.e.*, at $t = 0$, $i = i_0$.

Applying this we get

$$0 = -\frac{L}{R}\log_e i_0 + A \quad \text{or} \quad A = \frac{L}{R}\log_e i_0$$

so that

$$t = -\frac{L}{R}\log_e i + \frac{L}{R}\log_e i_0$$

$$-\frac{R}{L}t = \log_e i - \log_e i_0 = \log_e \frac{i}{i_0}$$

or $$e^{-(Rt/L)} = \frac{i}{i_0}$$

or $$i = i_0\, e^{-(Rt/L)} \qquad \ldots(2)$$

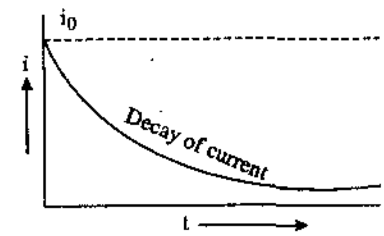


Fig. 6

We put $t = \frac{1}{R}$

$$i = i_0\, e^{-1}$$

$$i = 10\,(0.368) = 0.368\, L_0 = 36.8\%\, L_0$$

Therefore "the time constant of an L–R circuit may also be defined as the time in which the *current decays from maximum to* 0.368 (or 36.8%) of its maximum value".

From eq. (2) we note that the current in the circuit decays exponentially as shown in Fig. (3).

• 2.4. CHARGING OF A CAPACITOR

Consider a circuit consisting of a condenser of capacity C in series with a resistance R and a battery of constant e.m.f. E shown in Fig. (7). When the circuit is closed by connecting S' to a, the condenser plates start to receive the charge until the potential difference across them becomes equal to E. In this process of charging, the battery tries to introduce more and more charge but the charge already attained by the plates of the condenser opposes the introduction of further charge. In other words, a back e.m.f. is developed in the circuit due to charge already attained by condenser plates at any instant. The back e.m.f. naturally decreases the rate of charging of condenser.

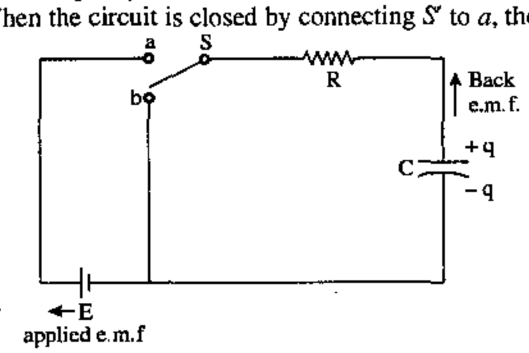


Fig. 7

Let q be the charge across condenser plates at any instant t. According to definition of capacity, potential difference between them will be $= q/C$. This acts opposite to the applied e.m.f. E so that the effective e.m.f. at instant t will be equal to $(E - q/C)$ which must be equal to Ri by Ohm's law. Thus we can write that

$$E - \frac{q}{C} = Ri \qquad \ldots(1)$$

Further by definition

$$i = \frac{dq}{dt} \qquad \ldots(2)$$

Since maximum possible potential difference at condenser plates is equal to E, the maximum possible charge q_0 on them will be given by

$$q_0 = CE \quad \text{or} \quad \frac{q_0}{C} = E \qquad \ldots(3)$$

Hence eq. (1) may be written as :

$$\frac{q_0}{C} - \frac{q}{C} = R\frac{dq}{dt}$$

or $$(q_0 - q) = CR\frac{dq}{dt}$$

or $$dt = \left(\frac{dq}{q_0 - q}\right)CR$$

On integration, we get $$\int dt = \int\left(\frac{dq}{q_0 - q}\right)CR$$

or $$t = -CR\log_e(q_0 - q) + A$$

where A is the integration constant and since in this circuit at

$$t = 0,\ q = 0$$

so $$0 = -CR\log_e(q_0 - 0) + A$$

or $$A = CR\log_e(q_0)$$

Hence we have $$t = -CR\log_e(q_0 - q) + CR\log_e q_0$$

or $$-\frac{t}{CR} = \log_e(q_0 - q) - \log_e q_0 = \log_e\left(\frac{q_0 - q}{q_0}\right)$$

or $$e^{-t/CR} = \frac{q_0 - q}{q_0} = \left(1 - \frac{q}{q_0}\right)$$

or $$\frac{q}{q_0} = 1 - e^{-t/CR}$$

or $$q = q_0(1 - e^{-t/CR}) \qquad \ldots(4)$$

This shows that charge in CR circuit increases exponentially as shown in Fig. (8).

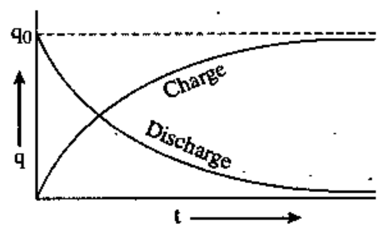


Fig. 8

The rate of growth of charge is given by

$$\frac{dq}{dt} = q_0 \cdot \frac{1}{CR} \cdot e^{-t/CR}$$

$$= q_0 \cdot \frac{1}{CR}\left(1 - \frac{q}{q_0}\right)$$

$$= \frac{1}{CR}(q_0 - q) \qquad \ldots(5)$$

which shows that smaller is the product CR, more rapid is the growth of charge on the condenser.

The product CR is called the capacitive time constant of the circuit and may be defined as follows :

Putting $t = CR$ in equation (4), we get

$$q = q_0(1 - e^{-t/t}) = q_0\left(1 - \frac{1}{e}\right)$$

$$= q_0\left(1 - \frac{1}{2.718}\right) = 0.632\, q_0$$

Time constant may be defined as the time in which condenser plates acquire charge 0.632 of its maximum possible value q_0 (= CE).

Discharge of the Condenser

When the condenser is fully charged then if the Morse key is released (*i.e.*, K is connected to b, Fig. 7) the condenser naturally starts to discharge at a definite rate which can be found out as follows :

Let q be the charge remained on condenser plates at any time t during discharge, then q/C will be potential difference between plates at that instant. Since battery has been disconnected so q/C will be the only source of e.m.f. and hence it must be equal to Ri.

Thus we can write that

$$\frac{q}{C} = Ri$$

or $$-\frac{q}{C} = Ri \text{ (–ve since e.m.f. will be a back e.m.f.)}$$

or $$-\frac{q}{C} = R\frac{dq}{dt}$$

or $$\frac{dq}{q} = -\frac{1}{CR}\cdot dt$$

Integrating it

$$\int \frac{dq}{q} = -\frac{1}{CR}\int dt$$

or $$\log_e q = -\frac{t}{CR} + B$$

where B is the integration constant. Now during discharge we have $q = q_0$ at $t = 0$ (*i.e.* condenser is fully charged in the beginning), so that

$$\log_e q_0 = -0 + B$$

or $$B = \log_e q_0$$

Hence, we have $$\log_e q = -\frac{t}{CR} + \log_e q_0$$

or $$\log_e q - \log_e q_0 = -\frac{t}{CR}$$

or $$\log_e \frac{q}{q_0} = -\frac{t}{CR}$$

or $$\frac{q}{q_0} = e^{-t/CR}$$

or $$q = q_0\, e^{-t/CR} \qquad \ldots(6)$$

If we put $$t = CR$$

$\therefore$ $$q = q_0\, e^{-1}$$

$$q = q_0\,(0.368) = q_0\,(36.8\%)$$

Therefore "capacitive time constant of a CR-circuit may also be defined as the time during which the charge on capacitor decays from maximum to 0.368 (36.8%) of its maximum value.

This shows that discharge in CR circuit is also exponential as shown in Fig. (9).

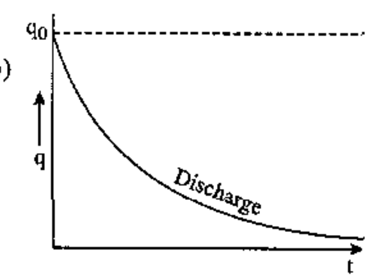


Fig. 9

Now the rate of discharge is given by

$$\frac{dq}{dt} = -q_0\cdot\frac{1}{CR}\,e^{t/CR}$$

$$= -q_0\frac{1}{CR}\frac{q}{q_0} \qquad \text{[from eq. (6)]}$$

or $$-\frac{dq}{dt} = \frac{q}{CR}$$

which shows that smaller is the time constant CR, the quicker is the discharge of the condenser.

• 2.5. DISCHARGE OF A CAPACITOR THROUGH INDUCTANCE

Let a capacitor be connected with an inductance and a cell through a Morse key K as shown in Fig. (10). Let the condenser be first charged to the maximum possible value q_0 (= CE) by connecting S to a and then discharged through the inductance L by connecting S to b. Let during discharge, i, be the current in the circuit and q be the charge left on condenser plates at any instant t. Then evidently at this instant, t, the potential difference across the condenser plates will be $= q/C$ and back e.m.f. induced in the inductance will

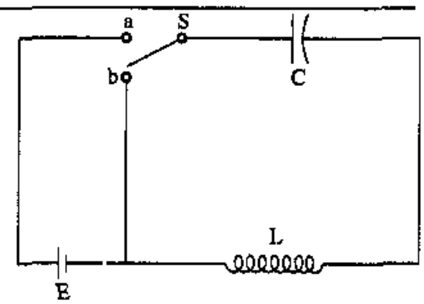


Fig. 10

be $\left(-L\frac{di}{dt}\right)$. Now since there is no resistance in the circuit and no external e.m.f. during discharge we may write

$$\frac{q}{C}=-L\frac{di}{dt} \quad \text{or} \quad L\frac{di}{dt}+\frac{q}{C}=0$$

or $$L\frac{d^2q}{dt^2}+\frac{q}{C}=0 \qquad \left(\because\ i=\frac{dq}{dt},\ \text{ so } \frac{di}{dt}=\frac{d^2q}{dt^2}\right)$$

or $$\frac{d^2q}{dt^2}+\frac{q}{LC}=0$$

This is a simple differential equation representing a simple harmonic motion and if we let

$$\frac{1}{LC}=\omega^2 \quad \text{then} \quad \frac{d^2q}{dt^2}+\omega^2 q=0 \qquad \ldots(1)$$

Now let the trial solution of this equation be of the form $q=Ae^{\alpha t}$ where A and α are arbitrary constants.

so that $$\frac{dq}{dt}=A\alpha e^{\alpha t} \quad \text{and} \quad \frac{d^2q}{dt^2}=A\alpha^2 e^{\alpha t}$$

Substituting these values is equation (1), we have

$$A\alpha^2 e^{\alpha t}+\omega^2 A e^{\omega t}=0$$

or $$\alpha^2+\omega^2=0$$

or $$\alpha=\pm i\omega \quad \text{where} \quad i=\sqrt{(-1)}.$$

Thus the equation (1) has two solutions, given by

$$q=A\,e^{+i\omega t} \quad \text{and} \quad q=Ae^{-i\omega t}$$

The general solution can be written as

$$q=A_1e^{+i\omega t}+A_2e^{-i\omega t} \qquad \ldots(2)$$

where A_1 and A_2 are the two constants to be determined as follows:

We know that during discharge at $t=0$, $q=q_0$ so that equation (2) gives

$$q_0=A_1+A_2 \qquad \ldots(3)$$

Further on differentiating eq. (2), we get

$$i=\frac{dq}{dt}=A_1e^{i\omega t}\,(i\omega)+A_2e^{-i\omega t}\,(-\,i\omega)$$

And since at $t=0$ $\qquad \left(\text{charge is constant so that } \frac{dq}{dt}=0\right)$

We have $0=A_1\,(i\omega)-A_2\,(i\omega)$ or $A_1=A_2$

Therefore equation (3) gives

$$A_1=A_2=\frac{q_0}{2}$$

Substituting this in eq. (2) we get

$$q=\frac{q_0}{2}\left(e^{i\omega t}+e^{-i\omega t}\right)$$

or $$q=q_0\cos\omega t \qquad \ldots(4)$$

From this equation we can say that :

(i) This discharge of the condenser is oscillatory and simple harmonic in nature.

(ii) The time period of oscillatory discharge is given by

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{LC} \qquad \ldots(5)$$

(iii) The frequency of oscillatory discharge is given by

$$f=\frac{1}{T}=\frac{1}{2\pi\sqrt{LC}}$$

The principle for oscillatory discharge in LC circuit is used in wireless telegraphy in producing the electromagnetic oscillation and thereby transmitting the message from one place to another :

From eq. (4) current is given by

$$i = \frac{dq}{dt} = -\, q_0\omega \sin \omega t$$

$$i = -\, q_0 \sin \omega t$$

where maximum current amplitude is

$$\mathbf{i_0 = q_0\ \omega = \frac{q_0}{\sqrt{LC}} = \frac{CE}{\sqrt{LC}}}.$$

• 2.6. CHARGING OF A CAPACITOR IN LCR CIRCUIT

Consider a circuit of inductance L, resistance R and condenser C connected in series with a battery of e.m.f. E as shown in Fig. (11). Let on pressing the key S, we have the current in the circuit as i and charge on condenser plates as q at any instant t. The value of e.m.f. *in different parts of the circuit will be as follows :*

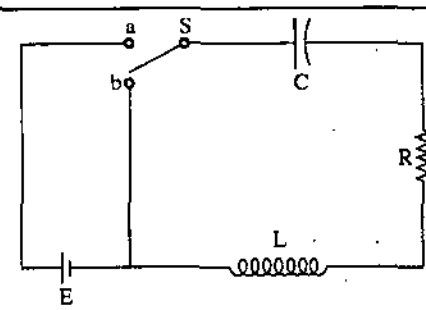


Fig. 11

(i) across the inductance $= L\dfrac{di}{dt} = L\dfrac{d^2q}{dt^2}$

(ii) across the resistance $= Ri = R\,.\,\dfrac{dq}{dt}$

(iii) across the condenser $= q/C$

and since the external e.m.f. is E, the equation of electromotive force may be written as :

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E$$

or

$$\frac{d^2q}{dt^2} + \frac{R}{L}\cdot\frac{dq}{dt} + \frac{1}{LC}\cdot q = \frac{E}{L}$$

Now if we put, $\dfrac{R}{L} = 2k$ and $\dfrac{1}{LC} = \omega^2$, we get

$$\frac{d^2q}{dt^2} + 2k\frac{dq}{dt} + \omega^2 q = \frac{E}{L}$$

or

$$\frac{d^2q}{dt^2} + 2k\frac{dq}{dt} + \omega^2\left(q - \frac{E}{\omega^2 L}\right) = 0 \qquad \ldots(1)$$

Let $x = q - \dfrac{E}{\omega^2 L}$ or $\dfrac{dx}{dt} = -\dfrac{dq}{dt}$ and $\dfrac{d^2x}{dt^2} = \dfrac{d^2q}{dt^2}$, we get

$$\therefore \qquad \frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega^2 x = 0 \qquad \ldots(2)$$

Now let the trial solution of eq. (2) be of the form $x = Ae^{\alpha t}$

so that $\qquad \dfrac{dx}{dt} = A\alpha e^{\alpha t}$ and $\dfrac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t}$

Substituting these values in equation (2) we get

$$\alpha^2 e^{\alpha t} + 2k\alpha e^{\alpha t} + \omega^2 e^{\alpha t} = 0$$

or

$$\alpha^2 + 2k\alpha + \omega^2 = 0$$

which on solving gives

$$\alpha = -\,k \pm \sqrt{(k^2 - \omega^2)}$$

Therefore the general solution may be written as

$$x = A_1 e^{[-k + \sqrt{(k^2-\omega^2)}]\,t} + A_2\, e^{[-k - \sqrt{(k^2-\omega^2)}]\,t}$$

where, A_1 and A_2 are arbitrary constants, the values of which can be determined by boundary condition.

Further $$x = q - \frac{E}{\omega^2 L}$$

or $$q = \frac{E}{\omega^2 L} + x$$

$$= \frac{E}{\omega^2 L} + \left[A_1 e^{[-k + \sqrt{(k^2 - \omega^2)}]t} + A_2 e^{[-k - \sqrt{k^2 - \omega^2}]t}\right]$$

But $$\omega = \frac{1}{LC} \text{ or } \frac{E}{\omega^2 L} = \frac{ELC}{L} = EC = q_0$$

so that $$q = q_0 + \left[A_1 e^{[-k + \sqrt{(k^2 - \omega^2)}]t} + A_2 e^{[-k - \sqrt{(k^2 - \omega^2)}]t}\right] \quad ...(3)$$

To evaluate the values of A_1 and A_2, let us apply the following boundary conditions :

We know that at $t = 0$, $q = 0$. Hence from eq. (3) we have

$$0 = q_0 + A_1 + A_2$$

or $$A_1 + A_2 = -q_0 \quad ...(4)$$

Further differentiating equation (3) with respect to t, we get

$$\frac{dq}{dt} = i = A_1\left[-k + \sqrt{(k^2 - \omega^2)}\right] e^{[-k + \sqrt{(k^2 - \omega^2)}]t}$$
$$+ A_2\left[-k - \sqrt{(k^2 - \omega^2)}\right] e^{[-k - \sqrt{k^2 - \omega^2}]t}$$

and putting at $t = 0$, $i = 0$. we get

$$0 = A_1\left[-k + \sqrt{(k^2 - \omega^2)}\right] + A_2\left[-k - \sqrt{(k^2 - \omega^2)}\right]$$
$$= -k(A_1 + A_2) + [A_1 - A_2]\sqrt{(k^2 - \omega^2)}$$
$$= kq_0 + (A_1 - A_2)\sqrt{(k^2 - \omega^2)} \qquad (\because A_1 + A_2 = -q_0)$$

Hence $$A_1 - A_2 = -\frac{kq_0}{\sqrt{k^2 - \omega^2}} \quad ...(5)$$

Now solving eqs. (4) and (5), we get

$$A_1 = \frac{q_0}{2}\left(1 + \frac{k}{\sqrt{k^2 - \omega^2}}\right)$$
$$A_2 = -\frac{q_0}{2}\left(1 - \frac{k}{\sqrt{k^2 - \omega^2}}\right)$$

and eq. (3) may be written as

$$q = q_0 - \frac{q_0}{2}\left(1 + \frac{k}{\sqrt{k^2 - \omega^2}}\right) e^{[-k + \sqrt{(k^2 - \omega^2)}]t} - \frac{q_0}{2}\left(1 - \frac{k}{\sqrt{k^2 - \omega^2}}\right) e^{[-k + \sqrt{(k^2 - \omega^2)}]t} \quad ...(6)$$

This represents the expression of charge on condenser in *LCR* circuit at any constant t. We consider the following three cases :

Case I. When $k^2 > \omega^2$ $\left(i.e.\ \frac{R^2}{4L^2} > \frac{1}{LC}\right)$

In this case the equation (6) for q tells directly that the charge goes on increasing till it finally attains the Steady value q_0. This is shown in Fig. (12) by curve a. The charge is neither dead beat nor oscillatory.

Case II. When $k^2 = \omega^2$ $\left(i.e.\ \frac{R^2}{4L^2} = \frac{1}{LC}\right)$

For this case we again get the result as in case I except that here the rate of growth will be greater. It is shown in Fig. (12) by curve b. The case is a critical one, when the charge is neither dead beat nor oscillatory.

Case III. When $k^2 < \omega^2$.

In this case, the roots of equation (6) will be imaginary since

$$k^2 - \omega^2 = -\text{ ive}$$

Now if we let $\sqrt{(-1)}\,.\,\sqrt{\omega^2 - k^2} = i\beta$ then we may write

$$q = q_0\left[1 - \frac{1}{2}\left(1 + \frac{k}{i\beta}\right)e^{(-k+i\beta)t} - \frac{1}{2}\left(1 - \frac{k}{i\beta}\right)e^{(-k-i\beta)t}\right]$$

$$= q_0\left[1 - e^{-kt}\left\{\left(\frac{e^{i\beta t} + e^{-i\beta t}}{2}\right) + \frac{k}{\beta}\left(\frac{e^{i\beta t} - e^{-i\beta t}}{2i}\right)\right\}\right]$$

$$= q_0\left[1 - e^{-kt}\left(\cos\beta t + \frac{k}{\beta}\sin\beta t\right)\right]$$

$$= q_0\left[1 - \frac{e^{-kt}}{\beta}(\beta\cos\beta t + k\sin\beta t)\right]$$

Further if we put $\beta = r\cos\theta$ and $k = r\sin\theta$

so that $r = \sqrt{(\beta^2 + k^2)} = \omega$ and $\tan\theta = \frac{k}{\beta} = \frac{k}{\sqrt{(\omega^2 - k^2)}}$

then we have

$$q = q_0\left[1 - \frac{e^{-kt}}{\beta}r(\cos\beta t\cos\theta + \sin\beta t\sin\theta)\right]$$

$$= q_0\left[1 - \frac{e^{-kt}}{\beta}\,.\,r\cos(\beta t - \theta)\right]$$

$$q = q_0\left[1 - \frac{\omega e^{-kt}}{\sqrt{\omega^2 - k^2}}\cos\{\sqrt{(\omega^2 - k^2)}\,t - \theta\}\right] \quad \ldots(7)$$

This equation represents a damped oscillatory growth of charge with amplitude decreasing with time as shown in Fig. 12 by curve (c).

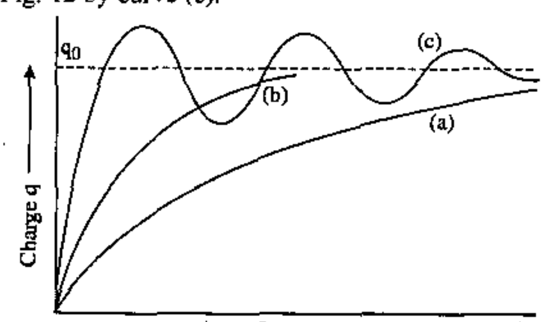


Fig. 12

Also it is clear by eq. (7) that time period of it is given by

$$T = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{(\omega^2 - k^2)}} \quad \text{or} \quad T = \frac{2\pi}{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}}$$

and the frequency by $n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}$

Here it may be noted that on differentiating the eq. (7) and then on solving, we get

$$i = \frac{dq}{dt} = q_0\frac{\omega^2 e^{-kt}}{\sqrt{(\omega^2 - k^2)}}\sin[\sqrt{(\omega^2 - k^2)}\,.\,t]$$

i.e. the current in the circuit during growth is damped oscillatory.

• 2.7. DISCHARGE OF CAPACITOR THROUGH INDUCTANCE AND RESISTANCE

Suppose a condenser is connected as shown in Fig. (13) with a resistance R, and inductance L and a battery through a Morse key. Let it be first charged by connecting S to a till it receives the maximum possible charge $q_0 = CE$. It is then discharged through R and L by connecting S to b. Let during discharge, i be the current in the circuit and q is charge remained on capacitor at any instant t then the values of e.m.f. in different parts of the circuit will be as follows :

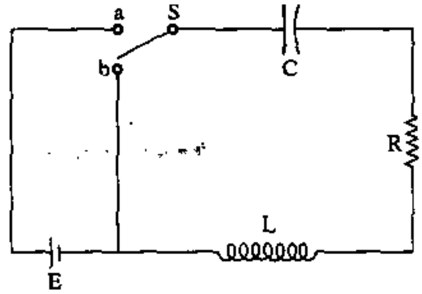


Fig. 13

(i) across inductance, $L\frac{di}{dt} = L\frac{d^2q}{dt^2}$

(ii) across the resistance, $Ri = R\frac{dq}{dt}$

(iii) across the condenser, q/c.

Since the external e.m.f. is zero, the circuit equation may be written as

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

or
$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

Now on putting $\frac{R}{L} = 2k$ and $\frac{1}{LC} = \omega^2$, we get

$$\frac{d^2q}{dt^2} + 2k\frac{dq}{dt} + \omega^2 q = 0 \qquad \ldots(1)$$

Further let the trial solution of this equation be of the form

$$q = Ae^{\alpha t}$$

so that
$$\frac{dq}{dt} = A\alpha\, e^{\alpha t}$$

and
$$\frac{d^2q}{dt^2} = A\alpha\, e^{\alpha t}$$

Substituting in eq. (1), we get

$$A\alpha^2 e^{\alpha t} + 2kA\alpha e^{\alpha t} + \omega^2 Ae^{\alpha t} = 0$$

or
$$\alpha^2 + 2k\alpha + \omega^2 = 0$$

which on solving gives $\quad \alpha = -k \pm \sqrt{(k^2 - \omega^2)}$.

Hence, the general sol. of eq. (1) may be written as

$$q = A_1 e^{[-k + \sqrt{(k^2 - \omega^2)}\,]\,t} + A_2 e^{[-k - \sqrt{(k^2 - \omega^2)}\,]\,t} \qquad \ldots(2)$$

where A_1 and A_2 are two arbitrary constants to be determined by boundary conditions as follows :

We know that at $t = 0$, $q = q_0$, hence eq. (2) gives

$$q_0 = A_1 + A_2 \qquad \ldots(3)$$

Now differentiating (2), we get

$$i = \frac{dq}{dt} = \left\{A_1\,(-k + \sqrt{(k^2 - \omega^2)})\right\} e^{[-k + \sqrt{(k^2 - \omega^2)}\,]\,t}$$

$$+ \left\{A_2\,(-k + \sqrt{(k^2 - \omega^2)})\right\} e^{[-k + \sqrt{(k^2 - \omega^2)}\,]\,t}$$

On applying the condition $i = 0$ at $t = 0$, we get

$$0 = A_1\left\{-k + \sqrt{(k^2 - \omega^2)}\right\} + A_2\left\{-k - \sqrt{(k^2 - \omega^2)}\right\}$$

$$= -k\,(A_1 + A_2) + \sqrt{(k^2 - \omega^2)}\,(A_1 - A_2)$$

$$= -kq_0 + \sqrt{(k^2 - \omega^2)}\,(A_1 - A_2) \qquad [\because A_1 + A_2 = q_0]$$

$$\therefore \qquad A_1 - A_2 = \frac{kq_0}{\sqrt{(k^2 - \omega^2)}} \qquad \ldots(4)$$

Now solving eqs. (3) and (4), we have

$$A_1 = \frac{q_0}{2}\left[1 + \frac{k}{\sqrt{(k^2 - \omega^2)}}\right]$$

and
$$A_2 = \frac{q_0}{2}\left[1 - \frac{k}{\sqrt{(k^2 - \omega^2)}}\right]$$

Now putting these values of A_1 and A_2 in eq. (2), we have

$$q = \frac{q_0}{2}\left[1 + \frac{k}{\sqrt{k^2 - \omega^2}}\right] e^{[-k + \sqrt{k^2 - \omega^2}\,]\,t} + \frac{q_0}{2}\left[1 - \frac{k}{\sqrt{k^2 - \omega^2}}\right] e^{[-k - \sqrt{k^2 - \omega^2}\,]\,t} \qquad \ldots(5)$$

This eq. represents the expressions for charge on condenser at any instant t during discharge. Here it is worthwhile to consider the following three cases :

Case I. When $k^2 > \omega^2$.

In this case, the roots will be real and negative. Hence equation (5) shows that q decreases asymptotically to zero [fig. 14, curve a]. This is called 'over damped' non-oscillatory or dead beat discharge.

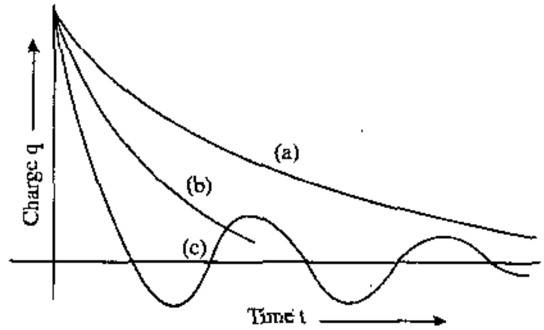


Fig. 14

Case II. When $k^2 = \omega^2$.

The case is a critical one. When the discharge is neither dead beat nor oscillatory. In this case, the discharge is shown in [Fig. 14, curve b].

Case III. When $k^2 < \omega^2$.

In this case $\sqrt{(k^2 - \omega^2)}$ is an imaginary quantity. Now for convenience, we put

$$\sqrt{(k^2 - \omega^2)} = i\sqrt{(\omega^2 - k^2)} = i\beta \text{ where } \beta = \sqrt{(\omega^2 - k^2)}$$

Hence eq. (5) becomes

$$q = \frac{q_0}{2}\left(1 + \frac{k}{i\beta}\right)e^{(-k+i\beta)t} + \frac{q_0}{2}\left(1 - \frac{k}{i\beta}\right)e^{(-k-i\beta)t}$$

$$= \frac{q_0}{2}e^{-kt}\left(e^{i\beta t} + \frac{k}{i\beta}e^{i\beta t} + e^{-i\beta t} - \frac{k}{i\beta}e^{-i\beta t}\right)$$

$$= q_0 e^{-kt}\left[\left(\frac{e^{i\beta t} + e^{-i\beta t}}{2}\right) + \frac{k}{\beta}\left(\frac{e^{i\beta t} - e^{-i\beta t}}{2i}\right)\right]$$

$$= q_0 e^{-kt}\left[\cos\beta t + \frac{k}{\beta}\sin\beta t\right]$$

$$= q_0\frac{e^{-kt}}{\beta}[\beta\cos\beta t + k\sin\beta r]$$

Now let $\beta = r\cos\theta$ and $k = r\sin\theta$

so that $\quad r = \sqrt{(\beta^2 + k^2)} = \omega$ and $\tan\theta = \dfrac{k}{\beta} = \dfrac{k}{\sqrt{\omega^2 - k^2}}$

Then we get

$$q = q_0\frac{e^{-kt}}{\beta}[r\cos\theta\cos\beta t + r\sin\theta\sin\beta t]$$

$$= q_0 \cdot \frac{re^{-kt}}{\beta}[\cos\beta t\cos\theta + \sin\beta t\sin\theta]$$

$$= q_0\frac{\omega \cdot e^{-kt}}{\beta}(\cos(\beta t - \theta))$$

$$= q'_0 e^{-kt}\cos(\beta t - \theta) \qquad \left[\because q'_0 = \frac{q_0\,\omega}{\beta}\right]$$

This represents a simple harmonic curve of which the amplitude $q'_0 e^{-kt}$ diminished exponentially with increasing time. That is, we can say that the discharge of condenser under such a condition is damped oscillatory in which the amplitude goes on decreasing with time as shown in [Fig. (14), curve (c)].

The period of discharge is given by

$$T = \frac{2\pi}{\beta}$$

$$= \frac{2\pi}{\sqrt{(\omega^2 - k^2)}} = \frac{2\pi}{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}}$$

and the frequency of discharge is given by

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}$$

$$= \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right)} \qquad \text{when } R \text{ is small.}$$

• 2.8. MEASUREMENT OF HIGH RESISTANCE BY LEAKAGE METHOD

We know that if a condenser is first charged and then discharged through a resistance R then during discharge, the charge q remained on plates of condenser, is given by

$$q = q_0\, e^{-t/CR}$$

where C is capacity of condenser and q_0 is the charge on condenser before discharging. Further we know that if R is high, CR will be high and the rate of discharge of condenser will be very slow. This theory provides a method of measuring high resistance as follows :

Formula : From eq. (1), we have

$$\frac{q_0}{q} = \frac{1}{e^{-t/CR}} = e^{t/CR}$$

or $$\log_e \frac{q_0}{q} = \frac{t}{CR}$$

or $$R = \frac{t}{C \log_e (q_0/q)} = \frac{t}{2.3026\, C \log_{10} (q_0/q)}$$

Thus by determining the value of (q_0/q), R can be determined.

The circuit is shown in the fig. 15. C is the standard capacitor, R is the high resistance which is to be determined, B.G. is a ballistic galvanometer and E is the battery of e.m.f. E.

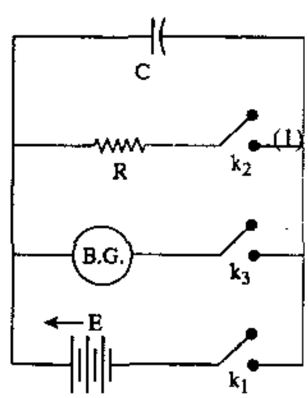


Fig. 15

First of all capacitor C is charged by pressing the key k_1 when it is fully charged then it is discharged through B.G. on k_1 is raised and k_3 is pressed at ones. In this position θ_0 is calculated which is $q_0 \propto \theta_0$.

Now capacitor is again charged by pressing k_1. When it is fully charged then it is discharged through resistance R by pressing k_2 for a time t. In this position some of the charge of the capacitor leaks through resistance R then remaining charge on the capacitor is discharged the galvanometer by pressing key k_3 and θ is noted then

$$q \propto \theta$$

$$\therefore \quad \frac{q_0}{q} = \frac{\theta_0}{\theta}$$

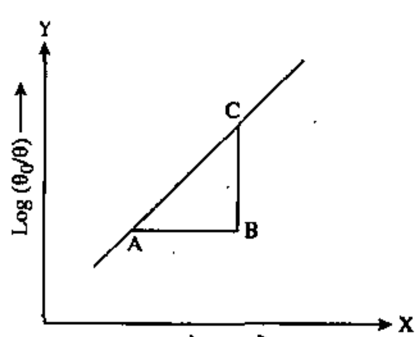


Fig. 16

so by eq. (1)

$$R = \frac{t}{2.3026\, C \log_{10}\left(\frac{\theta_0}{\theta}\right)} \qquad \dots (2)$$

A graph is plotted between t and $\log_{10}\left(\frac{\theta_0}{\theta}\right)$. In the graph a straight line is obtained.

As C is known the value of R can be determined from equation (2).

• 2.9. ENERGY STORED IN AN INDUCTOR

Let a source of e.m.f. is connected to an inductor L. As the current starts growing, an induced e.m.f. is set up in the inductor and the induced e.m.f., then, opposes the growth of current through it. The energy spent by the source of e.m.f. is stored in the inductor in the form of magnetic field.

The rate of growth of current at that time is equal to $\frac{di}{dt}$ then

induced e.m.f. produced in the inductor, $e = L\frac{di}{dt}$

Small amount of work done by the source is given by

$$dW = ei\,dt = \left(L\frac{di}{dt}\right)i\,dt = L\,i\,di$$

Total amount of work done from $i = 0$ to $i = i_0$

$$W = \int_0^{i_0} Li\,di = L\int_0^{i_0} i\,di = \frac{1}{2}Li_0^2$$

Therefore, energy stored in the

$$U = \frac{1}{2}Li_0^2$$

This is the required expression.

Energy stored in a capacitor : Let source of e.m.f. E is connected to a capacitor of capacitance C. As the changing of the capacitor takes place, the potential difference across the plates increases. The work done by the source is stored in the capacitor in the form of its energy of the potential difference across the plates of that time is V.

$$V = \frac{q}{C}$$

Small amount work done by the source of e.m.f.

$$dW = V\,dq = \frac{q}{C}\,dq$$

Total amount of work done by the source of e.m.f. so as to increase charge from the initial value $q = 0$ to the final value q_0 is given by

$$W = \int_0^{q_0} \frac{q}{C}\,dq = \frac{1}{C}\int_0^{q_0} q\,dq = \frac{1}{2}\cdot\frac{q_0^2}{C}$$

Therefore, energy stored in the capacitor

$$U = \frac{1}{2}\cdot\frac{q_0^2}{C} \qquad \ldots (1)$$

When the capacitor acquires charge q_0, the potential difference across the plates of the capacitor will become equal to V_0, the e.m.f. of the source of e.m.f.

$$\therefore \qquad q_0 = CV_0$$

Substituting for q_0 (= CE) in equation (1), we have

$$U = \frac{1}{2}CV_0^2 \qquad \ldots (2)$$

Substituting for $C\left(=\frac{q_0}{V_0}\right)$ in equation (1), we have

$$U = \frac{1}{2}q_0 V_0 \qquad \ldots (3)$$

The equations (1), (2) and (3) give the energy stored in a charged capacitor. The energy is stored in the capacitor at the expense of the energy of the source of e.m.f. and it resides in the capacitor in the form of electric field.

• STUDENT ACTIVITY

1. What are the dimensions of R/L ?

2. What are the dimensions of CR ?

• 2.10. MEAN VALUE OF ALTERNATING CURRENT

We know that the value of current at any instant in A.C. is given by

$$I = I_0 \sin \omega t$$

where I_0 is the peak value.

The average value of current over a complete cycle in A.C. circuit is zero. Hence it has no significance and therefore the mean value of alternating current is defined as its average over half a cycle. The half period of A.C. is $\frac{\pi}{\omega}\left(\because T = \frac{2\pi}{\omega}\right)$ and since integral of any quantity in any limits means summation of that quantity in those limits. Hence means current over half the cycle may taken as given by

$$I_{mean} = \frac{\int_0^{T/2} I dt}{T/2}$$

$$= \frac{\int_0^{\pi/\omega} I_0 \sin \omega t \, dt}{\pi/\omega}$$

$$= -\frac{I_0 \omega}{\pi}\left[\frac{-\cos \omega t}{\omega}\right]_0^{\pi/\omega}$$

$$= -\frac{I_0}{\pi}[\cos \pi - \cos 0]$$

$$= -\frac{I_0}{\pi}[-1-1]$$

$$= \frac{2I_0}{\pi}$$

$$\boxed{I_{\text{mean}} = \frac{2}{\pi} \times \text{Peak value of current}}$$

similarly, we can write

$$\boxed{E_{\text{mean}} = \frac{2}{\pi} \times \text{Peak value of e.m.f.}}$$

Root mean square value of an alternating current : It is defined as the square root of the average of I^2 during a complete cycle may be taken as

$$I^2 = \frac{\int_0^{2\pi/\omega} I^2 dt}{2\pi/\omega}$$

$$= \frac{\int_0^{2\pi/\omega} I_0^2 \sin^2 \omega t \, dt}{2\pi/\omega}$$

$$= \frac{i_0^2\omega}{2\pi} \int_0^{2\pi/\omega} \frac{1}{2}(1 - \cos 2\omega t)\, dt$$

$$= \frac{I_0^2\omega}{4\pi}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^{2\pi/\omega}$$

$$= \frac{I_0^2\omega}{4\pi}\left[\frac{2\pi}{\omega}\right] = \frac{I_0^2}{2}$$

so $$I_{rms} = \sqrt{I^2} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

similarly $$E_{rms} = \frac{E_0}{\sqrt{2}}$$

Relation between Mean Value and R.M.S. value : We have

$$I_{mean} = \frac{2I_0}{\pi} \quad \text{and} \quad I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$\therefore \quad \frac{I_{rms}}{I_{mean}} = \frac{I_0/\sqrt{2}}{2I_0/10} = \frac{\pi}{2\sqrt{2}}$$

$$\frac{I_{rms}}{I_{mean}} = 1.11$$

I_{rms}/I_{mean} is known as **form factor**. This form factor is only true for A.C. r.m.s. value of alternating current is also known as "effective" or "virtual" value of current.

$$i_{virtual} = \frac{I_0}{\sqrt{2}} = I_{r.m.s.}$$

Similarly,

$$E_{virutal} = \frac{E_0}{\sqrt{2}} = E_{r.m.s.}$$

• 2.11. LAG AND LEAD OF CURRENT

In A.C. circuit it is found that the frequency of voltage and current remains same but they do not remain in same phase, in general. In some circuits, maximum of current is found to occur a little after the maximum of applied voltage while in same, a little before. This is called respectively the lag and lead of current over applied e.m.f.

(a) Circuit containing the pure resistance : Let an alterning e.m.f. $E = E_0 \sin \omega t$ be applied to a circuit containing resistance R [Fig. 17(a)]. Let I be the current in the resistance R at any instant t. Now by Ohm's law, the potential difference across the resistance R, at that instant, will be RI and it must be equal to applied e.m.f. E at that instant *i.e.*,

$$IR = E = E_0 \sin \omega t \qquad ...(1)$$

$$I = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t$$

or
$$I = I_0 \sin \omega t \qquad ...(2)$$

where I_0 = maximum or peak value of current = E_0/R.

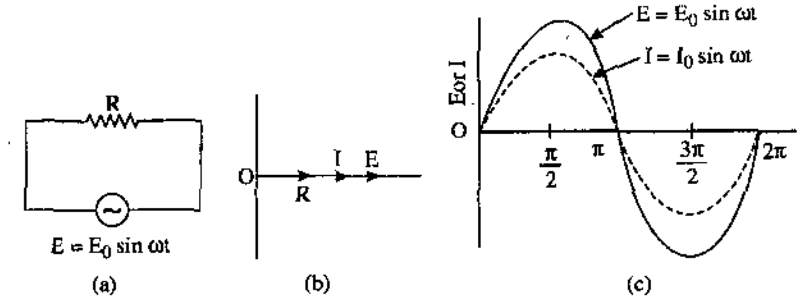


Fig. 17

A comparison of equations (1) and (2) shows that the current and voltage rise and fall simultaneously, or they are in phase with each other. This fact is illustrated geometrically in Fig. 17(c). where E and I are shown in same phase at each instant.

Vector or Phase diagram : It is also useful to represent the e.m.f. and current in a vector diagram representing the current I and voltage E as vectors. Obviously then the angle between E and I vectors will represent the phase difference between them.

Since in circuit of resistance only E and I and in same phase so they may be represented along the same line as shown in Fig. 17(b).

(b) Circuit containing pure inductance : Let an alternating e.m.f. $E = E_0 \sin \omega t$ be applied across an inductance [Fig. 18(a)]. Since A.C. is varying at every instant, so a back e.m.f. at every instant is produced due to self inductance. This e.m.f. will be equal to $-L\frac{dI}{dt}$ (by definition). Now as there is no other e.m.f. in the circuit, so applying Kirchhoff's Law we can say that this back e.m.f. must be equal and opposite to applied e.m.f. *i.e.*, we may write

$$\therefore \qquad E = E_0 \sin \omega t = -\left(-L\frac{dI}{dt}\right).$$

or
$$\frac{dI}{dt} = \frac{E_0}{L} \sin \omega t$$

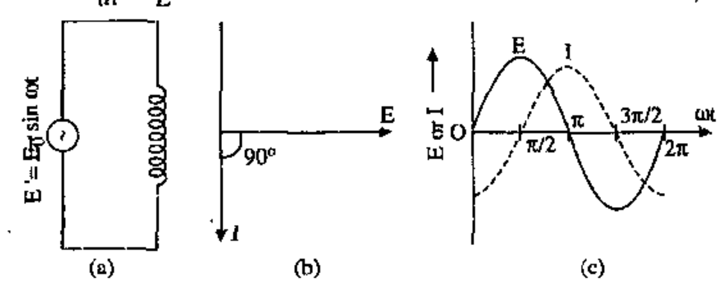


Fig. 18

Integrating both the sides, we have

$$I = -\frac{E_0}{\omega L} \cdot \cos \omega t$$

which may also be written as

$$I = -\frac{E_0}{\omega L} \cdot \cos \omega t = \frac{E_0}{\omega L} \sin (\omega t - \pi/2)$$

$$I = I_0 \sin (\omega t - \pi/2) \qquad ...(3)$$

where $I_0 \left(= \frac{E_0}{\omega L}\right)$ is the maximum current. Comparing this equation with $E = E_0 \sin \omega t$, we see that

(i) The current lags behind the e.m.f. by $\pi/2$. This is illustrated geometrically in Fig. 18(c).

(ii) Law has the same effect in A.C. circuit as resistance R in B.C. circuit. This is called the inductive reactance and is denoted by X_L. Thus we may write

$$X_L = \omega L = 2\pi f.L$$

where f is the frequency of A.C. It may be noted that when L is in henrys and f is in cycles per second, X_L will have the unit ohm.

Vector or phase diagram : In this Case E and I may be represented in vector form as shown in [Fig 18(b)] in which it is shown that I lags behind E by an angle of 90°.

(c) Circuit Containing pure capacitance : Let an alternating e.m.f. $E = E_0 \sin \omega t$ be applied across the plates of a perfect condenser of capacitance C. The condenser is charged first in one direction and then in the opposite direction. The result of to and fro motion of electron round the circuit is an alternating current flows through the condenser. But when the voltage across the condenser plates has reached its maximum value in one direction, the condenser is fully charged and no current flows in it. Since there is no resistance in the circuit, the potential difference $V = q/C$ across the plates, at any instant, must be equal to the applied e.m.f. at that instant *i.e.*, we may write

$$\frac{q}{C} = E = E_0 \sin \omega t$$

or

$$q = CE_0 \sin \omega t.$$

Further we know that through the condenser at that instant will be given by

$$I = \frac{dq}{dt} = CE_0\, \omega \cos \omega t$$

$$= \frac{E_0}{\frac{1}{C\omega}} \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots(4)$$

where $I_0 = \frac{E_0}{I/C\omega}$ is the maximum current. Comparing the equation with the $E = E_0 \sin \omega t$ we see that :

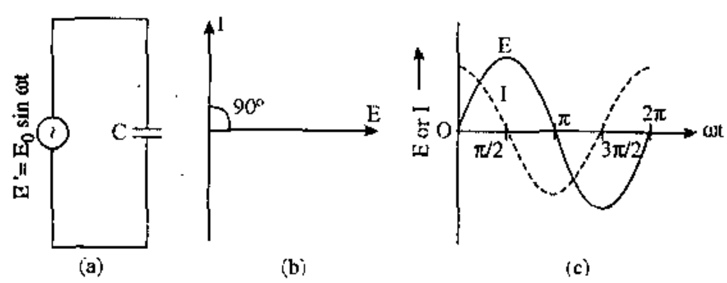


Fig. 19

(i) the current leads the e.m.f. by $\pi/2$. It is illustrated geometrically in Fig. 19(c).

(ii) $\frac{1}{C\omega}$ has the same effect in A.C. circuit, as resistance R in a D.C. circuit. It is called the capacitive and is denoted by X_C.

Thus we may write

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Also note that if C is in farad and f is in cycles per second, X_C will have the unit ohm.

Vector or Phase diagram : In this case the E and I may be represented in vector form as shown in [Fig. 19(b)] in which it is illustrated and I leads over E by an angle of 90°.

• 2.12. A.C. CIRCUIT CONTAINING INDUCTANCE AND RESISTANCE IN SERIES

Let us consider a circuit having a resistance, R and an inductance, L, in series and shown in Fig. (20). Let an alternating e.m.f. $E = E_0 \sin \omega t$ be applied to the circuit. If I be the instantaneous value of the current through the circuit, the rate of change of current in the circuit is $\frac{dI}{dt}$. Due to the changing current, an induced e.m.f. is set up in the inductance, which oppose the applied e.m.f. and is given by $-L\frac{dI}{dt}$.

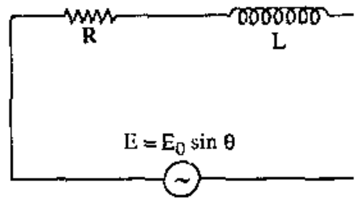


Fig. 20

Thus, the effective e.m.f. $= E - L\frac{dI}{dt}$

and applying Ohm's law to the circuit, we may write

$$E - L\frac{dI}{dt} = RI$$

or $$L\frac{dI}{dt} + RI = E = E_0 \sin \omega t \qquad \ldots(1)$$

Now in A.C. we know that I varies simple harmonically with the same frequency as the applied e.m.f. but differing in amplitude and phase, hence the trial solution of equation (1) may be taken of the form,

$$I = I_0 \sin(\omega t - \phi) \qquad \ldots(2)$$

where I_0 and ϕ are constants, to be determined as follows :

Differentiating equating (2), we get

$$\frac{dI}{dt} = I_0\, \omega \cos(\omega t - \phi) \qquad \ldots(3)$$

Substituting eq. (2) and (3) in eq. (1), we have

$$LI_0\omega \cos(\omega t - \phi) + RI_0 \sin(\omega t - \phi) = E_0 \sin \omega t$$

or $$LI_0\omega\,[\cos \omega t \cos \phi + \sin \omega t \sin \phi] + RI_0\,[\sin \omega t \cos \phi - \cos \omega t \sin \phi] = E_0 \sin \omega t \qquad \ldots(4)$$

At $\omega t = 0$, $\cos \omega t = 1$ and $\sin \omega t = 0$, so from eq. (4), we get

$$L\omega\, I_0 \cos \phi - RI_0 \sin \phi = 0$$

or $$\tan \phi = \frac{\omega L}{R} \qquad \ldots(5)$$

and from Fig. (21) drawn on the basis of equation (5), we can write that

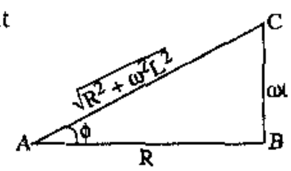


Fig. 21

$$\sin \phi = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

and $$\cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

Further since at $\omega t = \pi/2$, $\cos \omega t = 0$ and $\sin \omega t = 1$, equation (4) gives

$$\omega L\, I_0 \sin \phi + RI_0 \cos \phi = E_0$$

and substituting the value of $\sin \phi$ and $\cos \phi$, we have

$$I_0\left[\frac{\omega^2 L^2}{\sqrt{(R^2 + \omega^2 L^2)}} + \frac{R^2}{\sqrt{(R^2 + \omega^2 L^2)}}\right] = E_0$$

or $$I_0 \sqrt{(R^2 + \omega^2 L^2)} = E_0$$

or $$I_0 = \frac{E_0}{\sqrt{(R^2 + \omega^2 L^2)}} \qquad \ldots(6)$$

Now substituting for I_0 in equation (2), we have :

$$I = \frac{E_0}{\sqrt{(R^2 + \omega^2 L^2)}} \sin(\omega t - \phi) \qquad \ldots(7)$$

where $$\phi = \tan^{-1}\frac{\omega L}{R}$$ (from 5)

It may be noted that :

(i) The amplitude *i.e.*, the maximum current is given by

$$I_0 = \frac{E_0}{\sqrt{(R^2+\omega^2L^2)}}$$

(ii) The current lags behind the voltage by an angle

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

(iii) $\sqrt{(R^2+\omega^2L^2)}$ plays the same role in this A.C. circuit as resistance R in a D.C. circuit. It is called the impedance and is denoted by Z. Now since ωL is known as the inductive reactance X_L. We can write

$$Z = \sqrt{R^2 + X_L^2} \quad \ldots(8)$$

Since $I_0/\sqrt{2} = I_{R.M.S.}$ or I_υ (Virtual current)

and $E_0/\sqrt{2} = E_{R.M.S.}$ or E_υ (Virtual e.m.f.)

We may write that $$I_\upsilon = \frac{E_\upsilon}{\sqrt{(R^2+\omega^2L^2)}} \quad \ldots(9)$$

and also that $$\frac{E_\upsilon}{I_\upsilon} = \frac{E_{R.M.S.}}{I_{R.M.S.}} = \sqrt{(R^2+\omega^2L^2)} = Z \quad \ldots(10)$$

which tells that effective resistance = effective e.m.f./effective current.

Vector or Phase diagram : The vector diagram for above circuit may be plotted on the basis of the fact that the potential difference across the resistance ($E_R = RI_0$) always remains in phase with the current by across inductance ($E_L = X_L I_0 = \omega L I_0$) always lead over current by 90°. So E_R may be represented along current line of the circuit and E_L at 90° ahead to E_R shown in Fig (22) representing vector diagram of LR circuit by law of parallelogram of vector additions, we write that :

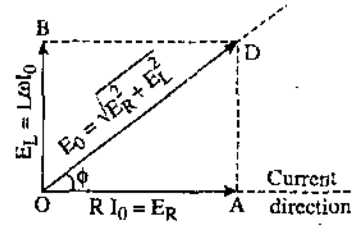


Fig. 22

(i) The diagonal OD will represent the sum of E_L and E_R *i.e.*, the resultant e.m.f. E_0 of the circuit.

(ii) The magnitude of E_0 will be given by

$$OD^2 = OA^2 + AD^2$$

or $$E_0^2 = (RI_0)^2 + (L\omega I_0)^{1/2}$$

or $$I_0 = \frac{E_0}{\sqrt{(R^2+\omega^2L^2)}}$$

which is same as equation (6).

(iii) The angle of E_0 from current line OA (*i.e.*, phase angle between current and voltage) will be given by

$$\tan\phi = \frac{AD}{OA} = \frac{I_0\omega L}{RI_0} = \frac{\omega L}{R}$$

which is same as eq. (7).

Thus we found I_0 and ϕ by vector diagram method.

2.13. A.C. CIRCUIT CONTAINING A RESISTANCE AND CAPACITANCE IN SERIES

Let an alternating e.m.f. $E = E_0 \sin \omega t$ be applied to a circuit containing capacity, C, and a resistance, R in series, [Fig. (23)] at any instant (t). Let I be the instantaneous value of the current and q the charge on the condenser so that the potential difference across the condenser at this instant is q/C. Since this acts opposite to the applied e.m.f. the effective e.m.f. in the circuit will be $\left(E_0 \sin \omega t - \frac{q}{C}\right)$, which by Ohm's law must be equal to resistance × current *i.e.*, we may write

$$E_0 \sin \omega t - \frac{q}{C} = RI$$

or
$$RI + \frac{q}{C} = E_0 \sin \omega t$$

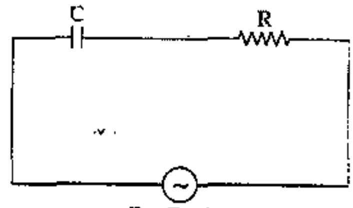


Fig. 23

Now, differentiating it, we get

$$R\frac{dI}{dt} + \frac{1}{C}\frac{dq}{dt} = E_0 \omega \cos \omega t$$

As
$$\frac{dq}{dt} = I,$$

we have
$$R\frac{dI}{dt} + \frac{I}{C} = E_0 \omega \cos \omega t \quad \text{...(1)}$$

Now let the trial sol. of eq. (1) be of the form

$$I = I_0 \sin (\omega t + \phi) \quad \text{...(2)}$$

where I_0 and ϕ are constants to be determined as follows :

Differentiating eq. (2), we get

$$\frac{dI}{dt} = I_0 \omega \cos (\omega t + \phi)$$

Substituting for I and $\frac{dI}{dt}$ in eq. (1), we have

$$\omega R I_0 \cos (\omega t + \phi) + \frac{I_0}{C} \sin (\omega t + \phi) = E_0 \omega \cos \omega t$$

$$= E_0 \omega \cos (\omega t + \phi - \phi)$$

$$= E_0 \omega [\cos (\omega t - \phi) \cos \phi + \sin (\omega t - \phi) \sin \phi]$$

Equating the coefficients of $\cos (\omega t - \phi)$ and $\sin (\omega t - \phi)$ on both sides, we get

$$R I_0 \omega = E_0 \omega \cos \phi \quad \text{...(3)}$$

and
$$I_0 / C = E_0 \omega \sin \phi \quad \text{...(4)}$$

Further squaring and adding eq. (3) and (4), we get

$$I_0^2 \left(R^2 \omega^2 + \frac{1}{C^2} \right) = E_0^2 \omega^2$$

or
$$I_0^2 \left(R^2 + \frac{1}{\omega^2 C^2} \right) = E_0^2$$

or
$$I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \quad \text{...(5)}$$

and dividing eq. by (4) by (3) we get

$$\tan \phi = \frac{1}{R\omega C} = \frac{1/\omega C}{R} \quad \text{...(6)}$$

Putting the value of I_0 from (5) in eq. (2), we get

$$I = \frac{E_0}{\sqrt{R^2 + 1/\omega^2 C^2}} \sin (\omega t + \phi) \quad \text{...(7)}$$

where
$$\phi = \tan^{-1} \frac{1/\omega C}{R}.$$

It may be noted that :

(i) The amplitude *i.e.*, the maximum current is given by

$$I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

(ii) The current leads over the e.m.f. by an angle $\phi = \tan^{-1} \frac{1/\omega C}{R}$.

(iii) $\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$ play the same role in this A.C. circuit as resistance R in a D.C. circuit. It is called impedance and denoted by Z. Further since $1/\omega C$ is called the capacitive reactance, X_C, so we can write

$$Z = \sqrt{(R^2 + X_C^2)} = \sqrt{\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]} \qquad \ldots(8)$$

Vector diagram : *The value of I_0 and phase, ϕ, may also be obtained by vector diagram* [Fig. (24)]. It may be plotted and described in a manner similar to [Fig. (22)] of LR circuit but here as shown $E_C = X_R I_0 = \frac{I_0}{\omega C}$ will lag behind $E_R = RI_0$. From figure, we find

$$OD^2 = OA^2 + AD^2 \quad \text{or} \quad E_0^2 = (RI_0)^2 + \left(\frac{I_0}{\omega C}\right)^2$$

or

$$I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

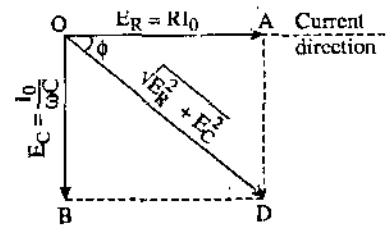


Fig. 24

which is same as eq. (5).

(ii) $$\tan\phi = \frac{AD}{OA} = \frac{I_0/\omega C}{I_0 R} = \frac{1/\omega C}{R}.$$

which is same as eq. (6).

• 2.14. A.C. CIRCUIT CONTAINING INDUCTANCE AND CAPACITANCE IN SERIES

Let an alternating e.m.f. $E = E_0 \sin \omega t$ be applied to a circuit containing an inductance, L, and capacitance, C, in series, [Fig. (25)].

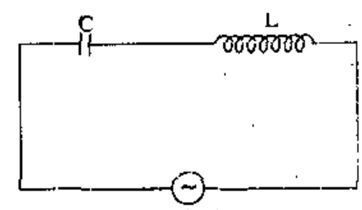


Fig. 25

The eqn. of e.m.f. is

$$L\frac{dI}{dt} + \frac{q}{C} = E = E_0 \sin \omega t$$

Differentiating it,

$$L\frac{d^2I}{dt^2} + \frac{1}{C}\frac{dq}{dt} = E_0\,\omega \cos \omega t$$

$$L\frac{d^2I}{dt^2} + \frac{I}{C} = E_0\,\omega \cos \omega t \qquad \ldots(1)$$

Let the trial sol. of eqn. be of the form

$$I = I_0 \sin(\omega t - \phi) \qquad \ldots(2)$$

where I_0 and ϕ are the two constants, to be determined as follows:

From eq. (2) we may write

$$\frac{d^2I}{dt^2} = -I_0\,\omega^2 \sin(\omega t - \phi)$$

Now substituting for I and $\frac{d^2I}{dt^2}$ in eqn. (1), we get

$$-LI_0\omega \sin(\omega t - \phi) + \frac{I_0}{C}\sin(\omega t - \phi) = E_0\omega \cos \omega t$$

$$= E_0\omega \cos(\omega t - \phi + \phi)$$

$$= E_0\omega\,[\cos(\omega t - \phi)\cos\phi - \sin(\omega t - \phi)\sin\phi]$$

Equating the coefficients of $\sin(\omega t - \phi)$ and $\cos(\omega t - \phi)$ on both the sides, we get

$$-LI_0\omega + \frac{I_0}{C} = -E_0\omega \sin\phi \qquad \ldots(3)$$

$$0 = E_0\,\omega \cos\phi \qquad \ldots(4)$$

Squaring and adding eqns. (3) and (4), we get

$$\left(-L\omega+\frac{1}{C}\right)^2 I_0^2 = E_0^2\,\omega^2$$

$$\left(\omega L-\frac{1}{\omega C}\right)^2 I_0^2 = E_0^2$$

or
$$I_0=\frac{E_0}{\left(\omega L-\frac{1}{\omega C}\right)} \qquad \ldots(5)$$

and dividing eq. (3) by (4) we get

$$\tan\phi=\infty$$

$$\phi=\frac{\pi}{2} \qquad \ldots(6)$$

Putting the value of I_0 in eq. (2), we get

$$I=\frac{E_0}{\left(\omega L-\frac{1}{\omega C}\right)}\sin(\omega t-\pi/2) \qquad \ldots(7)$$

It may be noted that :

(i) The current, at all instants, lags behind the e.m.f. by an angle $\pi/2$.

(ii) The amplitude *i.e.*, the maximum current is given by

$$I_0=\frac{E_0}{\left(\omega L-\frac{1}{\omega C}\right)}$$

(iii) $\left(\omega L-\frac{1}{\omega C}\right)$ plays the same role in this A.C. circuit as resistance R in a D.C. circuit. Thus here impedance is $Z=\left(\omega L-\frac{1}{\omega C}\right)$. Further as ωL and $\frac{1}{\omega C}$ are known respectively as inductive reactance X_L and capacitive reactance X_C, we may write that in this case

$$Z=X_L-X_C=\left(\omega L-\frac{1}{\omega C}\right) \qquad \ldots(8)$$

(iv) The current will be infinite (*i.e.*, maximum) when,

$$\left(\omega L-\frac{1}{\omega C}\right)=0 \;\text{ or }\; \omega=\frac{1}{\sqrt{LC}} \;\text{ or }\; f=\frac{1}{2\pi/\sqrt{LC}}$$

Since $\frac{1}{2\pi\sqrt{(LC)}}$ is also the resonant frequency of an LC circuit, we can infer that the amplitude of the current in LC circuit will be maximum when the frequency of applied e.m.f. will be equal to the natural frequency of the circuit. This is called the condition of resonance.

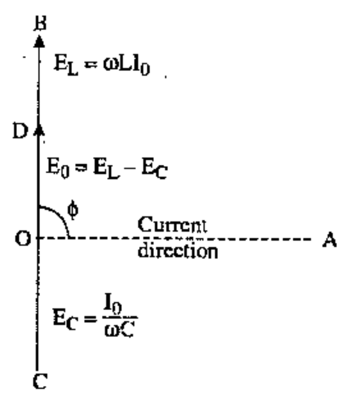


Fig. 26

Vector diagram : The vector diagram of LC circuit is illustrated in Fig. (26). Here the direction $E_L = I_0\omega L$ will be 90° ahead of direction of current and direction of $E_C = I_0/\omega C$ will lag by 90°. It is evident from figure that

(i) $OD = OB - OC$ or $E_0 = I_0\omega_L - \frac{I_0}{C\omega}$

or
$$I_0=\frac{E_0}{L\omega-\frac{1}{\omega C}}$$ which is same as eq. (5).

(ii) Direction of resultant e.m.f. will be 90° ahead of the current which proves eq. (6).

• 2.15. A.C. CIRCUIT CONTAINING RESISTANCE, INDUCTANCE AND CAPACITANCE (SERIES RESONANT CIRCUIT)

Let an alternating e.m.f. $E = E_0 \sin \omega t$ applied to a circuit containing an inductance L, capacitance, C, and resistance R as shown in Fig. (27). Let at any instant, I, be the current in the circuit and q be the charge on the condenser, then the e.m.f. in *different parts of the circuit will be as follows :*

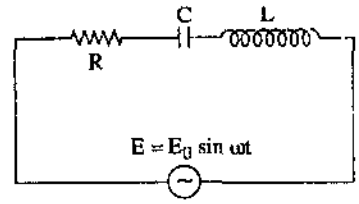


Fig. 27

(i) potential difference across the resistance $= RI$

(ii) potential difference across the capacitance $= q/C$

(iii) back e.m.f. due to self-inductance in $L = L\dfrac{dI}{dt}$.

The equation of e.m.f. therefore may be written as

$$L\frac{dI}{dt} + RI + q/C = E_0 \sin \omega t$$

Differentiating, we get

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}\frac{dq}{dt} = E_0\omega \cos \omega t$$

Putting $\dfrac{dq}{dt} = I$, we get

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C} = E_0\omega \cos \omega t \quad ...(1)$$

Let the trial solution be of the form

$$I = I_0 \sin (\omega t - \phi) \quad ...(2)$$

where I_0 and ϕ are constants to be determined as follows.

Differentianting (2), we get

$$\frac{dI}{dt} = I_0\omega \cos (\omega t - \phi)$$

and

$$\frac{d^2I}{dt} = -I_0\omega^2 \sin (\omega t - \phi)$$

Putting I, $\dfrac{dI}{dt}$ and $\dfrac{d^2I}{dt^2}$ in eq. (1), we get

$$-LI_0\omega^2 \sin (\omega t - \phi) + RI_0\omega \cos (\omega t - \phi) + \frac{I_0}{C} \sin (\omega t - \phi) = E_0\omega \cos \omega t$$

$$-LI_0\omega^2 [\sin \omega t \cos \phi - \cos \omega t \sin \phi] + RI_0\omega$$

$$[\cos \omega t \cos \phi + \sin \omega t \sin \phi] + \frac{I_0}{C} [\sin \omega t \cos \phi - \cos \omega t \sin \phi] = E_0\omega \cos \omega t \quad ..(3)$$

Now when $\omega t = \pi/2$, $\sin \omega t = 1$ and $\cos \omega t = 0$.

Eq. (3) becomes,

$$-LI_0\omega^2 \sin \phi + RI_0\omega \sin \phi + \frac{I_0}{C} \cos \phi = 0$$

$$\therefore \quad \tan \phi = \frac{\omega L - 1/\omega C}{R} \quad ...(4)$$

and if we draw Fig. (28) satisfying eq. (4), we may also write

$$\sin \phi = \frac{\omega L - 1/\omega C}{\sqrt{\left[R^2 + \left(\omega L - \dfrac{L}{\omega C}\right)^2\right]}}$$

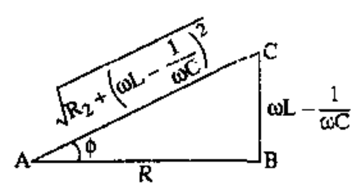


Fig. 28

and

$$\cos \phi = \frac{R}{\sqrt{\left[R^2 + \left(\omega L - \dfrac{L}{\omega C}\right)^2\right]}}$$

Further when $\omega t = 0$, $\sin \omega t = 0$ and $\cos \omega t = 1$, then eq. (3) becomes

$$LI_C\omega^2 \sin \phi + RI_0\omega \cos \phi - \frac{I_0}{C} \sin \phi = E_0\omega$$

or

$$\left(\omega L + \frac{1}{\omega C}\right) I_0 \sin \phi + RI_0 \cos \phi = E_0 \quad ...(5)$$

Substituting the value of $\sin\phi$ and $\cos\phi$ as above, we have

$$\left(\omega L-\frac{1}{\omega C}\right)I_0\frac{\left(\omega L-\frac{1}{\omega C}\right)}{\sqrt{\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]}}+RI_0\frac{R}{\sqrt{\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]}}$$

$= E_0$

or $$I_0\left[\frac{\left(\omega L-\frac{1}{\omega C}\right)^2}{\sqrt{\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]}}+\frac{R^2}{\sqrt{\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]}}\right]=E_0$$

or $$I_0\sqrt{\left[R^2+\left(\omega L-\frac{L}{\omega C}\right)^2\right]}=E_0$$

or $$I_0=\frac{E}{\sqrt{\left[R^2+\left(\omega L-\frac{L}{\omega C}\right)^2\right]}} \quad \text{...(6)}$$

Substituting this value of I_0 in eq. (2), we get

$$I_0=\frac{E}{\sqrt{\left[R^2+\left(\omega L-\frac{L}{\omega C}\right)^2\right]}}\sin(\omega t-\phi) \quad \text{...(7)}$$

where $$\phi=\tan^{-1}\left[\frac{\left(\omega L-\frac{1}{\omega C}\right)}{R}\right] \quad \text{by eq. (4).}$$

(i) The maximum current is given by $I_0=\dfrac{E_0}{\sqrt{\left[R^2+\left(L\omega-\frac{1}{C\omega}\right)^2\right]}}$

and it is evident from it that $\sqrt{\left[R^2+\left(\omega L-\frac{1}{C\omega}\right)^2\right]}$ plays the same role in this A.C. circuit as a resistance in a d.c. circuit. Thus this represents the impedance Z of the circuit. Further since $L\omega$ and $\frac{1}{C\omega}$ respectively represent inductive reactance X_L and capacitive reactance X_C, we may write

$$Z=\sqrt{\left[R^2+\left(L\omega-\frac{L}{C\omega}\right)^2\right]}=\sqrt{[R^2+(X_L-X_C)^2]} \quad \text{...(8)}$$

(ii) The current lags in phase behind e.m.f. by an angle

$$\phi=\tan^{-1}\frac{\omega L-1/\omega C}{R}$$

so that

(a) when $\omega L>\frac{1}{\omega C}$, ϕ is positive, *i.e.*, current lags behind the applied e.m.f.

(b) when $\omega L=\frac{1}{\omega C}$, $\phi=0$ *i.e.*, the current is in phase with the e.m.f.

(c) when $\omega L<\frac{1}{\omega C}$, ϕ is negative, *i.e.*, the current leads the applied e.m.f.

Vector Diagram : A vector diagram of a series LCR circuit is shown in Fig. (29a). It predicts that

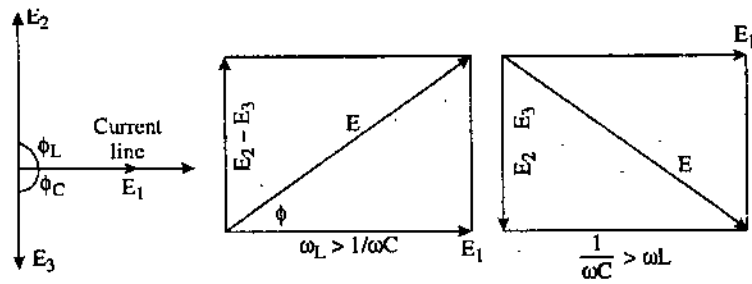


Fig. 29

(i) the vector $E_1 = I_0 R$ is in phase with the current.

The vector $E_2 = \omega L I_0$ is 90° is in advance of current.

The vector $E_3 = I_0/\omega C$, is 90° behind the current.

(ii) $$E_0 = \sqrt{[(E_1)^2 + (E_2 - E_3)^2]}$$

$$= I_0 \sqrt{\left[R^2 + \left(\omega L - \frac{L}{\omega C}\right)^2\right]}$$

(iii) $$\tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}, \text{ if } \omega L > \frac{1}{\omega C} \text{ [Fig. 29(b)]}$$

$$\text{and } \tan\phi = \frac{-\left(\omega L - \frac{1}{\omega C}\right)}{R}, \text{ if } \omega L < \frac{1}{\omega C} \text{ [Fig. 29(c)]}$$

Condition of resonance : When the inducting reactance X_L is equal to the capacitive reactance X_C, the impedance Z of the circuit will be minimum and hence the current will be minimum. This is the case of electrical resonance. Hence at resonance

$$X_L = X_C$$

or $$\omega L = \frac{1}{\omega C}$$

or $$\omega = \frac{1}{\sqrt{LC}}$$

$\Rightarrow$ $$f = \frac{1}{2\pi\sqrt{LC}}.$$

• 2.16. PARALLEL RESONANT CIRCUIT

Let an alternating voltage $E = E_0 \sin \omega t$ is applied to the circuit having an inductance L and resistance R in parallel with a capacitance C. In this position the current in the inductance is given by

$$i_1 = \frac{E_0}{\sqrt{R^2 + X_L^2}}$$

where $X_L = \omega L$ and the current lags behind E_0 by an angle θ where

$$\tan\theta = \frac{X_L}{R} = \frac{\omega L}{R}$$

The current in capacitance is

$$i_2 = \frac{E_0}{X_C}$$

where $X_C = \dfrac{1}{\omega C}$ and current lead E_0 by 90°.

In the fig. (30) i_1 and i_2 may be represented by OA and OB.

The impedance of the parallel circuit will be given by

$$Z = \frac{E_0}{i_0}$$

where i_0 is resultant current.

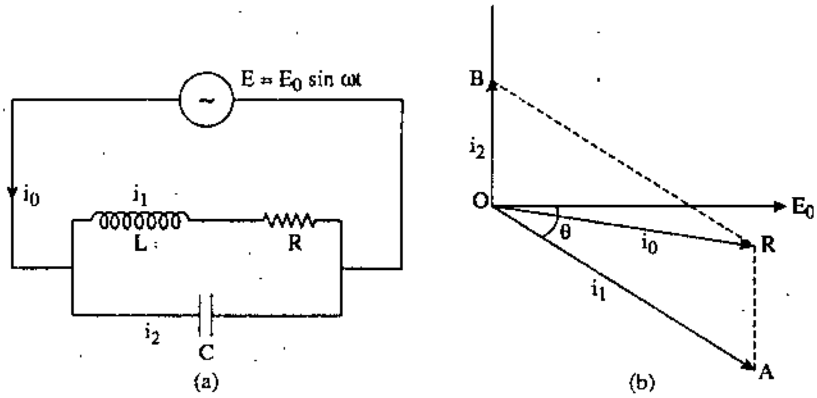


Fig. 30

At resonance : In this position the diagonal OR lies along the direction of E_0 which is shown in the below fig. 31.

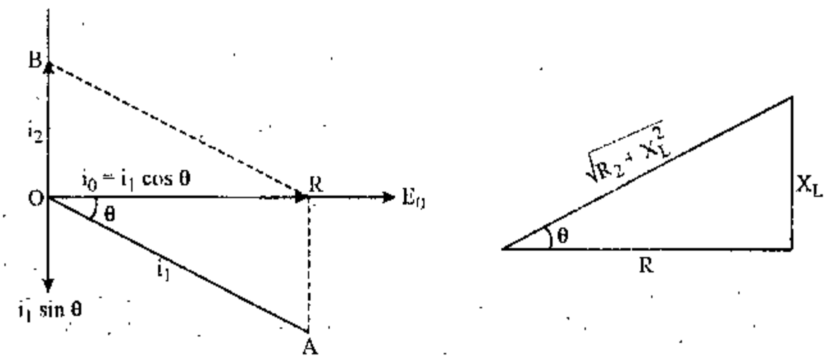


Fig. 31

From fig. 31

$$i_2 = i \sin \theta \qquad \dots (1)$$

$$i_0 = i_1 \cos \theta \qquad \dots (2)$$

$\therefore$
$$i_2 = \frac{E_0}{X_C}$$

and
$$i_1 = \frac{E_0}{\sqrt{R^2 + X_L^2}}$$

and
$$\sin \theta = \frac{X_l}{\sqrt{R^2 + X_L^2}}$$

By eq. (1)

$$\frac{E_0}{X_C} = \frac{E_0}{\sqrt{R^2 + X_L^2}} \cdot \frac{X_L}{\sqrt{R^2 + X_L^2}}$$

$$X_L X_C = R^2 + X_L^2 \qquad \dots (3)$$

$$\frac{\omega L}{\omega C} = R^2 + \omega^2 L^2$$

$$\frac{1}{LC} = \frac{R^2}{L^2} + \omega^2$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

Resonant frequency is given by

$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}}$$

This is the required expression for the resonant frequency of the circuit. If $R << L$ then resonant frequency is

$$\boxed{f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}}$$

Impedance of the circuit is

$$Z = \frac{E_0}{i_0} \qquad \ldots (4)$$

$$\because \quad i_1 = \frac{E_0}{\sqrt{R^2 + X_L^2}} \text{ and } \cos\theta = \frac{R}{\sqrt{R^2 + X_L^2}}$$

So by eq. (2)

$$i_0 = \frac{E_0}{\sqrt{R^2 + X_L^2}} \cdot \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{E_0 R}{R^2 + X_L^2}$$

$$i_0 = \frac{E_0 R}{X_L X_C} = \frac{E_0 R}{\omega L / \omega C} = \frac{E_0 RC}{L}$$

$$\therefore \quad \frac{E_0}{i_0} = \frac{L}{RC}$$

So by eq. (4), we get

$$\boxed{Z = \frac{L}{RC}}$$

This is expression for impedance.

From this expression it is clear that if R is negligible then the impedance is infinite at resonance.

• 2.17. QUALITY FACTOR (Q) OF A SERIES RESONANT CIRCUIT

We know that if the value of resistance in the LCR-series circuit is very low then a large current flows, when angular frequency of the a.c. source is close to the resonant frequency ω_0. In this position such an LCR-series circuit is said to be **more selective** or **more sharp**. Hence for a sharp resonance in LCR circuit the resistance of the circuit should be low.

For the selectivity or sharpness of a resonant circuit is measured by **Q-factor**. This is called **quality factor**. The quality factor may be defined as *"the ratio of the voltage drop through inductor or capacitor to the applied voltage is known as quality factor Q"* i.e.,

$$\boxed{Q = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}}}$$

In the condition of resonance the voltage across L is

$$V_L = I X_L$$

and applied voltage is $V = IR$. So quality factor is

$$Q = \frac{IX_L}{IR} = \frac{\omega_0 L}{R}$$

since

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

so

$$Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

This is the required expression for the quality factor in LCR circuit.

Q is just a number and also known as *voltage multiplication* factor of the circuit.

• 2.18. SHARPNESS OF RESONANCE

When an alternating e.m.f., $E_0 \sin \omega t$ is applied to an LCR circuit, electrical oscillations occur in the circuit with the frequency ω of the applied e.m.f. The amplitude of these oscillations *i.e.*, current amplitude is given by

$$I_0 = \frac{E_0}{\sqrt{\left[R^2 + \left(\omega L - \frac{L}{\omega C}\right)^2\right]}}$$

[where $\sqrt{\left[R^2+\left(\omega L-\frac{L}{\omega C}\right)^2\right]}$ is the impedance Z of the circuit.]

At resonance when the frequency ω of the applied e.m.f. is equal to the natural frequency $\omega_0\left(=\frac{1}{\sqrt{LC}}\right)$ of the circuit, i_0 is maximum and is equal to E_0/R. The impedance Z of the circuit is now equal to R. At other values of ω, i_0 is smaller and hence Z will be larger than R. The variation of i_0 with frequency ω of applied e.m.f. is shown in Fig. (32).

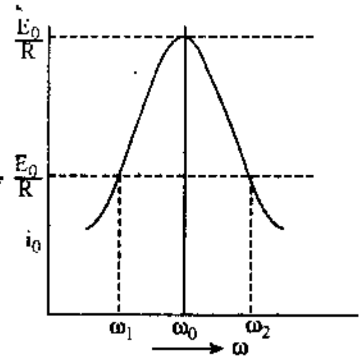


Fig. 32

The rapidity with which the current falls from it's resonant value (E_0/R) with change in applied frequency is known as 'sharpness of resonance'. It is measured as the ratio of resonant frequency ω_0 to the difference of two frequencies ω_1 and ω_2 *i.e.*, sharpness of resonance $=\frac{\omega_0}{\omega_2-\omega_1}$, ω_1 and ω_2 are known as 'half power frequencies' because at these frequencies the power in the circuit reduces to half its maximum value. The difference of half power frequencies $\omega_2-\omega_1$ is called as 'band-width'. The smaller is the band width, the sharper is the resonance.

Relation between sharpness of resonance and Q : At resonant frequency ω_0 the impedance is R, therefore at ω_1 and ω_2 it must be $\sqrt{2}\,.\,R$ *i.e.*,

$$Z=\sqrt{\left[R^2+\left(\omega L-\frac{L}{\omega C}\right)^2\right]}=\sqrt{2}\,.\,R$$

or
$$R^2+\left(\omega L-\frac{L}{\omega C}\right)^2=2R^2$$

or
$$\left(\omega L-\frac{L}{\omega C}\right)^2=R^2$$

or
$$\left(\omega L-\frac{L}{\omega C}\right)=\pm R$$

Thus if $\omega_2>\omega_1$, we can write

$$\omega_1 L-\frac{1}{\omega_1 C}=-R.$$

and
$$\omega_2 L-\frac{1}{\omega_2 C}=+R$$

Adding and subtracting, we get

$$\omega_1\omega_2=\frac{1}{LC}\quad\text{and}\quad\omega_2-\omega_1=\frac{R}{L}.$$

This is the expression for the bandwidth.

The Q of LCR circuit is

$$Q=\frac{\omega_0 L}{R}\Rightarrow\frac{R}{L}=\frac{\omega_0}{Q}$$

Hence
$$\omega_2-\omega_1=\frac{\omega_0}{Q}\quad\text{or}\quad Q=\frac{\omega_0}{\omega_2-\omega_1},$$

but $\frac{\omega_0}{\omega_2-\omega_1}$ is a measure of sharpness of resonance. Thus the quality factor Q directly measures the sharpness of resonance.

• 2.19. CHOKE COIL

The choke coil is the electrical device which is used for controlling current in an a.c. circuit without wasting electrical energy in the form of heat.

Principle : A choke coil is based upon the principle that when a.c. flows through an inductor then the current lags the e.m.f. by a phase angle $\frac{\pi}{2}$.

Construction : It consists of a coil of wire in which large number of turns of insulated copper wire wound over a soft iron core. In it a laminated iron core is used which minimum the loss of electrical energy due to production of eddy current.

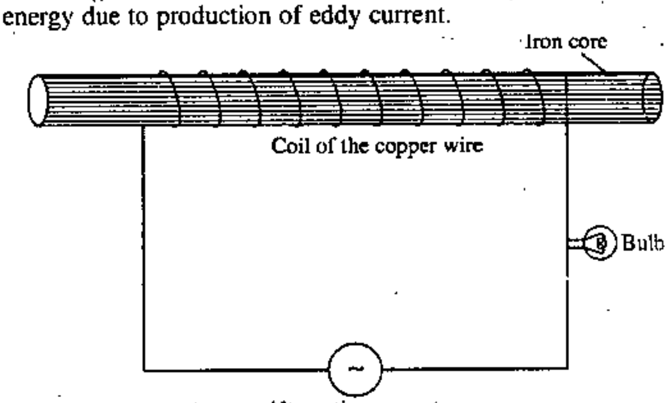


Fig. 33

In practice, a low frequency choke coil is made of insulated copper wire wound on a soft iron core, while a high frequency choke coil has air in place of core material.

Working : In the fig. 33 a choke coil is connected in series with the electrical device. The inductive reactance decreases the current. Since the alternating e.m.f. leads the current by phase angle $\pi/2$. In this position the average power consumed by the choke coil is given by

$$P = E_0 I_0 \cos\frac{\pi}{2} = 0$$

$$\boxed{P = 0}$$

However, a practical inductance consists a small resistance *i.e.*, a practical inductance may be considered as a series combination of inductance L and small resistance R so average power will be

$$P = E_0 I_0 \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

where $\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$ which is known as power factor for particle inductance.

Uses :

(1) The choke coil is used to control the current in place of resistance. If, we use a resistance to controlled the current then electrical energy will be wasted in the form of heat.

(2) A choke coil is used to control the current without wasting electrical energy in the form of heat.

Preference of choke coil over a resistance : The current in an a.c. circuit can also be reduced by using an ordinary resistance (rheostat). But this method is not economical because the electrical energy supplied (i^2Rt) by the source is wasted as heat. Hence choke coil is preferred over the resistance.

• SUMMARY

- Kirchhoff's second law shows the electrostatic force, which is a conservative force and the work done by it in any closed path is zero.
- According to Wheatstone's bridge. $\frac{P}{Q} = \frac{R}{S}$.
- In charging of a capacitor, the product C_k is is called the capacitive time constant of the circuit.

$$i_0 = g_0\omega = \frac{q_0}{\sqrt{LC}} = \frac{CE}{\sqrt{LC}}$$

- Energy stored in an inductor, $U = \frac{1}{2} L\, i_0^2$.

- Energy stored in a capacitor, $U = \frac{1}{2} q_0 V_0$.
- The value of current at any instant in A.C. is given by

$$I = I_0 \sin \omega t$$

where I_0 is the peak value.

- Resonant frequency; $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$ if $R << L$ then $f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}$.
- Quality factor (Q) of a series resonant circuit,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \text{(in } LCR \text{ circuit)}$$

Q is just a number and also known as voltage multiplication factor of the circuit.

- The choke coil is an electrical device which is used for controlling current in A.C. circuit without wasting heat-energy.

• STUDENT ACTIVITY

1. Define mean value of alternating current.

2. Define root mean square value of alternating current.

3. Give the relation between mean value and r.m.s. value of current.

4. Explain lag and lead of current.

5. Explain acceptor circuit.

6. Why a parallel resonant circuit is called a rejector circuit ?

• TEST YOURSELF

1. State and explain Kirchhoff's laws for the distribution of current in a network.
2. Applying Kirchhoff's laws to deduce the condition of balance in a Wheatstone's Bridge.
3. Explain how the presence of resistance and an inductance affects the growth and decay of current in a circuit.
4. Discuss the charging and discharging of a condenser through resistance.
5. Show that when a condenser is discharged through an inductance, the discharge is oscillatory. What will be its time period and frequency ?
6. Derive an expression for the growth of charge in a circuit containing a condenser, an inductance and a resistance. Under what conditions the charging will be oscillatory and then what will be its frequency ?
7. Give the theory of oscillatory discharge of a condenser through a circuit containing an inductance and a resistance. Obtain an expression for the frequency of discharges.
8. Describe with necessary theory, the method of measuring high resistance by leakage method.
9. Obtain an expression for the energy stored in an inductor and in a capacitor.
10. An alternating e.m.f. is applied to (a) Pure resistance (b) Pure inductance and (c) Pure capacitance. Investigate the phase relationship of the alternating current with alternating e.m.f. in each case.
11. Obtain an expression for current and phase diagram in A.C. circuit containing a resistance and an inductance.
12. An alternating e.m.f. is aplied to a circuit containing a resistance and capacitance in series. Derive an expression for the impedance, current and phase in the circuit. Draw a vector diagram.
13. Obtain an expression for the current and phase angle in a circuit containing a capacitance and an inductance in series.
14. An e.m.f. $E = E_0 \sin \omega t$ is applied to a circuit containing resistance R, inductance L, and capacitance C in farlef. Calculate the current at any instant.
15. Explain Parallel resonant circuit.
16. What do you understand by the quality factor Q of an oscillating circuit ?
17. What do you mean by sharpness of resonance ? How it is related with the quality factor Q of the circuit ?
18. Explain the principle, construction and working of choke coil.
19. In the equation of A.C., $I = I_0 \sin \omega t$, the current amplitude and frequency will respectively be:

   (a) $I_0, \dfrac{\omega}{2\pi}$ (b) $\dfrac{I_0}{2}, \dfrac{\omega}{2\pi}$ (c) $I_{rms}, \dfrac{\omega}{2\pi}$ (d) I_0, ω

20. If the phase difference between the e.m.f. and the current in an A.C. circuit is ϕ then the R.M.S. value of wattless current will be :
(a) $I_{rms} \cos \phi$ (b) $I_{rms} \sin \phi$ (c) $I_{rms} \tan \phi$ (d) zero
21. The impedance of a series L-C-R circuit is
(a) $R+\left(\omega L-\frac{1}{\omega C}\right)$ (b) $\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$
(c) $\sqrt{R^2+\left(\omega L+\frac{1}{\omega C}\right)^2}$ (d) $R+\left(\omega L+\frac{1}{\omega C}\right)$
22. The range of values of power factor is :
(a) 0 to 1 (b) 0 to -1 (c) 0 to ∞ (d) 2 to ∞
23. The unit of susceptance is :
(a) ohm (b) ohm^{-1} (c) ohm-cm (d) ohm-m
24. The R.M.S. value of effective current is :
(a) $I_{rms} \cos \theta$ (b) $I_{rms} \sin \theta$ (c) $I_0 \cos \theta$ (d) $I_0 \sin \theta$
25. In an A.C. circuit $X_L = X_C$ then the value of power factor will be :
(a) 1 (b) 0 (c) Infinity (d) $\frac{1}{2}$
26. The phase difference between alternating e.m.f. and current in capacitive circuit will be :
(a) zero (b) π (c) $-\pi/2$ (d) $+\pi/2$
27. The power-factor of wattless current is :
(a) infinity (b) one (c) zero (d) $\frac{1}{2}$
28. The quality factor of an A.C. circuit is related to band width as :
(a) inversely proportional (b) directly proportional
(c) directly proportional to log (d) inversely proportional to log
29. Choke-coil in an A.C. circuit is used for :
(a) decreasing AC (b) decreasing AV
(c) increasing AC (d) increasing AV
30. In series LCR resonant circuit, to increase the resonant frequency :
(a) L will have to be increased
(b) C will have to be increased
(c) LC will have to be decreased
(d) LC will have to be increased

True/False Type Questions

31. The power loss in choke coil is negligible.
32. The R.M.S. value of wattless current is $\frac{I_0}{2} \sin \theta$.
33. Power loss due to effective current is
$$2\, E_{rms} I_{rms} \cos \theta.$$
34. Apparent power is equal to maximum value of average power.
35. Quality factor represents sharpness of resonance.
36. LCR parallel resonant circuit are used for current amplification.
37. In series LCR resonant circuit, the current and voltage are in same phase in the case of resonance.
38. The reciprocal of resistance is called reactance.
39. In purely resisitive A.C. circuit, the value of Z and X are zero and one respectively.
40. The square root of peak value of A.C. is defined as the R.M.S. value of A.C.

ANSWERS

19. (a) 20. (b) 21. (b) 22. (a) 23. (b) 24. (a) 25. (a) 26. (c) 27. (c) 28. (a)
29. (a) 30. (c) 31. True 32. False 33. False 34. True 35. True
36. True 37. True 38. False 39. False 40. False

UNIT

3

MAGNETICS AND MAGNETIC PROPERTIES OF MATTER

STRUCTURE

LEARNING OBJECTIVES

After going this unit you will learn :

- Lorentz force due to electric field and magnetic field.
- Tracks of the electrons in the bubble chamber are spirals.
- The principal and construction of cyclotron and its limitations.
- Biot-Savart's law and its applications.
- Magnetic intensity, magnetic flux, magnetic susceptibility, magnetic permeability and their relations.
- Classification of different substances on the basis of their magnetic properties.

• 3.1. MAGNETIC INDUCTION OR MAGNETIC FLUX DENSITY

We know that a magnetic field is the space around a magnet or around a current carrying conductor in which magnetic force can be experienced.

We see that when the switch of current is open then no current flows through the circuit *i.e.*, the magnetic field disappears and hence the motion of electrons in the wire produces a magnetic field *i.e., a moving charge is responsible for the magnetic field.*

Now we have to calculate the expression for the force on a moving charge in a magnetic field due to this expression we can define magnetic field or magnetic induction or magnetic flux density.

Let us consider a positive charge q which is moving in a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$. Let θ is the angle between $\vec{v}$ and $\vec{B}$. In this position the force acting on a moving charge depends upon the following factor :

(i) The magnitude of force $\vec{F}$ is directly proportional to the charge q *i.e.*, $F \propto q$

(ii) The magnitude of $\vec{F}$ is directly proportional to the component of velocity acting perpendicular to the direction of magnetic field.

i.e., $$F \propto v \sin\theta$$

(iii) The magnitude of $\vec{F}$ is directly proportional to the magnitude of $\vec{B}$ *i.e.*,

$$F \propto B$$

From above

$$F \propto qvB\sin\theta$$

$$\therefore \quad F = kqvB\sin\theta$$

where k is constant and its value is 1, *i.e.* $k = 1$

$$\therefore \quad F = qvB\sin\theta$$

$$\Rightarrow \quad B = \frac{F}{qv\sin\theta}$$

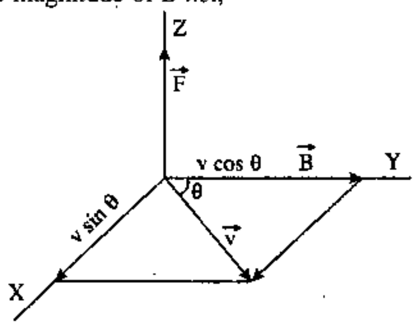


Fig. 1

if $q = 1$, $v = 1$, $\sin\theta = 1$ *i.e.* $\theta = 90°$

then $$\boxed{B = F}$$

Thus *"magnetic field induction at any point in the field is equal to the force which acts on a unit charge moving with a unit velocity perpendicular to the direction of magnetic field at that point".*

Now the following cases may arise :

(i) When $v = 0$ then $\boxed{F = 0.}$ This means that if a charge is at rest then in a magnetic field, no force act on it.

(ii) If $\theta = 0$ or $180°$ then $\boxed{F = 0.}$ This means that if a charged particle is moving parallel to the direction of magnetic field then no force act on it.

(iii) When $\theta = 90°$ then $F = qvB$ *i.e.*, max.

This means that force will be maximum. The direction of this force can be determined by using **Fleming's Left Hand Rule.**

Unit of $\vec{B}$: In S.I. system the unit of $\vec{B}$ is tesla (T) or weber/meter^2.

From $F = qvB\sin\theta \Rightarrow B = \dfrac{F}{qv\sin\theta}$

if $q = 1C$, $v = 1\ \text{ms}^{-1}$ and $\theta = 90°$ and $F = 1\ N$.

then $$\boxed{B = 1\ \text{T}}$$

Thus, the magnetic field induction at a point is said to be one tesla if a charge of one coulomb which moving at right angle to a magnetic field with a velocity 1 ms^{-1} experiences force 1 newton at that point.

Hence $$T = \frac{1\ \text{N}}{1\ \text{C} \times 1\ \text{ms}^{-1}} = \frac{1\ \text{N}}{1\ \text{As} \times 1\ \text{ms}^{-1}}$$

$$= 1\ \text{NA}^{-1}\ \text{m}^{-1} \qquad (1\ \text{C} = 1\ \text{Ampere} \times 1\ \text{second})$$

Dimension of $\vec{B}$: As

$$B = \frac{F}{qv\sin\theta}$$

$\therefore$ Dimension of $$B = \frac{[\text{MLT}^{-2}]}{[\text{AT}]\,[\text{LT}^{-1}]} = [\text{MA}^{-1}\ \text{T}^{-2}]$$

Hence the dimensions of B is $[MA^{-1}T^{2}]$.

• 3.2. LORENTZ FORCE

The Lorentz force may be defined as *"The force experienced by a charged particle moving in a space where both electric and magnetic fields exist is known as Lorentz force".*

Now we will calculate the expression for the Lorentz force.

(i) Force due to electric field : If a charged particle carrying charge $+q$ and let E is the electric field strength then force on charge is

$$\vec{F}_e = q\vec{E} \quad \text{... (1)}$$

(ii) Force due to magnetic field : We know the magnetic force on moving charged particle is

$$\vec{F}_m = q\,(\vec{v} \times \vec{B}) \quad \text{... (2)}$$

where $\vec{B}$ = magnetic field, $\vec{v}$ = velocity, q = charge.

Therefore, total force acting on a charged particle is equal to the electric force plus magnetic force.

i.e., $$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q\,(\vec{v} \times \vec{B})$$

$$\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$$

This is the required expression for the Lorentz force. Now the following cases may arise :

Case I : *When $\vec{v}, \vec{E}$ and $\vec{B}$, all the three are collinear :* In this position, the charged particle is moving parallel or antiparallel to the field and the magnetic force on the charged particle is zero while the electric force on the charge particle will produce acceleration along the direction of electric field. *i.e.,*

$$\because \quad \vec{F} = q\vec{E}$$

$$m\vec{a} = q\vec{E}$$

$$\boxed{\vec{a} = \frac{q\vec{E}}{m}}$$

Case II : *When $\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular to each other :* In this position, if $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0$$

then acceleration in the particle is

$$\vec{a} = \frac{\vec{F}}{m} = 0 \quad \Rightarrow \boxed{\vec{a} = 0}$$

This means that when a particle will pass through the fields then there is no change in its velocity.

Here $$F_e = F_m$$

i.e., $$qE = qvB$$

or $$\boxed{v = \frac{E}{B}}$$

This concept has been used in velocity selector to get a charged beam having a definite velocity.

• 3.3. MOTION OF A CHARGE IN A MAGNETIC FIELD

Consider a uniform magnetic field of magnetic induction $\vec{B}$ (same magnitude and direction at all points). Let a charged particle q is moving in the magnetic field $\vec{B}$ in a direction perpendicular to the field with velocity $\vec{v}$. The magnetic force on the charged particle is given by

$$\vec{F} = q\vec{v} \times \vec{B}.$$

Its magnitude is $F = qvB$ and direction perpendicular to both v and $\vec{B}$ and is given by right hand screw rule.

As the force is perpendicular to the velocity, therefore, it changes simply the direction of velocity without changing its magnitude. That is, a uniform circular motion is described by the particle (magnitude of force is constant while direction changes). The acceleration is then centripital and the centripital force is given by $F = mv^2/r$, where m is the mass of the particle and r the radius of the circle described.

$$\therefore \qquad F = qvB = \frac{mv^2}{r}$$

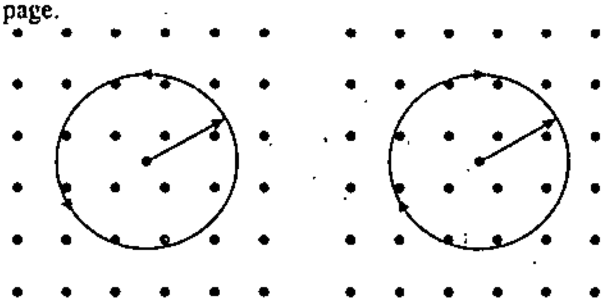
or
$$r = \frac{mv}{qB}$$

$$\therefore \qquad r \propto p\ (= mv)$$

The radius r of the path is proportional to the momentum mv of the charged particle.

If the direction of the initial velocity $\vec{v}$ of the particle is not perpendicular to the magnetic field $\vec{B}$, then the path of the particle is a helix.

A field perpendicular to the paper is represented by a dot (·). Fig. 2 (a) represents the path of a positive and fig. 2 (b) of a negative charge moving perpendicularly to a uniform magnetic field perpendicular to the page.



q positive; B upward (a) Fig. 2 q negative; B upward (b)

Tracks in a Bubble Chamber $r = mv/qB$, r is the radius of the circle of the electron (mass m and charge e) of velocity $\vec{v}$ in a magnetic field $\vec{B}$.

The electron suffers collisions with the liquid particles in the bubble chamber. Therefore, their energy goes on decreasing with decrease of velocity, the radius of the circle described by the electron also decreases. Thus the tracks of the electrons in the bubble chamber are spirals.

• 3.4. CYCLOTRON

It is a device which is used to accelerate the positive charged particles like protons, deutrons etc., so that they may be used for radio active disintegration.

It was invented by Lawrence and it is also known as **magnetic resonance accelerator.**

Principle : It is based upon the principle that *a positive ion or particle can acquire large energy with a smaller alternating potential difference by making them to cross the same electric field and again by making use of strong magnetic field".*

Construction : It consists of two metals plates D_1 and D_2 in the form of dees so they are known as dees. They are placed in a uniform magnetic field parallel to the axis and they are electrically insulated to each other. The dees are connected to an oscillator so that a potential of 10^5 volt and frequency about 10^7 cycle/sec may be established between the dees. The electric field in gap between dees will be once directed towards one dee and then towards the other *i.e.*, dees become positive and negative alternately at half period of the oscillator voltage.

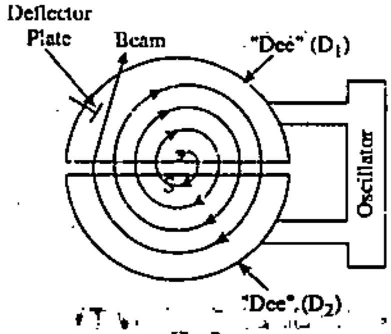


Fig. 3

An ion source S is placed in the gap between dees.

Theory and working : Let positive ion be emitted from the ion source at any instant and faces the dees D_2 which is at that instant negative. This positively charged ion will be accelerated by the electric field within the gap between the dees and will enter the dees with negative charge. Since the region inside dees will be free from electric field, the ion will move under the influence of magnetic field which is perpendicular to the dee and so it will travel in a circular path of radius

$$F = qvB$$

$$F = \frac{mv^2}{r}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\,(d\vec{l} \times \vec{r})}{r^3}}$$

Magnetic field due to current through entire wire at point P is

$$\boxed{\vec{B} = \int \frac{\mu_0}{4\pi} \cdot \frac{I\,d\vec{l} \times \vec{r}}{r^3}}$$

This is the required expression for the Biot-Savart's law.

Some important features of Biot-Savart's law are as follows:

(i) Biot-Savart's law is valid for a symmetrical current distribution.

(ii) Biot-Savart's law is applicable only to very small length of current carrying conductor.

(iii) This law can not be verified easily experimentally as the current carrying conductor of very small length can not be obtained practically.

(iv) This law is similar to the Coulomb's law in electrostatics.

(v) The direction of $d\vec{B}$ is perpendicular to $Id\vec{l}$ as well as $\vec{r}$.

(vi) If $\theta = 0°$ then $dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin \theta°}{r^2} = 0$

This means that there is no magnetic field induction at any point on the thin linear current carrying conductor.

(vii) If $\theta = 90°$ then $dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}$ which is maximum.

(viii) If $\theta = 0°$ or $180°$ then $dB = 0$ *i.e.*, minimum.

• 3.6. APPLICATION OF BIOT-SAVART'S LAW

Magnetic field due to a current carrying long, straight conductor : Let us consider a infinitely long conductor which is placed in vacuum and carrying a current i amperes. Let P is the point at which the magnetic field $d\vec{B}$ is to be determined.

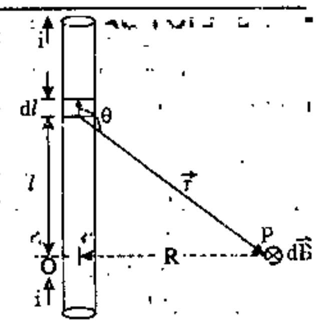


Fig. 5

Here $OP = R$ and $OA = l$. Let $d\vec{l}$ is the current element at A then according to Biot-Savart's law the magnetic induction $d\vec{B}$ at P due to current element $d\vec{l}$ is

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i\,d\vec{l} \times \vec{r}}{r^3} \quad \text{... (1)}$$

Its magnitude is given by

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl \sin \theta}{r^2} \quad \text{... (2)}$$

$d\vec{B}$ is right angles to the paper and directed into it.

$$\tan PAO = \tan (180° - \theta)$$

$$= -\tan \theta = \frac{R}{l}$$

or $$l = -R \cot \theta.$$

differentiating it, we get

$$dl = R \operatorname{cosec}^2 \theta \, d\theta. \quad \text{... (3)}$$

Also $$\sin PAO = \sin (180° - \theta) = \sin \theta = \frac{R}{r}$$

or $$r = R \operatorname{cosec} \theta. \quad \text{... (4)}$$

Putting the values of dl and r from eq. (3) and (4) in eq. (2), we get

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i\,R \operatorname{cosec}^2 \theta \, d\theta \sin \theta}{R^2 \operatorname{cosec}^2 \theta} = \frac{\mu_0}{4\pi} \cdot \frac{i \sin \theta \, d\theta}{R} \quad \text{... (5)}$$

The direction of the field due to each element is the same, so the resultant field at P due to the whole conductor will be obtained by integrating eq. (5). As the conductor is infinitely long, therefore the limits of integration will be from $\theta = 0$ (at the lower end) to $\theta = +\pi$ (at the upper end) of the conductor.

$\therefore$ The magnitude of the magnetic field at P is given by

$$B = \frac{\mu_0 i}{4\pi R}\int_0^{+\pi} \sin\theta \, d\theta$$

$$= \frac{\mu_0 i}{4\pi R}\Big[-\cos\theta\Big]_0^{+\pi}$$

$$= \frac{\mu_0 i}{4\pi R}[-\cos\pi + \cos(0)]$$

$$= \frac{\mu_0 i}{4\pi R}[1+1]$$

$$= \frac{\mu_0}{4\pi}\left(\frac{2i}{R}\right)$$

or $$B = \frac{\mu_0 i}{2\pi R} \text{ weber/meter}^2.$$

This is the expression for the magnetic induction or magnetic field intensity near a long straight conductor.

• 3.7. FORCES BETWEEN TWO PARALLEL CURRENT-CARRYING CONDUCTORS

Fig. 6 shows two long parallel straight conductors M and N carrying currents i_1 and i_2 respectively. They are placed in vacuum at a distance R apart. The magnitude of the magnetic induction at any point on conductor N due to the current i_1 in M is

$$B = \frac{\mu_0 i_1}{2\pi R}$$

The direction of B is perpendicular to the paper and directed into the paper (right-hand screw rule).

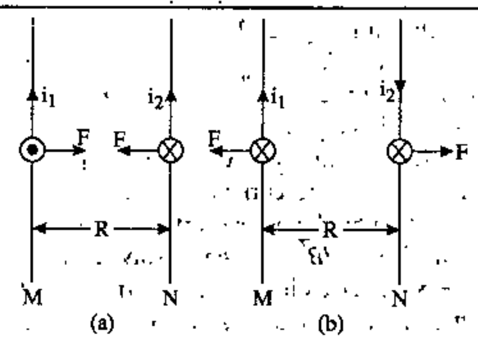


Fig. 6

Thus the current carrying conductor N is situated in a magnetic field B perpendicular to its length, therefore it experiences a magnetic force. The magnitude of the magnetic force acting on a length l to N is

$$F = i_2 B l = i_2\left[\frac{\mu_0 i_1}{2\pi R}\right] l. \qquad [\text{from } \vec{F} = i\,\vec{l}\times\vec{B}]$$

and the magnetic force per unit length of N is

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi R}$$

The direction of this magnetic force F is given by right hand screw-rule. Fig. 6(a) shows that the force is towards M (i_1 and i_2 are in the same direction) and fig. 6 (b) shows that the force is away from M (i_1 and i_2 are in opposite direction).

Similarly, the conductor M is situated in a magnetic field due to N. The force per unit length of M due to the current in N is $\frac{\mu_0 i_1 i_2}{2\pi R}\left(\text{magnetic field } \frac{\mu_0 i_2}{2\pi R}\right)$. The force is towards the conductor M for currents in the same direction and away from conductor M for currents in opposite directions. Thus the conductors attract each other if the currents in them are in the same direction, and repel each other if the currents are in opposite directions.

Definition of ampere : The force of attraction (or repulsion) between two long parallel current carrying straight conductors in vacuum has been used to define the *ampere* in the M.K.S. system of units. The magnetic force per unit length of the conductor is given by

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi R} = \frac{\mu_0 2 i_1 i_2}{4\pi R}$$

If $i_1 = i_2 = i$, then

$$dB = \frac{\mu_0 (n\,dx)\, ia^2}{2(a^2 + x^2)^{3/2}} \text{ weber/metre}^2 \qquad \ldots (1)$$

If r is the distance of the coil from P and $d\theta$ is the angle subtended by this coil at P, then

In ΔABC

$$\sin\theta = \frac{BC}{AB} = \frac{r\,d\theta}{dx}$$

$$\because \qquad dx = \frac{r\,d\theta}{\sin\theta}$$

In ΔAPO

$$a^2 + x^2 = r^2$$

or $$(a^2 + x^2)^{3/2} = r^3$$

Putting the values of dx and $(a^2 + x^2)^{3/2}$ in eq. (1), we get

$$dB = \frac{\mu_0 n\,(r\,d\theta)\, ia^2}{2r^3 \sin\theta} = \frac{\mu_0\, n\, ia^2\, d\theta}{2r^2 \sin\theta}$$

But from the figure,

$$\frac{a^2}{r^2} = \sin^2\theta.$$

$$\therefore \qquad dB = \frac{1}{2}\mu_0\, ni \sin\theta\, d\theta. \qquad \ldots (2)$$

The total magnetic field induction B at P due to the whole solenoid will be obtained by integrating eq. (2) between the limits θ_1 to θ_2, the semi-vertical angles subtended at P by the first and last turn of the solenoid respectively.

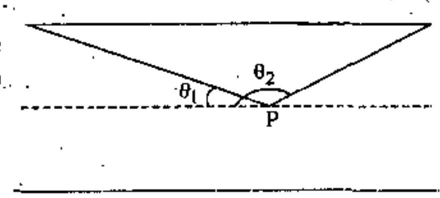


Fig. 10

$$\therefore \qquad B = \int_{\theta_1}^{\theta_2} dB = \int_{\theta_1}^{\theta_2} \frac{1}{2}\mu_0\, ni \sin\theta\, d\theta$$

$$= \frac{1}{2}\mu_0\, ni \Big[-\cos\theta\Big]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2}\mu_0\, ni\, (\cos\theta_1 - \cos\theta_2) \text{ weber/metre}^2 \qquad \ldots (2)$$

If P is well inside a very long solenoid, then $\theta \cong 0°$ ($\cos\theta_1 = 1$) and $\theta_2 \cong 180°$. ($\cos\theta_2 = -1$).

$$\therefore \qquad B = \frac{1}{2}\mu_0\, ni\, \{1 - (-1)\} = \mu_0\, ni \text{ weber/metre}^2. \qquad \ldots (3)$$

Magnetic Induction Field at the ends of Solenoid : At the end of the last turn, $\theta_1 = 90°$ and $\theta_2 = 90°$ so that $\cos\theta_1 = 1$ and $\cos\theta_2 = 0$. Then from eq. (2), we get

$$B = \frac{1}{2}\mu_0\, ni \text{ weber/metre}^2,$$

At the ends

$$\theta_1 = 90° \text{ and } \theta_2 = 180°$$

Again from eq. (2),

$$B = \frac{1}{2}\mu_0\, ni$$

Thus the magnetic induction at the ends is half of that the centre.

• 3.10. AMPERE'S LAW

According to this law, "*the line integral of magnetic field induction $\overrightarrow{B}$ around any closed path in vacuum is equal to μ_0 times of the total current passing through the closed path, i.e.,*"

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 I$$

Proof : Let us consider a long straight conductor XY which is lying in the plane of paper. Let I be the current which flowing in the direction as shown in the fig. 12. Due to this current a magnetic field is produced around the conductor. The magnetic lines of force are concentric circles perpendicular to the plane of paper which are shown by dotted lines.

We have the magnitude of the magnetic field induction produced at point P at a distance r from the conductor is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

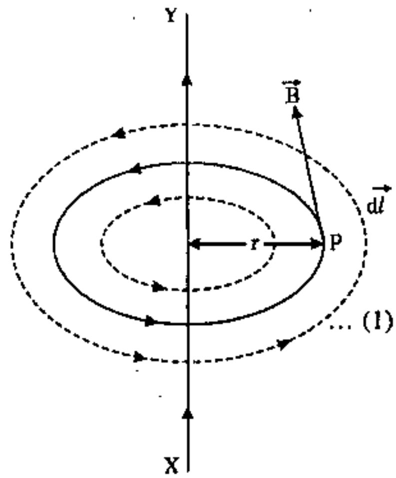


Fig. 11

Since the direction of $\vec{B}$ at every point is along the tangent to the circle. Consider a small element $\vec{dl}$ of the circle of radius r at P, then the angle between the direction of $\vec{B}$ and $\vec{dl}$ will be zero. In this position, we have

$$\oint \vec{B} \cdot \vec{dl} = \oint B dl \cos 0°$$

$$= \oint B\, dl$$

$$= \oint \frac{\mu_0}{4\pi} \cdot \frac{2I}{r} \cdot dl$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \oint dl$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \cdot 2\pi r$$

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$$

Hence proved.

From above it is clear that it does not depend upon the radius of circle *i.e.*, size of the path.

• 3.11. MAGNETIC INDUCTION DUE TO A STRAIGHT CONDUCTOR

Fig. 12 shows a long wire with current i. Direction of magnetic field $\vec{B}$ will be along the finger tips of right hand when thumb is kept in the direction of current. We draw a circle of radius R through point P at which $\vec{B}$ is to be calculated. $\vec{B}$ will be constant at all points on this circle and is parallel to the circle element $\vec{dl}$. Then according to Ampere's law

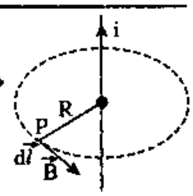


Fig. 12

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i$$

But $$\oint \vec{B} \cdot d\vec{l} = \oint \vec{B}\vec{dl} \cos 0° = B \oint \vec{dl}$$

The integral $$\oint \vec{dl} = 2\pi R$$

This is the circumference of the circle. Therefore. magnetic induction at point P distance R from the wire is

$$\vec{B} \oint \vec{dl} = \mu_0 i$$

or $$B(2\pi R) = \mu_0 i$$

or $$\boxed{B = \frac{\mu_0 i}{2\pi R}}.$$

• 3.12. MAGNETIC INDUCTION DUE TO A LONG SOLENOID

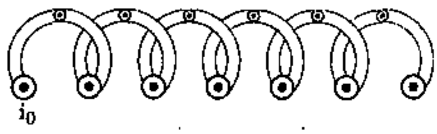


Fig. 13

We consider a solenoid very long compared with its diameter and carrying a current i_0. It is experimentally noted that magnetic field outside a solenoid is very small in comparison with the field inside. Also that the lines of induction inside the solenoid are straight and parallel. It means that magnetic field inside a solenoid is uniform except near the edges Fig. (14).

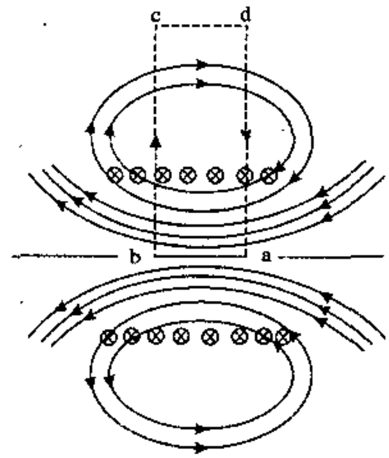


Fig. 14

Let us consider a rectangular path *abcd* Fig. (14). Side *ab* is parallel to the axis of the solenoid. Sides *bc* and *ad* are taken long so that *cd* side is far away from the solenoid and we can take that at such a far distance, field due to solenoid is negligible. That is, $\vec{B}$ at *cd* is zero. Further when solenoid is long, the field at *ab* will be uniform and is perpendicular to side *bc* and *ad* *i.e.*, angle between $\vec{B}$ and these sides is 90°.

Applying Ampere's law to the closed path *abcd*, we have

$$\oint_{abcd} \vec{B}.\vec{dl} = \mu_0 i \quad \ldots(1)$$

where i is the net current enclosed by the rectangle and not the current i_0 flowing in the wire of solenoid. We can say that i is the 'amount' of current enclosed in the rectangle *abcd*. If n be the number of turns per unit length of the solenoid, then number of turns in length *ab* equal to x meter say, will be nx. Now the current through each turn is i_0 therefore the amount of current enclosed in the rectangle will be

$$i = nx\, i_0 \quad \ldots(2)$$

The left hand side of eq. (1) can be written as

$$\oint_{abcd} \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl} \quad \ldots(3)$$

Since side bc and da, are perpendicular to **B**, we have

$$\int_b^c \vec{B}.\vec{dl} = \int_b^c B\,.\,d\,l \cos 90° = 0$$

Similarly $$\int_d^a \vec{B}.\vec{dl} = 0$$

Further $\vec{B}$ at *cd* is zero so that

$$\int_c^d \vec{B}.\vec{dl} = 0$$

Therefore eq. (3) becomes

$$\oint_{abcd} \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl}$$

Since $\vec{B}$ is parallel to *ab*, angle between them will be 0°. Therefore

$$\oint_{abcd} \vec{B}.\vec{dl} = \int_a^b B\,d\,l \cos 0°$$

$$= \int_a^b B\,d\,l = Bx \quad \ldots(4)$$

Puting eq. (4) and (2) into eq. (1) we get

$$Bx = \mu_0 (nx) i_0$$

$$B = \mu_0 n i_0$$

-hich is the field at a point well inside a long solenoid and is independent of length and diameter ˉ the solenoid. However, field at a point due to finite solenoid is different and will be calculated ısing Biot-Savart's law.

3.13. MAGNETIC INDUCTION OF AN ENDLESS SOLENOID (TOROID)

A toroid (a long solenoid bent in the shape f circular ring) is shown in Fig. (15). i_0 is the ırrent in its wire. By symmetry, direction of $\vec{B}$ at ıy point is tangential to a circle drawn through ıat point with same centre as that of toroid. The ıagnitude of $\vec{B}$ on any point of such a circle will ː constant. Let us consider a point P with in the ɔroid. Let us draw a circle of radius r through it. pplying Ampere's law to this circle we have

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i, \qquad \ldots(1)$$

ʼhere i, is the amount of current through the circle ınd not the current i_0. Suppose the total number f turns in the toroid is N and current in each turn ː i_0, then total current through the circle will be

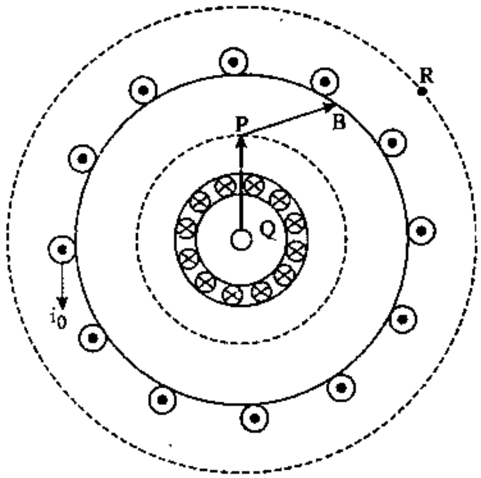


Fig. 15

$$i = Ni_0 \qquad \ldots(2)$$

Also $$\oint \vec{B} \cdot \vec{dl} = B\,(2\pi r) \qquad \ldots(3)$$

Putting eq. (3) and (2) in eq. (1) we get

$$B\,(2\pi r) = \mu_0 N i_0$$

$$B = \frac{\mu_0 N i_0}{2\pi r}$$

That is the field varies with r. It means that if the circumference of the toroid is l then $= 2\pi r$, so that

$$B = \frac{\mu_0 N i_0}{l}$$

The field $\vec{B}$ at an inside point such as Q is zero because there is no current enclosed by the ircle through Q.

The field $\vec{B}$ at an outside point such as R is also zero because net amount of current enclosed n the circle through R will be zero. This is because each turn of the winding passes twice through ıis area enclosed by the circle, carrying equal currents in opposite directions.

3.14. MAGNETIC FIELD DUE TO A LONG, CYLINDRICAL CURRENT CARRYING WIRE

(i) Inside the wire $\left(r < \frac{d}{2}\right)$ Let us consider a long cylindrical wire of diameter d and let i urrent is flowing in it. Draw a circle of radius $r\left(< \frac{d}{2}\right)$ inside the wire. If $\vec{B}$ is the magnetic field nduction then by Ampere's law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i' \qquad \ldots(1)$$

vhere $\vec{dl}$ = circle element at that point.

i' = the current enclosed by the circle of radius r.

Since the current i is **uniformly** distributed so

$$i' = \left(\frac{i}{\pi R^2}\right)\pi r^2 = \frac{ir^2}{R^2} \quad \ldots (2)$$

where $R = \frac{d}{2}$ (say)

Since $\vec{B}$ is constant and parallel to the circle element $\vec{dl}$ so

$$\oint \vec{B}.\vec{dl} = \oint B.dl = B\oint dl = B(2\pi r)$$

$$\oint \vec{B}.\vec{dl} = B(2\pi r) \quad \ldots (3)$$

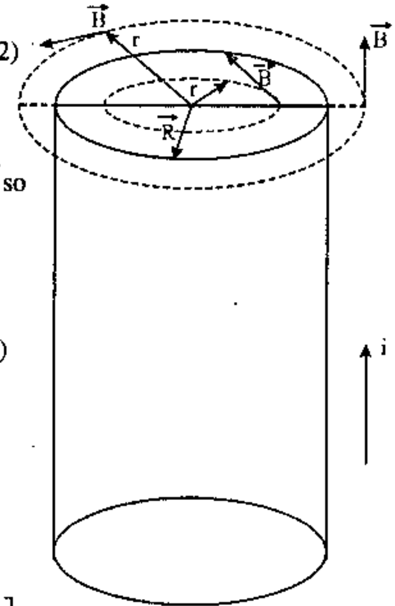


Fig. 16

Putting the values of eqs. (2) and (3) in eq. (1), we get

$$B(2\pi r) = \mu_0\left[\frac{ir^2}{R^2}\right]$$

$$B = \frac{\mu_0 ir}{2\pi R^2}$$

$$\therefore \quad \boxed{B = \frac{2\mu_0 ir}{\pi d^2}} \quad \left[\because R = \frac{d}{2}\right]$$

This is the required expression for the magnetic field induction inside the cylindrical wire. From this expression it is clear that if $r = 0$ then $B = 0$. This means that the magnetic field induction at the centre of cylindrical wire is zero.

At the surface $\boxed{B = \frac{\mu_0 i}{\pi d}}$ $\left[\because \text{ at surface } r = \frac{d}{2}\right]$

(ii) Outside the wire $\left(r > \frac{d}{2}\right)$: In this position Ampere's law becomes

$$\oint \vec{B}.\vec{dl} = \mu_0 i$$

because now the circle of radius r enclosed the total current i so

$$B(2\pi r) = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

This is the required expression of the magnetic field induction outside the cylindrical wire. From above expression it is clear that the magnetic field induction outside the cylindrical wire is inversely proportional to the radius r.

• 3.15. MAGNETIC FIELD OF A LONG, HOLLOW CYLINDRICAL CURRENT CARRYING CONDUCTOR

Let us consider a hollow cylinder of inner and outer radii a and b respectively and let this hollow cylinder conductor carrying a uniformly distributed current i. Let P be the point at a distance r from the axis at which the magnetic field is to be determined.

Now the following three cases may be arised :

(i) When $0 \leq r < a$: This means that the point P is inside the hollow region which is shown in the fig. 17. Draw a circle of radius r with centre O. Since the magnetic field $\vec{B}$ at any point on the circle is tangential to the circle and equal in magnitude every where so by Ampere's law, we have

$$\oint \vec{B}.\vec{dl} = \mu_0 i' \quad (1)$$

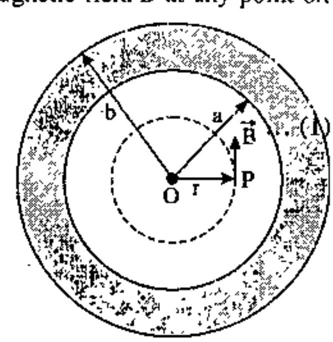


Fig. 17

where $\vec{dl}$ = circle element at P

i' = current enclosed by the circle of radius $r < a$.

Here $i' = 0$ since there is no current within the conductor. So by eq. (1)

$$\oint \vec{B}.\,d\vec{l} = B\,(2\pi\,r) = \mu_0 \times 0$$

$$\boxed{B = 0}$$

Thus the magnetic field due to a hollow current carrying cylinder is zero everywhere with in the hollow region.

(iii) When $a \le r < b$: This situation is shown in the fig. 18. In this position the Ampere's law becomes

$$\oint \vec{B}.\,d\vec{l} = \mu_0\, i' \qquad \ldots (2)$$

where i' = current enclosed by the circle of radius r such that $a < r < b$.

Since i is uniformly distributed so

$$\frac{i}{4\pi\,(b^2 - a^2)} = \frac{i'}{4\pi\,(r^2 - a^2)}$$

$$\therefore \qquad i' = i\left[\frac{r^2 - a^2}{b^2 - a^2}\right]$$

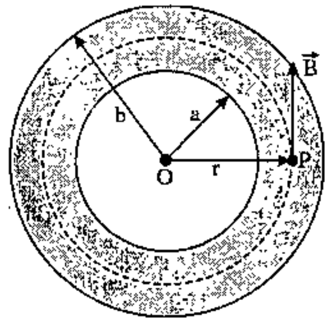


Fig. 18

Put this value in eq. (2), we get

$$B\,(2\pi\,r) = \mu_0\, i\left[\frac{r^2 - a^2}{b^2 - a^2}\right].$$

$$\boxed{B = \frac{\mu_0\, i}{2\pi\, r}\left[\frac{r^2 - a^2}{b^2 - a^2}\right]}$$

This is the required expression for the magnetic field when $a \le r < b$.

(iii) When $b < r < \infty$: This situation is shown in the following fig. 19. In this case P lies outside the conductor. In this condition Ampere's law becomes

$$\oint \vec{B}.\,d\vec{l} = \mu_0\, i' \qquad \ldots (3)$$

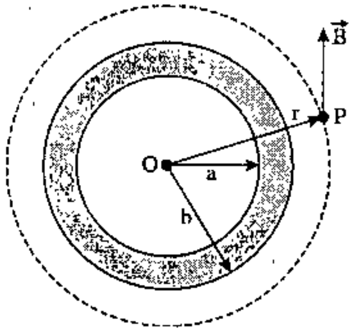


Fig. 19

where i' = current enclosed by the circle of radius $r > b$.

Here $i' = i$.

Put this value in eq. (3), we get

$$B\,(2\pi\,r) = \mu_0\, i$$

$$\boxed{B = \frac{\mu_0\, i}{2\pi\, r}}$$

This is the required expression for the magnetic field out side the hollow cylinder.

• 3.16. AMPERE'S LAW IN DIFFERENTIAL FORM

Let us consider a straight current carrying conductor, for this the Ampere's law is given by

$$\oint \vec{B}.\,d\vec{l} = \mu_0\, i \qquad \ldots (1)$$

This equation shows that the line integral of the magnetic induction around a closed curve is equal to μ_0 times of the total current which pass through the area bounded by the curve. This is Ampere's law in integral form.

Now we obtain the Ampere's law in differential form, for this let any close curve C in a region where currents are following. Let $\vec{j}$ is the current density which varies from place to place but it is constant in time.

The total current is

$$i = \int_s \vec{J}.\,d\vec{S}$$

where $d\vec{S}$ is a small element of area where the current density is $\vec{J}$ inside the closed curve C.

Put the value of i in eq. (1), we get

$$\oint \vec{B}.\,d\vec{l} = \mu_0 \int_s \vec{j}.\,d\vec{S} \qquad \ldots (2)$$

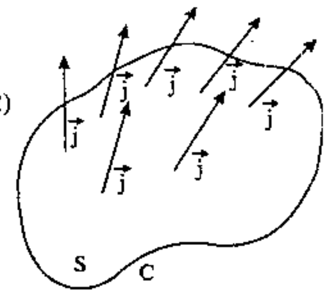


Fig. 20

On applying Stokes theorem

i.e.
$$\oint \vec{B}.\,d\vec{l} = \int_s \text{curl}\,\vec{B}.\,d\vec{S}$$

Put this value in eq. (2), we get

$$\int_s \text{curl}\,\vec{B}.\,d\vec{S} = \mu_0 \int_s \vec{j}.\,d\vec{S}$$

$$\text{curl}\,\vec{B} = \mu_0 \vec{j}$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{j}} \qquad \ldots (3)$$

This is the required expression for the Ampere's law in differential form.

The above equation is not enough to determine $\vec{B}$ at a point by the given value of $\vec{j}$ at that point. The reason of this problem is that many different vector fields could have the same curl. To remove this problem we need another equation. We have the Gauss's law which is as follows :

$$\boxed{\text{div}\,\vec{B} = 0}$$

This is the equation which together with eqn. (3) uniquely determine $\vec{B}$ if $\vec{j}$ is known.

Significance of the above equation : The equation div $\vec{B} = 0$ shows that there exists no isolated magnetic poles to act as a source or sinks for the lines of $\vec{B}$ which close into themselves.

• STUDENT ACTIVITY

1. Define ampere.

2. Prove that curl $\vec{B} = \mu_0 \vec{J}$.

3. When the force experienced by moving charge in a magnetic field will be maximum or minimum ?

• 3.17. FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN MAGNETIC FIELD (FORCE ON A CURRENT)

The current in a conductor is due to the motion of free electrons. When such a conductor is placed in a magnetic field, each electron moving in the magnetic field experience a force. Hence the current carrying conductor experiences a force when placed in a magnetic field.

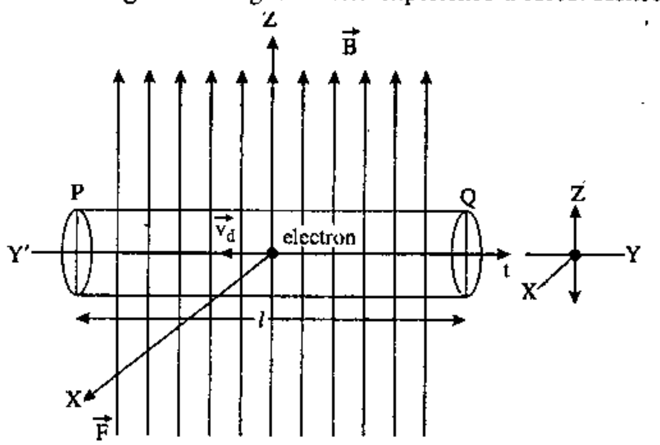


Fig. 21

Let us consider a current carrying conductor PQ of length l which is placed along Y-axis and a uniform magnetic field $\vec{B}$ acts along Z-axis *i.e.*, perpendicular to the length of conductor. Let i is the current flows through the conductor from P to Q. Due to the magnetic field electrons move with drift velocity v_d. Let A is the cross-section of conductor and let n are the number of electrons per unit volume so current flowing through the conductor is given by

$$i = nAev_d$$

$$\therefore \quad il = nAlev_d$$

Since $\vec{il}$ and $\vec{v_d}$ have opposite direction so

$$\vec{il} = -nA\,le\vec{v_d} \qquad \ldots (1)$$

We know that the Lorentz force is

$$\vec{f} = -e\,(\vec{v_d} \times \vec{B}) \qquad \ldots (2)$$

This force is only on one electron if N are the total number of electrons of conductor then total force

$$\vec{F} = N \times \vec{f} \qquad \ldots (3)$$

Since $\quad N = n \times$ volume of conductor $= nAl \qquad \ldots (4)$

Putting these value of eqns. (4) and (1) in eq. (3), we get

$$\vec{F} = nAl\,[-e\,(\vec{v_d} \times \vec{B})] = (-nAl\,e\vec{v_d}) \times \vec{B}$$

$$\vec{F} = \vec{il} \times \vec{B} \qquad [\because \text{ by eq. (1)}]$$

Magnitude of force is

$$\boxed{F = il\,B\sin\theta} \qquad \ldots (5)$$

where θ is the angle between the direction of $\vec{B}$ and the direction of $\vec{i}$.

Equation (5) is the required expression for the force acting on a current carrying conductor placed in a magnetic field.

From eqn. (5), we have

(i) if $\theta = 0°$ or 180 so $\boxed{F = 0}$ min (no force)

(ii) if $\theta = 90°$ then $F = i_l B$ *i.e.*, max. force.

• 3.18. FLEMING'S LEFT HAND RULE

According to this rule if the fore finger, central finger and thumb of left hand are stretched mutually perpendicular to each other such that the fore finger points in the direction of magnetic field $\vec{B}$ and central finger in the direction of current i then the thumb points in the direction of force $\vec{F}$ experienced by the conductor.

(b) Torque on a current carrying rectanguiar coil : Let us consider a rectangular coil $PQRS$ suspended in a uniform magnetic field $\vec{B}$ as shown in the fig. 22. Let $PQ = RS = l$ and $QR = SP = b$, i is the current which pass through the coil in the direction $PQRS$ and let θ be the angle between the plane of coil and the direction of magnetic field $\vec{B}$.

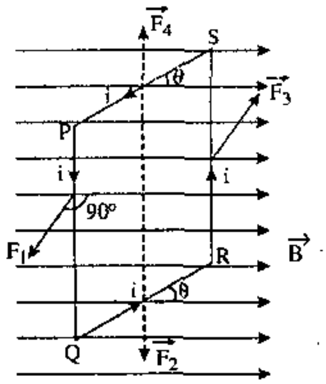


Fig. 22

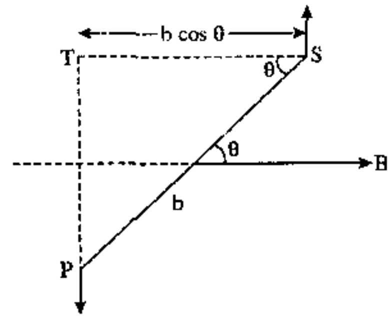


Fig. 23

The force on arm SP is

$$\vec{F_4} = i\,(\vec{SP} \times \vec{B})$$

$\therefore$ $$F_4 = i\,(SP)\,B \sin\theta$$

$$F_4 = i\,bB \sin\theta$$

For on QR is

$$F_2 = ibB \sin\theta$$

The direction of this force is in the plane of the coil directed downwards.

Since the forces F_2 and F_4 are equal and are opposite in direction so they cancel out each other.

Now the force on PQ is

$$F = i\,(PQ)\,B \sin 90^\circ$$

$$F_1 = i\,lB \qquad [\because\ \vec{PQ} \perp \vec{B}]$$

and force on theorem RS is

$$F_3 = i\,lB \sin 90$$

$$F_3 = ilB$$

Since the forces acting on the arms PQ and RS are equal, parallel and acting in opposite directions along different lines of action so they form a couple due to this couple the coil rotates in the anticlockwise direction.

The torque is given by

$$\tau = \text{Force} \times \text{arm of couple}$$

$$\tau = ilB \times b \cos\theta \qquad [ST = b\cos\theta \text{ by fig.}]$$

$$\boxed{\tau = iBA \cos\theta} \qquad [\because\ l \times b = A \text{ (area)}]$$

if reactangular coil has N turns then

$$\tau = NiBA \cos\theta$$

This is required expression for Torque acting on a current carrying rectangular coil.

Since $NiA = M$ [magnitude of dipole moment)

so $$\boxed{\tau = MB \cos\theta}$$

This is relationship between Torque and dipole moment.

• 3.19. MOVING COIL GALVANOMETER

Moving coil galvanometer is a device which is used for detection and measurement of small electric currents. Its working is based upon the torque acting on a current carrying coil placed in a magnetic field.

Principle : It is based upon the fact that when a current carrying coil is placed in a magnetic field then it experiences a couple.

Construction : It consists of a rectangular coil having large number of turns of insulated copper wire, wound over it. The coil is suspended by a phosphor bronze strip in a uniform magnetic field, which is produced by two strong cylindrical magnetic pole pieces N and S. The lower end of the coil is connected to one end of a spring of phosphor bronze. C is a soft iron core which is cylindrical. It is so held within the coil, that the coil can rotate freely without touching the iron core of pole pieces. This makes the magnetic field linked with the coil to be **radial field**. A small concave mirror is attached to lower portion. The whole arrangement is enclosed in a non-metallic case to avoid disturbance due to air etc.

The current to be measured enters at one terminal T_1 and passes through the suspension coil and spring and finally leaves at the second terminal T_2.

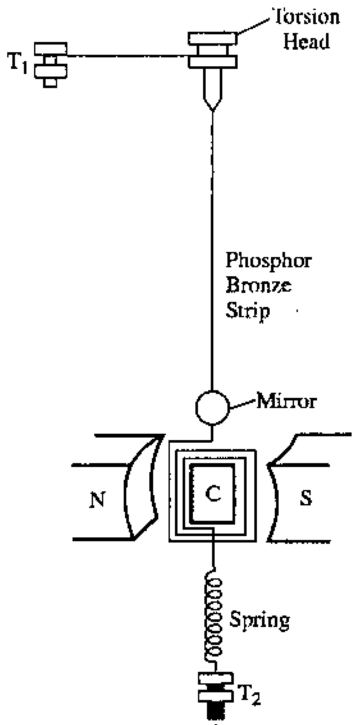


Fig. 24

Theory : Let the coil is suspended freely in the magnetic field.

Let l = length of the coil

b = breadth of the coil

N = number of turns in the coil.

Area of each turn of the coil $A = l \times b$.

Let i is the current passing through the coil then the torque

$$\tau = NiAB \sin \theta$$

Since magnetic field is radial *i.e.*, $\theta = 90°$.

$$\therefore \qquad \tau = NiAB$$

Due to this torque the coil rotates. The phosphor bronze strip gets twisted. As a result of it, a restoring torque begins to act in the phosphor bronze strip.

Let ϕ is the twist produced in the phosphore bronze strip due to rotation of the coil and C is the restoring torque (or couple) per unit twist of the phosphor bronze strip. then

Deflecting couple = Restoring couple

$$\tau = NiAB = C\phi$$

$$i = \frac{C}{NAB}\phi$$

$$i = k\phi$$

or $$i \propto \phi$$

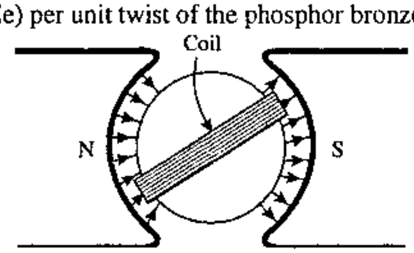


Fig. 25

where $k = \frac{C}{NAB}$ is constant of the instrument. Thus the current passed through the galvanometer is proportional to the deflection produced.

Advantages of Moving Coil Instruments

(1) They may be placed in any position *i.e.*, no need them to set in magnetic meridian.

(2) The galvanometer coil can be graduated permanently.

(3) The sensitivity of these galvanometers is very large.

• 3.20. BALLISTIC GALVANOMETER

It is a specially designed galvanometer which is used to measure the change of very short duration. In this maximum deflection is proportional to the charge which passed through the galvanometer.

Construction : It consists of a rectangular or circular coil of large number of turns of fine insulated copper wire wound on a non conducting frame made of ivory or bamboo and suspended between the pole piece of a permanent magnet by means of phosphor bronze strip. A soft iron cylinder is placed within the coil to make the field radial. The phosphor bronze is attached to a torsional head on the upper side. A small plane mirror is attached to the lower end of suspension fibre to record the deflection of the lamp by means of a lamp and scale arrangement. The lower end of the coil is attached to a spring of very fine phosphor bronze wire, coil and spring and leaves at the second terminal. In Ballistic Galvanometer, the moment of inertia of the coil or the time period of swing of the coil is made large and the coil is wounded on a non conducting material.

Theory : When a charge to be measured passes through the galvanometer, it gives an angular impulse to the coil and sets its oscillations. Let us obtain a relation between the charge and the first throw of the coil.

Let i be the current passed through the coil,

N be the number of turns in the coil,

B be the magnetic induction,

l be the length of the coil,

b be the breadth of the coil,

A be the area of the coil.

Force experienced by two vertical sides of coil is $= NiBl$

As these are two parallel and opposite forces, they constitute a couple of moment

$$= NiBl \times b = NiBA \qquad \text{(where } l \times b = A\text{)}$$

This couple acting for an infinitesimal time dt gives an angular impulse to the coil which is $NiBA.dt$ (because impulse = couple × times). If t is the total time then angular impulse

$$= \int_0^t NiBAdt$$

$$= NBA \int_0^t idt$$

$$= NBAq$$

where $$q = \int_0^t idt$$

Let us assume that the coil begins to rotate as this impulse is over. If ω be its angular velocity at start and I its moment of inertia about the axis of suspension, then the angular momentum produced in the coil due to this angular impulse is $I\omega$ *i.e.*,

$$I\,\omega = NBAq \qquad \text{...(1)}$$

Kinetic energy (K.E.) of the moving system in the initial position is $\frac{1}{2} I\,\omega^2$ and in the final position is zero. *i.e.*, decrease in $K = \frac{1}{2} I\,\omega^2$. This decrease in K.E. is equal to the work done in twisting the suspension fibre between initial and final position.

Let C be the couple per unit twist.

θ be the angle through which the coil has turned, *i.e.*, the first throw.

The couple for this twist, $\theta = C\,\theta$

The work done for a small twist, $d\theta = C\,\theta\,d\,\theta$

Work done in twisting the suspension fibre between the initial and final positions

$$= \int_0^{\theta_0} C\,\theta\,.\,d\,\theta$$

$$= \frac{1}{2} C\,\theta_0^2$$

where θ_0 is the angular deflection of the coil in the final position.

Thus $$\frac{1}{2} I\,\omega^2 = \frac{1}{2} C\,\theta_0^2 \qquad \text{...(2)}$$

Again the time period of oscillation of the system executing torsional oscillations is

$$T = 2\pi \sqrt{\frac{I}{C}}$$

or $$T^2 = \frac{4\pi^2 I}{C}$$

hence $$I = \frac{CT^2}{4\pi^2} \qquad \text{...(3)}$$

Multiplying eqs. (2) and (3), we have

$$I^2\omega^2 = \frac{C^2T^2\theta_0^2}{4\pi^2}$$

or $$I\omega = \frac{CT\,\theta_0}{2\pi} \qquad \text{...(4)}$$

Substituting the value of $I\omega$ in eq. (1), we get

$$\frac{CT\theta_0}{2\pi} = NABq$$

or $$q = \frac{T}{2\pi} \cdot \frac{C}{NAB} \cdot \theta_0 \qquad \ldots(5)$$

or $$q = K\theta_0$$

where $\frac{T}{2\pi} \cdot \frac{C}{NBA} = K$ is called the ballistic reduction factor or "ballistic constant".

This is the required relation. Thus, the charge q passed through the galvanometer is proportional to the maximum deflection θ_0 of the coil.

Correction for damping (Logarithmic decrement) : In deducting the relation (5), we assumed that the whole of the kinetic energy is used in overcoming.

(i) The electromagnetic damping : This is due to induced eddy currents in the frame and windings of the coil because when the coil carrying current moves in the magnetic field, eddy currents are generated in the coil which oppose the motion of the coil. This damping is reduced by winding the coil on a non-conducting frame.

(ii) The frictional damping : This is due to the air resistance experienced by the moving coil. So a correction is necessary for this damping.

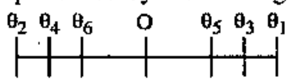


Fig. 26

Let θ_0 be the actual deflection in the absence of damping and $\theta_1, \theta_2, \theta_3$ etc. be the successive observed throws to the right and left continuously as shown in fig. (26).

It is found that the ratio of two successive throws is

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots = d \qquad \ldots(6)$$

where d is a constant, called the decrement per half vibration. $\log_e d$ is called the **logarithmic decrement** λ, so that

$$\log_e d = \lambda$$

or $$d = e^{\lambda} \qquad \ldots(7)$$

For half vibration, we write

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots = e^{\lambda}$$

and for full vibration

$$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda}$$

So that $$\frac{\theta_1}{\theta_{11}} = e^{10\lambda}.$$

To calculate the true throw in the absence of damping, we must note the first throw after the coil completes a quarter of a vibration. In this case the value of decrement would be $e^{\lambda/2}$.

$\therefore$ $$\frac{\theta_0}{\theta_1} = e^{\lambda/2}$$

or $$\theta_0 = \theta_1 e^{\lambda/2} = \theta_1\,[1 + \lambda/2 + \ldots]$$

Neglecting higher terms of λ as λ is very small

$$\theta_0 = \theta_1\,(1 + \lambda/2) \qquad \ldots(8)$$

Therefore correct expression for the quantity of charge passing through ballistic galvanometer is

$$q = \frac{T}{2\pi} \cdot \frac{C}{NBA}\,\theta_1\left(1 + \frac{\lambda}{2}\right) \qquad \ldots(9)$$

$$= K\,\theta_1\left(1 + \frac{\lambda}{2}\right) \qquad \ldots(10)$$

where $K = \frac{T}{2\pi} \cdot \frac{C}{NBA}$

Further as $$\frac{\theta_1}{\theta_{11}} = e^{10\lambda}$$

We have $\lambda = \frac{1}{10} \log_e \frac{\theta_1}{\theta_{11}}$

$$= \frac{1}{10} \times 2.3026 \log_{10} \frac{\theta_1}{\theta_{11}}$$

or $$\lambda = \frac{2.3026}{10} \log_{10} \frac{\theta_1}{\theta_{11}}$$

By noting the first and eleventh throw, the value of λ can be calculated.

Conditions when a Moving Coil Galvanomenter is Dead Beat : A galvanometer is said to be dead beat galvanometer if it's coil returns quickly to the rest position without oscillation after being deflected. The following conditions are required for the galvanometer to be dead beat :

(i) The period of oscillation of the coil must be small : The smaller moment of inertia of the coil and larger torsional constant of the supension ensure this condition.

(ii) The Damping must be heavy : Winding the coil on a conducting frame increases the damping. If a soft-iron core is placed between the pole-pieces of the permanent magnet, the electromagnetic damping is increased further.

• STUDENT ACTIVITY

1. What are the conditions for a moving coil galvanometer to be ballistic ?

2. What are the conditions for a moving coil galvanometer to be dead beat ?

3. What is charge sensitivity ?

4. What is current sensitivity ?

• 3.21. MAGNETIC INTENSITY ($\vec{H}$)

(i) Magnetic intensity ($\vec{H}$) : It is also known as H–field or magnetic field strength or magnetising force.

When a magnetic material is placed in a magnetic field then it gets magnetised. *The degree to which the magnetic field can magnetise the material is represented in terms of magnetic intensity* $\vec{H}$.

Magnetic intensity is defined by the relation

$$H = \frac{B_0}{\mu_0}$$

where $\mu_0 = 4\pi \times 10^{-7}$ weber meter^{-1} Ampere^{-1}. The unit of magnetic intensity is also **oersted**. It is related to **Gauss.**

$$1 \text{ oersted} = \frac{1 \text{ Gauss}}{\mu_0}.$$

(ii) Intensity of magnetisation (I) : When a magnetic material is placed in a magnetic field then it gets magnetised. The intensity of magnetisation of a magnetised material represents the extent to which the material is magnetised.

When an unmagnetised material is placed in a magnetic field then the field exerts a torque on the dipoles which align them parallel to the field. The material then acquires a magnetic moment and becomes magnetised. *The magnetic moment per unit volume of the material is known as intensity of magnetisation or magnetisation* ($\vec{I}$).

$$I = \frac{\text{Magnetic moment}}{\text{Volume}} = \frac{M}{V}$$

Unit = Ampere/meter (Am^{-1}).

(iii) Magnetic flux : The magnetic flux through a surface is defined as *"the number of magnetic lines of force passing normally through the surface."* It is denoted by ϕ and it is determined by the relation

$$\boxed{\phi = BA}$$

where B = magnetic field, A = surface area.

(iv) Magnetic induction or Flux density (B) : When a positive test charge q is fired with a velocity $\vec{v}$ through a point and the moving charge experiences a **sideways** force $\vec{F}$ then we can say that the magnetic field is present at that point. In this the Lorentz force is given by

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$

or $$F = qvB \sin\theta$$

where B is magnetic induction and it is defined as "the force experienced by a unit positive charge moving with unit velocity in a direction perpendicular to the magnetic field is known as magnetic induction or magnetic flux intensity.

Its unit is $\vec{B}$ weber/meter^2 or tesla (T).

(v) Magnetic susceptibility (χ_m) : It is the property which determines how easily a speciman can be magnetised.

The susceptibility of the magnetic material is defined as '*'the ratio of the intensity of magnetisation (I) induced the material to the magnetising force.* It is denoted by χ_m.

i.e., $$\boxed{\chi_m = \frac{I}{H}}$$

Since I is magnetic moment per unit *volume so* χ_m *is also be defined as the volume susceptibility of the material.*

We can classify materials in terms of χ_m, material with positive values of χ_m are paramagnetic and those which has negative values of χ_m are diamagnetic. For paramagnetic χ_m is positive and very large.

(vi) Magnetic permeability (μ) : The magnetic permeability of the material is the measure of its conduction of magnetic lines of force through it. *It is defined as the ratio of magnetic induction* $\vec{B}$ *within a magnetised material to the magnetic intensity* $\vec{H}$ *of the magnetising field i.e.,*

$$\mu = \frac{\vec{B}}{\vec{H}}$$

Its unit is $\frac{\text{weber}}{\text{Ampere meter}}$ (wb A^{-1} m^{-1})

Relative magnetic permeability (μ_r) : It is the ability of the material to permit the passage of magnetic lines of forces through it *i.e.*, the degree of extent to which magnetic field *can penetrate the material is called relative permeability and it is denoted by* μ_r.

Relative magnetic permeability of the material is defined as the ratio of the number of lines of magnetic induction per unit area *i.e.*, B in that material to the number of magnetic lines per unit area that would be present, if the medium were replaced by vacuum B_0 *i.e.*,

$$\boxed{\mu_r = \frac{B}{B_0}}$$

Relative permeability of the material may also be defined as "the ratio of the magnetic permeability of the material (μ) to the magnetic permeability of free space (μ_0) *i.e.*,

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

It has no dimension, for vacuum $\mu_r = 1$.

(b) Relation between $\vec{H}, \vec{B}$ and $\vec{I}$: We have the magnetic field induction B in the material medium due to the magnetic force or magnetic intensity $\vec{H}$ will the sum of the field induction $\vec{B_0}$ in vacuum produced by $\vec{H}$ and the field induction $\vec{B_1}$ due to the magnetisation of the medium *i.e.*,

$$\vec{B} = \vec{B_0} + \vec{B_1}$$

Since $\vec{B_0} = \mu_0 \vec{H}$ and $\vec{B_1} = \mu_0 \vec{I}$

so $$\boxed{\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{I}}$$

This is the relationship between $\vec{B}, \vec{H}$ and $\vec{I}$ in S.I. system.

We can obtained this relation in Gausian or C.G.S. by replacing $\vec{B}$ by $\frac{\vec{B}}{C}$ and $\vec{H}$ by $\frac{\vec{H}}{4\pi}$, $\vec{I}$ by $C\vec{I}$ and μ_0 is replaced by $\frac{4\pi}{C^2}$ so above equation. reduce to

$$\frac{\vec{B}}{C} = \frac{4\pi}{C^2}\left[\frac{C}{4\pi}\vec{H} + C\vec{I}\right]$$

or $$\boxed{\vec{B} = \vec{H} + 4\pi \vec{I}.}$$

(c) Relation between relative permeability (μ_r) and magnetic susceptibility (χ_m) : When a magnetic material is placed in a magnetic field whose magnetic intensity is $\vec{H}$ then the material gets magnetised. In this position the total magnetic induction $\vec{B}$ in the material is the sum of the magnetic induction $\vec{B_0}$ produced by the magnetic intensity and magnetic induction $\vec{B_m}$, due to the magnetisation of the material so

$$\vec{B} = \vec{B_0} + \vec{B_m} \quad \ldots (1)$$

Since $\vec{B_0} = \mu_0 \vec{H}$ and $\vec{B_m} = \mu_0 \vec{I}$ so eqn. (1) becomes

$$\boxed{\vec{B} = \mu_0 (\vec{H} + \vec{I})} \quad \ldots (2)$$

This is the relation in S.I. system and the relation in Gaussian *i.e.* C.G.S. system is given by

$$\vec{B} = \vec{H} + 4\pi \vec{I} \quad \ldots (3)$$

We know $$\chi_m = \frac{\vec{I}}{\vec{H}}$$

$$\therefore \quad \vec{I} = \vec{H}\chi_m$$

Putting this value in eq. (2), we get

$$\vec{B} = \mu_0 (\vec{H} + \chi_m \vec{H})$$

$$\vec{B} = \mu_0 \vec{H}(1 + \chi_m)$$

$$\because \quad \vec{B} = \mu\vec{H}$$

so $$\mu\vec{H} = \mu_0 \vec{H}(1+\chi_m)$$

$\therefore$ $$\frac{\mu}{\mu_0} = 1 + \chi_m$$

$\therefore$ $$\boxed{\mu_r = 1 + \chi_m}$$

This is the relationship between μ_r and χ_m in S.I. system.

But in Gaussian or C.G.S. system this relation is obtained by eqn. (3), *i.e.*,

$$\boxed{\mu_r = 1 + 4\pi\,\chi_m}$$

Note : 1. The C.G.S. unit of magnetic induction B is **Gauss** which is equal to $\mathbf{10^{-4}}$ **Tesla.**

2. The C.G.S. unit of magnetic intensity $\vec{H}$ is the **oersted.**

• 3.22. PARA, DIA AND FERRO-MAGNETIC SUBSTANCES

On the basis of their magnetic properties different substances have been classified into the following three categories (i) Paramagnetic substances (ii) Diamagnetic substancs (iii) Ferro-magnetic substances.

Their main characteristic are as follows :

(1) Paramagnetic substances : These are those substances which when placed in a strong magnetic field then they become weakly magnetised in the direction of field. Manganese, chromium, aluminium, copper sulphate, liquid oxygen and solutions of salts of iron and nickel are paramagnetic substances.

When a paramagnetic bar is placed in a magnetic field, the magnetic flux density B in it is greater than the magnetic flux density B_0 in vacuum. Thus the relative permeability μ_r (μ/μ_0) of paramagnetic substances is slightly greater than 1. The magnetic flux density due to magnetisation is a small but positive quantity. The susceptibility of these substances decreases with increase of temperature. Paramagnetics are used to the measurement of low temperature.

Properties : (i) A paramagnetic bar, suspended between the poles of a magnet, shows opposite poles to those of the magnet, as its end. It turns until it lies along the applied field, fig. 27.

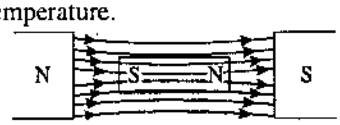


Fig. 27

(ii) Paramagnetic substances have the tendency to move from the *weaker to the stronger* parts in a non-uniform magnetic field. If a paramagnetic liquid is placed in a watch glass resting on two pole-pieces very near to each other, then the liquid accumulates in the middle where the field is strongest, fig. 28 (i). If the pole-pieces are far apart, the field is strongest near the poles and the liquid moves away from the centre giving a depression in the middle [fig. 28 (ii)].

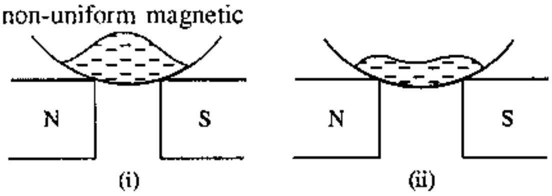


Fig. 28

(iii) If one limb of a narrow U-tube conaining a paramagnetic liquid is placed between the pole-pieces of an electromagnet in such a way that the level of the liquid is in line with the field, then on switching on the field, the level of the liquid rises in the limb.

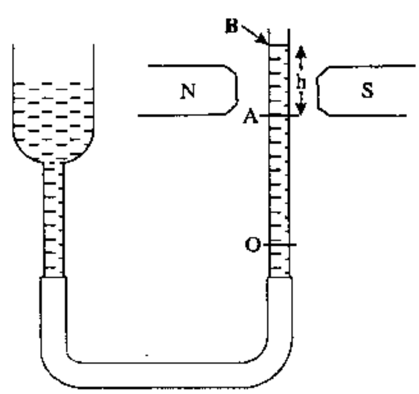


Fig. 29

(iv) When a paramagnetic gas is allowed to accent between the pole-pieces of an electromagnet, it spreads about the field.

(v) Paramagnetics obey Curie's law.

(ii) Diamagnetic substances are those which become weakly magnetised in a direction *opposite* to that of the external field. Bismuth, antimony, gold, water, alcohol, quartz, hydrogen are diamagnetic substances.

When a diamagnetic bar is placed in a magnetic field, the magnetic flux density B in it is less than the magnetic flux density B_0 in vacuum. Thus the relative permeability μ_r (μ/μ_0) of diamagnetic substances is less than 1. The magnetic flux density due to magnetisation is a small but negative quantity. The susceptibility of diamagnetic substances is independent of temperature.

Properties : (i) A diamagnetic bar, suspended between poles of a magnet, shows the similar poles to those of the magnet, at its ends. *It turns until it is right angle to the applied field, fig. 30.*

(ii) Diamagnetic substances have the tendency to move from the *stronger to the weaker* parts in a non-uniform magnetic field.

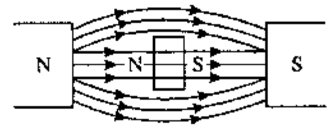


Fig. 30

If a diamagnetic liquid is placed in a watch glass resting on two pole-pieces very near to each other, then the liquid accumulates on the sides where the field is weaker and a depression in the middle is observed, fig. 31 (i). If the pole-pieces are far apart, the field is strongest near the poles and the liquid moves away from the ends to the middle, fig. 31 (ii).

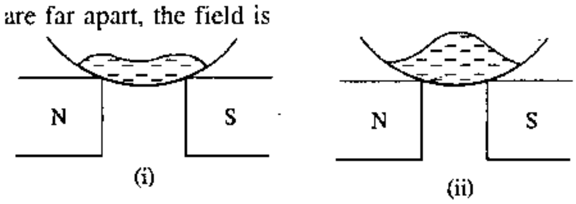


Fig. 31

(iii) A diamagnetic liquid shows a depression in the limb of a U-tube placed between the pole-pieces of an electromagnet.

(iv) When a diamagnetic gas is allowed to ascend between the pole-pieces of an electromagnet, it spreads across the field.

(v) Diamagnetics do not obey Curie's law.

(iii) Ferromagnetic substances are those which are strongly magnetised by relatively weak magnetic field in the direction of the external field. Iron, nickel, cobalt, gadolinium and their alloys are ferromagnetics.

Properties : Ferromagnetics show all the properties of paramagnetics of a much higher degree. For example, they have relative permeabilities of the order of the hundreds and thousands. The flux density in them may be hundreds or even thousands of times as great as that in the free space due to the same magnetising force.

The flux density B in a ferromagnetic substance is not directly proportional to the magnetising force H or the permeability $(= B/H)$ is not a constant.

The permeability μ of a ferromagnetic substance decreases with rise in temperature, and becomes practically equal to μ_0 at a certain temperature, called the ***Curie temperature*** (770°C for iron). Above the Curie temperature the ferromagnetic substance becomes paramagnetic.

• 3.23. VARIATION OF *B* WITH *H* : HYSTERESIS

Fig. 32 shows a specimen placed inside a long current-carrying solenoid. The current i through the solenoid is measured on the ammeter A. As $H = ni$, so the current i is directly proportional to the magnetising force H. C is a small search coil which is placed in contact with the end of the specimen and which is connected to a ballistic galvanometer G.

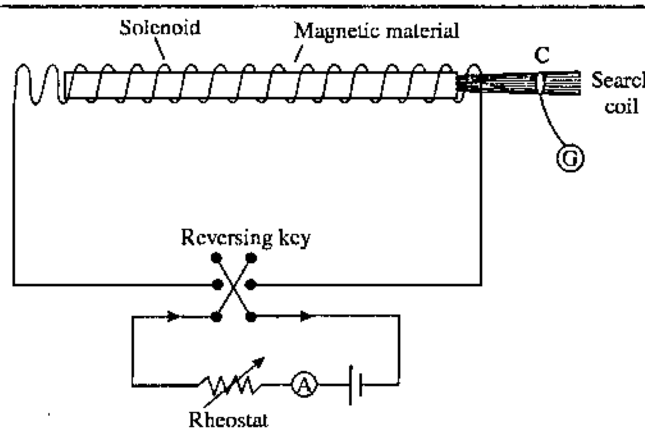


Fig. 32

The specimen is magnetised by passing a current i through the solenoid. The current i is now increased in steps from zero, and the corresponding deflections θ on the ballistic galvanometer are observed. Thus the specimen is taken through a *magnetic cycle* or cycle of magnetisation.

From above it is clear that (i) as the value of current is increased, the magnetising field H increases and the intensity of magnetisation I produced in the speciman also increases. A graph between I and H is shown in the fig. (33). At O when H is zero, I is also zero. When H increases I also increases but non uniformaly along OA.

Beyond A if H is further increase I remains constant. The substance in this condition is said to be saturated. If H is increased further I will not increase.

(ii) If H is now decrease the value of I decreases but at a much lower rate and when H is zero the value of I is not zero but has a value AB. Thus, I lags behind H.

(iii) If the value of H is further increased in the opposite direction, the value of I furher decreases along BC and becomes zero when the value of H is equal to OC. Thus, I still lagging behind H.

(iv) If the value of H is further increased, beyond OC, the substance is increasingly magnetised in opposite direction along CD. At D the substance is again fully saturated.

(v) On decreasing the value of H to zero *i.e.*, the H is made maximum in to its original direction, we get the portion BE of the curve. It is clear from the curve that I has a value equal to OE even when his zero.

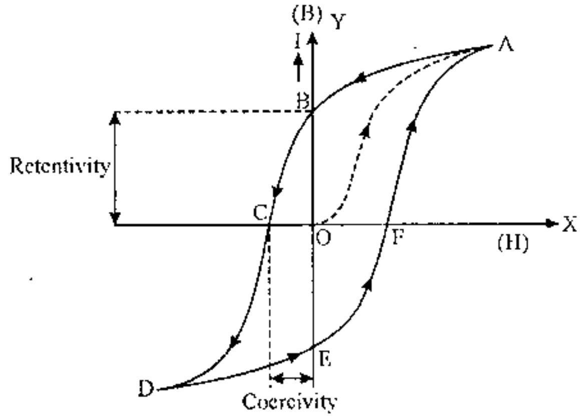


Fig. 33

(vi) On increasing the value of H further the curve EFA is obtained. This complete the cycle of magnetisation.

In the above experiment it is seen that the intensity of magnetisation (I) or magnetic induction does not become zero although the magnetising field is brought down to zero *i.e.*, the value of I or B always lags behind H. This shows that the magnetic conditions of iron tend to persist or oppose any change.

From above it is also clear that I (or B) always lags behind the field H. **This lagging of I (or B) behind H is called "hysteresis".** The closed curve $ABCDEFA$ which represents a cycle of magnetisation of the substance is known as the **"hysteresis curve or loop"** of the substance.

Hysteresis Loop : The graph showing how B increases with H from zero too maximum in one direction and then back through zero to a maximum in the opposite direction and inally back again through zero to the first maximum is called a hysteresis loop.

Remanence or Retentivity : When the specimen is taken through a magnetic cycle, the variation of B with H is shown in fig. 33. The point O represents the initial unmagnetised state of the substance ($B = 0$) and ($H = 0$). As H is increased B increases non-uniformly along OA. At A the substance is fully magnetised. If H is further increases than B does not increase.

If H is decreased the magnetisation B of the substance also decreases and takes a new path AB. Thus B lags behind to H. When H becomes zero B still has a value equal to OB. Thus the power of retaining the magnetism is known as "retentivity" or "remanence". Thus *the retentivity of a substance is a measure of the magnetisation remaining in the substance when the magnetising field is removed.* In fig. 9. OB represents the retentivity or remanence of the substance.

Coercivity : When the field is reversed, the residual magnetism is opposed. Each increase of magnetising field causes a decrease of flux density because domains are twisted out of alignment. Eventually, the flux density is reduced to zero, when the opposing field H has a certain value. This value of H is called *coercivity* of the material.

Thus "the limiting value of the force required to reduce retentivity to zero is known as coercivity". In fig. (33) OC represents the coercivity of the substance magnetic materials having small coercivity are known as soft while those having large coercivity are known as hard.

• 3.24. HYSTERESIS LOSS

When a specimen is magnetised by placing it in external magnetic field, the domains having the same direction as the external field grow in size, and also each domain rotates as a single unit. Thus direction of magnetisation of the domain becomes alinged with the direction of the field. The energy is taken by the specimen in this process. Making the external field zero, the domain boundaries do not move completely back to their original positions and the specimen is said to have retained some mangnetisation. This is hysteresis effect and the energy supplied to the specimen during magnetisation is not fully recovered. This difference of energy is called hysteresis loss. It is lost as heat in the material.

Hysteresis Loss per unit Volume per Cycle : Consider a ring of ferromagnetic material of permeability μ (weber/ ampere-metre), mean circumferential length l (metres), cross sectional area A (metre^2). Let the ring has a coil of N turns wound uniformly upon it. If a current i amperes is sent through the coil of produce a flux density B in the material, then the flux BA is linked with each turn of the magnetising coil.

∴ Total magnetic flux linked with the magnetising coil is given by

$$\Phi_B = NBA \text{ (weber-turns).} \qquad \ldots (1)$$

and the magnetising force (magnetic intensity) H in the material is given by

$$H = \frac{Ni}{l} \text{ ampere-turns/metre}$$

or

$$i = \frac{Hl}{N} \text{ ampere} \qquad \ldots (2)$$

Increasing the current i, the magnetic flux linked with the coil changes so that an induced e.m.f. ε is produced in the coil and is given by

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(NBA) \qquad \text{[from eq. (1)]}$$

$$= -NA\frac{dB}{dt} \text{ (volt)} \qquad \ldots (3)$$

Here. minus sign shows that the e.m.f ε oppose the increase in current (Lenz's law).

The work dW' (say) must be done by the current i against the induced e.m.f. ϵ in a time dt.

$$\therefore \quad dW' = -\varepsilon\, i\, dt$$

$$= \left(NA\frac{dB}{dt}\right)\left(\frac{Hl}{N}\right)dt \qquad \text{[from eq. (3) and (2)]}$$

$$= HAl\, dB \text{ joule}$$

This energy is stored in the magnetic field.

∴ The increase in energy per unit volume of the material is

$$dW = \frac{dW'}{Al} = H\, dB \text{ joule/metre}^3$$

where Al is the volume of the ring. If the flux density in the material is increased from B_1 to B_2, the required energy W per unit volume is given by

$$W = \int_{B_1}^{B_2} H\, dB \text{ joule/metre}^3. \qquad \ldots (4)$$

Fig. 34 shows a B–H curve.

At a point near Q

$$PQ = H \quad \text{and} \quad PR = dB$$

∴ Area of the strip $PQRS = HdB$.

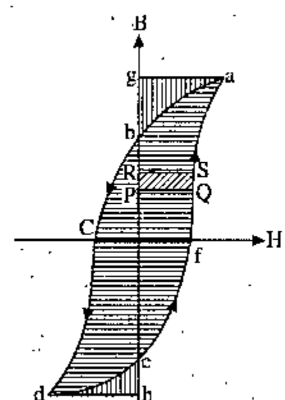


Fig. 34

The increase in energy per unit volume of the material in carrying it from the state f to the state a will be the sum of all such area, that is, the area *OfagO*. Now, the material is carried from the state a to the state b, then both H and B are reduced. In this process, an energy equal to the area *agba* per unit volume of the material is restored back. Thus the energy dissipated per unit volume of the material in carrying it from the state f to the state b (through a) is equal to the horizontally shaded area *OfabO*.

Considering the whole cycle, the net energy dissipated per unit volume of the material, in carrying it through one complete cycle of magnetisation, is equal to the area *fabcdef* (area of the *BH* loop).

∴ Energy dissipated per unit volume of the material per cycle of magnetisation

$$= \text{area of } B - H \text{ loop}$$

$$= \int H\, dB \text{ joule/metre}^3.$$

(b) Area of I–H loop : The magnetic induction is given by

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{I}$$

Differentiating, it, we get

$$dB = \mu_0\, dH + \mu_0\, dI$$

Multiplying both sides by H, we get

$$HdB = \mu_0\, HdH + \mu_0\, HdI$$

Integrating it for a complete cycle, we get

$$\oint HdB = \mu_0 \oint HdH + \mu_0 \oint HdI$$

But $\oint HdB$ the area of the B–H loop and $\oint HdI$ is the area of I–H loop.

$\oint HdH$ is zero because a graph of H plotted against H will give straight line and hence the enclosed area is zero.

$\therefore$ Area of the B–H loop = $\mu_0 \times$ area of the I–H loop.

Application : The knowledge of the energy dissipation helps in selecting the material giving the minimum hysteresis loss when it is carried through a cycle of magnetisation. Such materials are suitable for constructing the cores of the transformers and the armatures of dynamos and motors. Hysteresis curves also give an idea of the other magnetic properties of the material such as retentivity and coercivity so that the magnetic material may be selected for a particular purpose.

• SUMMARY

- The magnetic field induction at a point is said to be 1 tesla if a charge of 1 coulomb which moving at right angle to a magnetic field with a velocity of 1 ms^{-1}.
- The magnetic force on a charged particle,

  $$\vec{F} = q\vec{v}\vec{B}$$

  where, $\vec{B}$ magnetic field, q = charged particle, $\vec{v}$ = velociy.
- Cyclotron was invented by Lawrence and it is also known as magnetic resonance accelerator.
- Required expression for the Biot-Savart's law :

  $$\vec{B} = \int \frac{\mu_0}{4\pi} \cdot \frac{I\,\vec{dl} \times \vec{r}}{r^3}$$

- Biot-Savart's law is applicable to very small length of current carrying conductor and is valid for a symmetrical current distribution.
- Magnetic induction field at the ends of solenoid is half of that the centre,

  $$B = \frac{1}{2}\mu_0\, n_i$$

- Relationship between torque and dipole moment,

  $$\tau = MB\cos\theta$$

- Moving coil galvanometer is a device which is used for detection and measurement of small electric currents.

• STUDENT ACTIVITY

1. Explain magnetic intensity.

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2. Explain intensity of magnetisation.

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3. Explain magnetic flux.

4. What do you mean by magnetic susceptibility ?

5. Explain magnetic permeability.

6. Define para, di and ferromagnetic substances.

7. Explain hysterisis loss.

• TEST YOURSELF

1. Discuss the motion of a charged particle in uniform magnetic field.
2. What is Cyclotron ? Give its principle construction and working.
3. What is Biot-Savart's Law ? Find out magnetic field due to a current carrying long, straight conductor.
4. Show that two parallel conductors carrying currents in the same direction attract each other while those carrying currents in opposite directions repel each other.
5. Using Biot-Savart's law find an expression for the magnetic field at any point on the axis of a circular current carrying coil.
6. Obtain an expression for the intensity of magnetic field inside a long solenoid carrying a current I.

7. Use Ampere's law to find the magnetic induction B due to a straight current carrying conductor.
8. Use Ampere's law to find the magnetic induction B due to a long solenoid.
9. Using Ampere's law find out the magnetic induction B due to an endless solenoid or toroid.
10. State and prove Ampere's law in electromagnetism.
11. Derive an expression for the magnetic field resulting from a 8uniformly distributed current I in a long cylindrical wire of diameter d in the regions :

    (a) $0 \le r \le \frac{d}{2}$ (b) $\frac{d}{2} \le r \le \infty$.
12. Describe the principle, construction, theory and advantages of a moving coil galvanometer.
13. Give the construction and working of a moving coil ballistic galvanometer.
14. A charge is moving perpendicular to magnetic field. Its path is a :

    (a) parabola (b) circle (c) ellipse (d) straight line
15. If a charged particle enters a uniform magnetic field, neither perpendicular nor parallel then its path will be :

    (a) straight line (b) elliptical (c) hellical (d) parabola
16. Which of the following rays are not deflected by a magnetic field :

    (a) α-rays (b) β-rays (c) γ-rays (d) positive rays
17. An electron enters a magnetic field along perpendicular direction. Following quantity will remain constant :

    (a) momentum (b) kinetic energy (c) velocity (d) acceleration
18. A charged particle of charge q moving with velocity v enters along the axis of a current carrying solenoid. The magnetic force on the particle is : *(Meerut 2001)*

    (a) zero (b) qvB
    (c) finite but not qvB (d) infinite
19. Cyclotron is used to accelerate :

    (a) electrons (b) protons (c) neutrons (d) all of these
20. Maximum kinetic energy of the ion accelerated by cyclotron is given by :

    (a) $\frac{R^2q^2B^2}{2m}$ (b) $\frac{RqB}{m}$ (c) $\frac{Rq^2B^3}{m^2}$ (d) $\frac{R^2qB}{2m}$
21. A current i is flowing in a circular conductor of radius r. It is lying in a uniform magnetic field B such that its plane is normal to B. The magnetic force acting on the loop will be :

    (a) Zero (b) πirB (c) $2\pi irB$ (d) irB
22. The magnetic flux density at a point distant d from a long straight current carrying conductor is B, then its value at distance $\frac{d}{2}$ will be :

    (a) $4B$ (b) $2B$ (c) $\frac{B}{2}$ (d) $\frac{B}{4}$
23. The magnetic induction at the centre of square frame of a conducting wire, when its diagonally opposite centres are connected to a battery will be :

    (a) Zero (b) $\frac{\mu_0}{\pi a}$ (c) $\frac{2\mu_0 i}{\pi a}$ (d) $\frac{4\mu_0 i}{\pi a}$
24. The ratio of magnetic field on the axis of a circular current carrying coil of radius a to the magnetic field at its centre will be

    (a) $\frac{1}{\left(1+\frac{x^2}{a^2}\right)^{3/2}}$ (b) $\frac{1}{\left(1+\frac{a^2}{x^2}\right)^{3/2}}$ (c) $\frac{1}{\left(1+\frac{a^2}{x^2}\right)^{2}}$ (d) $\frac{1}{\left(1+\frac{a^2}{x^2}\right)^{3}}$
25. The following relation is correct

    (a) current sensitivity = charge sensitivity
    (b) current sensitivity = $\frac{2\pi}{T}\times$ charge sensitivity
    (c) current sensitivity = $\frac{1}{\text{charge sensitivity}}$
    (d) none of these
26. The value of ballistic reduction factor K is given by

    (a) $K=\frac{CE}{\theta_1\left(1+\frac{\lambda}{2}\right)}$ (b) $K=\frac{CE}{\theta_1\,(1+\lambda)}$ (c) $K=\frac{CE}{\theta_1\,(1+2\lambda)}$ (d) $K=\frac{2CE}{\theta_1\left(1+\frac{\lambda}{2}\right)}$

27. Which of the following condition is not necessary for the moving coil galvanometer to be ballistic
(a) the moment of inertia of moving system should be small
(b) suspension wire should be very fine
(c) air resistance should be small
(d) the damping should be small

28. The relative permeability of a substance is 0.9998. The substance is :
(a) dia-magnetic (b) para-magnetic
(c) ferro-magnetic (d) non-magnetic

29. The magnetism of atomic magnet is due to :
(a) only spin motion of electrons
(b) only orbital motion of electrons
(c) both spin and orbital motion of electrons
(d) the motion of protons

30. The resultant magnetic moment of neon atom will be :
(a) infinity (b) zero (c) μ_B (d) $\frac{\mu_B}{2}$

31. The unit of intensity of magnetic field H is : *(Meerut 2001)*
(a) Amp/m^2 (b) Amp/m (c) Ampere (d) $Amp\text{-}m^2$

32. The cause of paramagnetism is :
(a) unpaired electrons
(b) electron excess and spin motion of electrons
(c) paired electrons and orbital motion of electrons
(d) electrons and orbital motion of electrons

33. The value of earth's magnetic field is :
(a) 500 Gauss (b) 50 Gauss (c) 5 Gauss (d) 100 Gauss

34. The slope of B—H curve in magnetic saturation stage is :
(a) zero (b) infinity (c) μ_0 (d) $\frac{1}{\mu_0}$

35. The correct measure of magnetic hardness of a material is :
(a) ramnant magnetism (b) hysteresis loss
(c) coercivity (d) curie temperature

ANSWERS

14. (b) **15.** (c) **16.** (c) **17.** (b) **18.** (a) **19.** (b) **20.** (a) **21.** (a) **22.** (b) **23.** (a)
24. (a) **25.** (d) **26.** (a) **27.** (a) **28.** (a) **29.** (c) **30.** (b) **31.** (b) **32.** (b) **33.** (a)
34. (c) **35.** (c)

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UNIT

4

ELECTROMAGNETIC INDUCTION

STRUCTURE

LEARNING OBJECTIVES

After going this unit you will learn :

- Induced electromotive force, Neumann's law and Lenz's law.
- Self inductance and Rayleigh method to measure it.
- Mutual inductance and its measurement.
- Transformer's principle and the types of transformers with its uses.

• 4.1. ELECTROMAGNETIC INDUCTION

In 1831 Faraday discovered that "whenever magnetic lines of force are cut by a closed circuit then an induced current flows in the circuit and lasts only so long as the charge lasts". The induced e.m.f. due to which this such current is obtained in the circuit, is called the **induced electromotive force** and this phenomenon is called **electro- magnetic induction.**

Let us consider an experiment demonstrating electromagnetic induction.

Exp. : It consists a coil connected in series with a galvanometer as shown in the fig. (1). When there is no electromotive force in the circuit, we would not expect any deflection in the galvanometer. When a magnet is moved relative to the coil then the following facts are obtained :

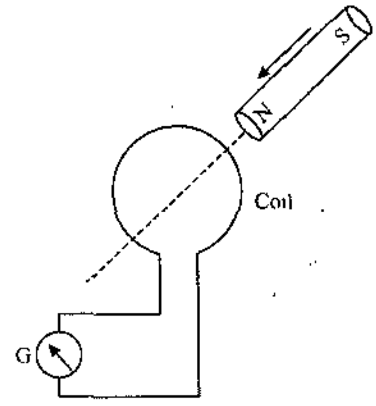

Fig. 1

(i) When the magnet is pushed towards to the coil, with its north pole facing the coil, then galvanometer shows a deflection in one direction. This means that a current is set up in the coil.

(ii) When the magnet is withdrawn from the coil then the galvanometer again shows a deflection but now in the opposite direction. This indicates that the current in the coil is now in opposite direction.

(iii) When the magnetic is held stationary with respect to the coil then the galvanometer does not show any deflection.

(iv) If the experiment is repeated with south pole facing towards the coil then the deflections are reversed in comparison to those observed in points (i) and (ii).

(v) It is further observed that when a magnet is moved fast, the deflection in the galvanometer is large and when it is moved slowly. The deflection is small. This shows that the deflection depends upon the rate at which the magnet is pushed or withdrawn from the coil.

(vi) Further experiment shows that is matter of relative motion of the magnet and the coil. It *does not make any difference whether the magnet is moved towards the coil or the coil is moved* torwards the magnet.

• 4.2. FARADAY'S LAWS OF ELECTRO-MAGNETIC INDUCTION

Faraday gave two laws of Electro-magnetic induction. They are :

(i) First Law (Neumann's Law) : According to first law "whenever the amount of magnetic *flux linked with a circuit changes then an e.m.f. is induced in the circuit. The induced e.m.f. so long* as the change in magnetic flux continues. *The magnitude of the induced e.m.f. is directly proportional to the rate of change of magnetic flux.* This is also known as Neumann's law.

i.e.,
$$e \propto \frac{d\Phi_B}{dt}$$

or
$$e = -\frac{d\Phi_B}{dt} \qquad \ldots(1)$$

The negative sign shows that the direction of e.m.f. is always opposite to the change in magnetic flux linked with the circuit.

If the rate of change of flux is in webers per second then induced e.m.f. e will be in volts. The two units are equivalent and equal to m^2 kg s^{-2} C^{-1}.

If the coil contains N turns, then the induced e.m.f. is

$$e = -N\frac{d\Phi_B}{dt} = -\frac{d\,(N\Phi_B)}{dt}$$

where $N\Phi_B$ is called the magnetic flux linked with the circuit.

(ii) Second Law : According to this law, "whenever an e.m.f. is set up by a change of magnetic flux through a circuit, its direction is always such as to oppose the change that causes it. This is also known as Lenz's Law".

To understand Lenz's law, it can be explained as, when the north pole of the magnet approaches the coil, then the direction of induced e.m.f. is such that the near face of the coil acts as a magnetic north pole. The repulsion between the two poles opposes the motion of the magnet towards the coil *i.e.* the coil oppose the flux through it. The work done in overcoming the repulsive force between two north poles appears as electrical energy in the circuit. Similarly when a magnet is withdrawn then the direction of induced current is such as to make the near face of the coil a south pole. The attraction of the two poles oppose the motion of the magnet away from the coils. The e.m.f. and the current is reversed and now coil attempts to maintain the flux *i.e.,* it oppose the decrease in flux. At the same time the work is done in overcoming the attractive force set up between the north pole of magnet and south polarity developed at the near end of the coil and it appears as electrical energy in the circuit. Thus in either case work has to be done in moving the magnet.

If the e.m.f. is caused by the change in flux through a closed circuit then the current resulting form the e.m.f. is in such a direction as to set up a flux of its own which oppose the original flux, if flux is increasing and in the same direction as the original flux, if later is decreasing.

• 4.3. DEDUCTION OF FARADAY'S LAW

$$e = -\frac{d\Phi_B}{dt}.$$

In Fig. (2), a wire of length l, and resistance R, resting on two parallel conducting rails of negligible resistance is shown. It is assumed that when wire slides on these rails, there will be no friction at all.

When a battery of e.m.f., E is connected to one end of the rails then a current i flows in the closed circuit formed through the wire. Due to the current i in wire, as it is free and situated in field of magnetic induction $\vec{B}$, it will experience a force

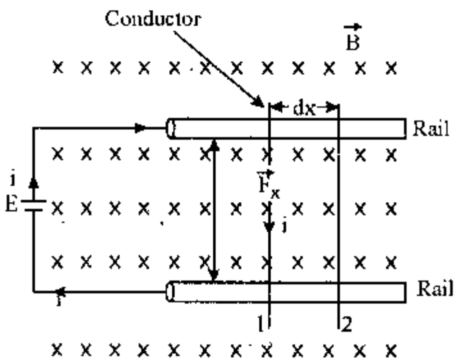

Fig. 2

$$\vec{F} = i\vec{l} \times \vec{B},$$

which act to the right of the wire as shown in the Fig. (2) according to Fleming's left hand rule. Suppose, under the action of this force, conductor moves to position 2 through a distance dx then,

$$\text{work done by the force} = F \,.\, dx. \qquad \ldots(1)$$

Further, if current flows for time dt, then joule heat in the wire will be

$$= Ri^2 dt \qquad \ldots(2)$$

By the conservation of energy principle both the work done of equation (1) and joule heat of equation (2) must be equal to the work done by the heat of e.m.f. in lifting the charge dq through it which is

$$= E \,.\, dq$$

Rate of doing the work by the heat

$$E \times \frac{dq}{dt} = di$$

Therefore work done in time dt is

$$= E\, i\, dt \qquad \ldots(3)$$

Hence from equations (1), (2) and (3), we can write

$$Ei\, dt = Ri^2 dt + F \,.\, dx$$

$$= Ri^2 dt + F \,.\, \frac{dx}{dt} \,.\, dt \qquad \ldots(4)$$

But $\frac{dx}{dt} = \upsilon$, velocity with which wire moves

and $F = ilB$ (from $F = ilB \sin\theta$, θ is $90°$ angle betweem $\vec{l}$ and $\vec{B}$)

Therefore on putting in equation (4), we get

$$Ei\, dt = Ri^2 dt + ilB\, \upsilon dt$$

or

$$i = \frac{E - lB\upsilon}{R} \qquad \ldots(5)$$

The additional term $(-lB\,\upsilon)$ is the e.m.f. induced in the closed circuit due to the motion of conductor from position 1 to position 2, cutting the magnetic flux through it. That is, induced e.m.f. is

$$e = -lB\,\upsilon. \qquad \ldots(6)$$

It is true because when conductor moves, change in flux is

$$d\Phi_B = BA = Bl\, dx$$

$$\frac{d\Phi_B}{dt} = Bl \,.\, \frac{dx}{dt} = Bl\,\upsilon \qquad \ldots(7)$$

From equations (6) and (7), we find

$$e = \frac{-d\Phi_B}{dt},$$

which is Faraday's law.

4.4. MEASUREMENT OF MAGNETIC FIELD

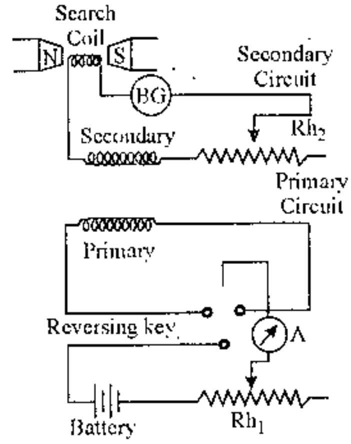

Fig. 3

Fig. (3) shows the experimental arrangement for the measurement of a magnetic field. Let N and S be the poles of an horse shoe electromagnet between which magnetic induction, $\vec{B}$, is to be measured.

A small coil called the search coil (of nearly 50 turns of fine copper wire and about one square cm. in area) is placed in between the pole pieces. The coil is placed in such a way that the plane of the coil is normal to the field, $\vec{B}$, so that maximum flux passes through it. The search coil is connected in series with a ballistic galvanometer (B.G.) through an adjustable resistance Rh_2 and a secondary coil.

The primary coil is connected through a reversing key to a battery, an ammeter A and adjustable resistance Rh_1. Initially the primary circuit is kept off and search coil is rapidly withdrawn from the field and the throw in ballistic galvanometer is noted.

Let B be magnetic induction between pole pieces.

A be area of cross-section of each turn of search coil.

N be total number of turns in search coil.

The flux Φ_B passing normally through each turn of coil

$$= \text{flux density} \times \text{area}$$
$$= B\,A$$

Hence total flux linked with coil $\Phi_1 = NBA$

If R is the resistance of the secondary circuit, then the quantity of electricity produced is

$$q_1 = \frac{\text{Change in magnetic flux } (\Phi_1 - \Phi_2)}{\text{Resistance}} = \frac{NBA}{R}$$

because when the search coil is rapidly withdrawn the magnetic flux linked with it is reduced to zero and therefore $\Phi_2 = 0$.

If θ_1 is the corresponding throw in ballistic galvanometer when charge q_1 passes through it then

$$q_1 = K\,\theta_1\,(1 + \lambda/2)$$

where 'K' is the ballistic constant and λ is the logarithmic decrement substituting for q_1 we get,

$$\frac{NBA}{R} = K\,\theta_1\,(1 + \lambda/2) \qquad \ldots(1)$$

If n_1 is the number of turns per unit length in primary of solenoid, the magnetic induction due to a current, i. in the primary is $\mu_0\, n_1 i$. If N_2 be the total number of turns in the secondary of solenoid and a be the area of cross-section of each turn, then flux linked with secondary coil is

$$\mu_0 n_1 i \times N_2 a$$

when the current in primary is reversed, the magnetic flux linked with secondary coil changes from $\mu_0 n_1 i\, N_2 a$ to $-\mu_0 n_1 i N_2 a$. Thus the total change of flux is $2\mu_0 n_1 i N_2 a$. If q_2 be the charge produced to change in magnetic flux and on passing it, through ballistic galvanometer, corresponding throw observed be θ_2, then

$$q_2 = \frac{\text{toal change in flux}}{\text{resistance}} = \frac{2\mu_0 n_1 i N_2 a}{R}$$
$$= K\,\theta_2\,(1 + \lambda/2) \qquad \ldots(2)$$

Dividing equation (1) by (2), we have

$$\frac{NBA}{2\mu_0 n_1 i N_2 a} = \frac{\theta_1}{\theta_2}$$

$$\boxed{B = \frac{2\mu_0\, n_1 i N_2 a}{NA} \cdot \frac{\theta_1}{\theta_2} \text{ web/met}^2}$$

which gives the magnetic induction in the space between pole pieces.

• 4.5. DIFFERENTIAL FORM OF FARADAY'S LAW

In a general case

$$e = \oint \vec{E} \cdot \vec{dl} \qquad \ldots(1)$$

Again $$e = -\frac{\partial \Phi_B}{\partial t} \qquad \ldots(2)$$

From eqs. (1) and (2), we get

$$\boxed{\oint \vec{E} \, \vec{dl} = -\frac{\partial \Phi_B}{\partial t}}$$

which is the form in which this law is expressed. Further the magnetic flux, Φ_B, can also be written as

$$\Phi_B = \int_S \vec{B} \cdot d\vec{S}$$

where S is any surface having loop as boundary. Therefore

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t}(\Phi_B) = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{dS}$$

Using Stoke's theorem to change line integral into surface integral, we get

$$\int_S \text{curl}\, \vec{E} \cdot \vec{dS} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{dS}$$

or $$\int_S \left(\text{curl}\, \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot \vec{dS} = 0$$

Since surface is arbitrary, therefore above eq. holds good only if the integrand vanishes, *i.e.*,

$$\text{curl}\, \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

or $$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

This is the differential form of Faraday's Law.

• 4.6. DISPLACEMENT CURRENT

We know that a *changing magnetic field, $\vec{B}$ gives rise to an electric field $\vec{E}$.* This is expressed in the mathematical relation as

$$\oint \vec{E} \cdot \vec{dl} = \frac{-d\Phi_B}{dt} \qquad \ldots(1)$$

Experimentally it has also been observed that a *changing electric field produces a magnetic field* and can be expressed mathematically

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \ldots(2)$$

In equation (2) the constant μ_0 and ε_0 appear to put the relation in proper units. We can write it as

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(\varepsilon_0 \frac{d\Phi_E}{dt} \right) = \mu_0 \, i_d \qquad \ldots(3)$$

where i_d is called the *displacement current* arising on account of changing electric field.

It can be seen that it is correct to call the term $\left(\varepsilon_0 \frac{d\Phi_E}{dt} \right)$ as current because it has the dimensions of current as shown below :

$$\varepsilon_0\left(\frac{d\Phi_E}{dt}\right)=\frac{\text{Coulomb}^2}{\text{newton}-\text{meter}^2}\times\frac{(\text{electric field}\times\text{area})}{\text{second}}$$

$$=\frac{\text{Coulomb}^2}{\text{newton}-\text{meter}^2}\times\frac{(\text{newton/coulomb})\ \text{meter}^2}{\text{second}}$$

$$=\frac{\text{Coulomb}}{\text{second}}=\text{ampere}.$$

We assign the name to this current as displacement current and can be expressed in terms of electric displacement $\vec{D}$ as shown below :

$$\varepsilon_0\frac{d\Phi_E}{dt}=\varepsilon_0\frac{d}{dt}(\vec{E}.\vec{A})=A\frac{d}{dt}(\varepsilon_0\vec{E})$$

$$=A\frac{d\vec{D}}{dt}$$

where $\frac{d\vec{D}}{dt}$ is called displacement current density. Thus displacement current is

$$i_d=A\frac{d\vec{D}}{dt}$$

$$=\varepsilon_0\left(\frac{d}{dt}\Phi_E\right)\quad\quad\ldots(4)$$

We also know that a magnetic field is also set up by a current i, in the wire, given by Ampere's law as

$$\oint\vec{B}.\vec{dl}=\mu_0 i,\quad\quad\ldots(5)$$

where i is current in the wire, called *conduction current.*

Combining equations (3) and (5), we get

$$\oint\vec{B}.\vec{dl}=\mu_0(i_d+i).\quad\quad\ldots(6)$$

It is displacement current, i_d in the gap between the plates of a capacitor Fig. (4) which allows the conduction current i to be continuous across the capacitor gap because $i_d = i$ as we shall show. Conduction current itself is not continuous across the capacitor gap because no charge is transported across the gap.

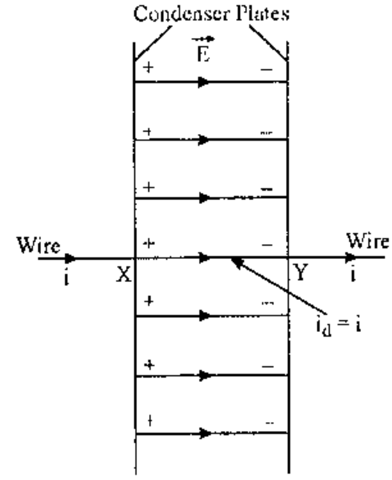

Fig. 4

Suppose A, is the area of condenser plates, q the charge then capacity is

$$C=\frac{\varepsilon_0 A}{d}$$

and the potential difference will be

$$V=\frac{q}{C}=\frac{qd}{\varepsilon_0 A}$$

so that electric field between plates is

$$E=\frac{V}{d}=\frac{qd}{\varepsilon_0 A}\cdot\frac{1}{d}=\frac{q}{\varepsilon_0 A}$$

Therefore

$$\frac{dE}{dt}=\frac{1}{\varepsilon_0 A}\frac{dq}{dt}=\frac{i}{\varepsilon_0 A}\quad\quad\ldots(7)$$

Now the displacement current is

$$i_d=\varepsilon_0\frac{d\Phi_E}{dt}=\varepsilon_0\frac{d}{dt}(EA)$$

$$=\varepsilon_0 A\frac{dE}{dt}$$

$$= \varepsilon_0 A \frac{i}{\varepsilon_0 A} \text{ using eq. (7)}$$

$$= i, \qquad \ldots(8)$$

which shows that the displacement current in the capacitor gap is identical with the conduction current and thus retains the continuity of current.

• STUDENT ACTIVITY

1. What is electromagnetic induction ?

2. Obtain differential form of Faraday's law.

3. Find out current induced in a loop.

• 4.7. SELF INDUCTANCE OR INDUCTION

The phenomenon of the self induction was discovered by Henry. We know when a current flows in a coil then magnetic field is set up in it as shown in the fig. 5 (a). If the current passing through the coil is changed, an induced e.m.f. is set up in the coil. According to Lenz's law, the direction of induced e.m.f. is such as to oppose the change in the current. When the current is increasing then the induced e.m.f. is against the current which is shown in fig. 5 (b) and when the current is decreasing then it is in the direction of current this is shown by fig. 5 (c). This phenomenon is called self induction.

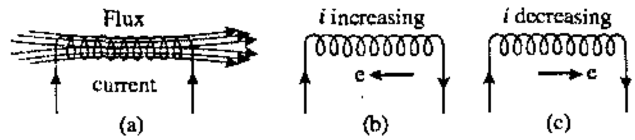

Fig. 5

In other words we can say that the property of the circuit by virtue of which any change in the magnetic flux linked with it, induces an e.m.f. in it is called self inductance and the induced e.m.f. is called back e.m.f.

When the current in a coil is switched on then self induction oppose the growth of current and when the current is switched off then the self induction opposes the decay of current.

Coefficient of self induction : Let us consider a coil of N turns carrying a current i. Let ϕ_B is the magnetic flux with each turns of the coil then total flux linked will be $N\phi_B$.

When no magnetic material near the coil then the flux linked with the coil will be proportional to the current flowing in it *i.e.*,

$$N\phi_B \propto i \quad \text{or} \quad N\phi_B = Li$$

where L is constant and this constant is known as coefficient of self induction so

$$\boxed{L = \frac{N\phi_B}{i}} \qquad \ldots (1)$$

When flux changes the induced back e.m.f. in coil is given by

$$e = -N\frac{d\phi_B}{dt}$$

$$e = -\frac{d}{dt}(N\phi_B$$

$$e = -\frac{d}{dt}(Li) \quad \Rightarrow \boxed{L = -\frac{e}{di/dt}} \qquad \ldots (2)$$

From eqns. (1) and (2) the coefficient of self induction may be defined in the following two ways :

(i) Self inductance of a coil is numerically equal to the magnetic flux linked with coil when unit current flows in it.

(ii) Self inductance of a coil is numerically equal to the induced e.m.f. in the coil when the rate of change of current in the coil is unity.

Units of self inductance are Henry : When e is in volt and di/dt in ampere per second, then the self-inductance is in *henry*. A circuit will have a self-inductance of 1 henry if an e.m.f. of 1 volt is induced by a current changing at the rate of 1 ampere per second.

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere/second}}$$

The smaller units are millihenry (mH) = 10^{-3} *henry and microhenry* (μH) = 10^{-6} *henry.*

Inductive circuit is a circuit having a winding whose self- inductance is appreciable compared with its resistance.

Rayleigh method to measure self inductance of a coil : The coil whose self inductance is to be determined is connected in the unknown arms of Wheatstone's bridge, P, Q, R are non-inductive resistances, r is a standard low resistance (of the order of .01 ohm or less).

From the Fig. (6), we keep $P = Q$ and short circuit the low resistance r (by closing Key K). Then we adjust R for no deflection by first pressing battery key K_1 and then galvanometer key K_2.

If now the galvanometer key K_2 is closed first and then the battery key K_1, then a throw θ_1 is observed in the galvanometer. This throw arises due to e.m.f. induced in the coil on account of momentary current that flows on pressing key K_2 and then K_1

$$\text{e.m.f. induced, } e = -L\frac{di}{dt}.$$

If G be galvanometer resistance then current through it due to induced e.m.f. is :

$$i' = \frac{kL}{G}\frac{di}{dt}$$

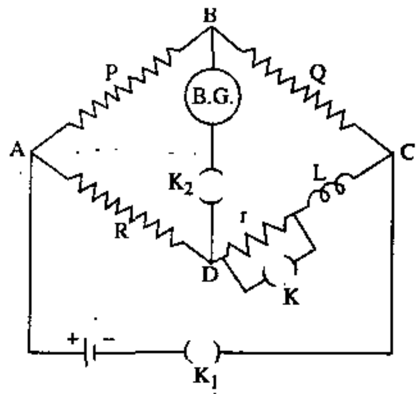

Fig. 6

where k is a constant which depend upon the relative resistance in the circuit.

Therefore corresponding charge passing through the galvanometer

$$q = \int_0^{i_0} i'\,dt = \frac{kL}{G}\int_0^{i_0}\frac{di}{dt}\,dt = \frac{kL}{G}\cdot i_0 \qquad \ldots(1)$$

where i_0 is the steady maximum value of the current that grows in the coil.

If θ_1 be the first throw of the galvanometer then we know that

$$q = K\,\theta_1\,(1+\lambda/2) \qquad \ldots(2)$$

where K is ballistic constant and $K = \frac{T}{2\pi}\cdot\frac{C}{NBA}$.

so that from (1) and (2)

$$\frac{kL}{G}\,i_0 = K\,\theta_1\,(1+\lambda/2)$$

$$= \frac{T}{2\pi}\cdot\frac{C}{NBA}\cdot\theta_1\,(1+\lambda/2) \qquad \ldots(3)$$

In order to eliminate k and i_0, we now introduce small resistance r, by opening key K. As r is small it does not affect the current i_0 in the CD arm. But it introduces, as it were, an extra e.m.f. $(r\,i_0)$ in this arm giving rise to steady current $\left(\frac{kr}{G}\right)i_0$ through the Galvanometer and if corresponding steady deflection is θ (keeping both K_1 and K_2 pressed), then

$$\left(\frac{kr}{G}\right)i_0 = \frac{C}{NBA}\cdot\phi \qquad \ldots(4)$$

From eqs. (3) and (4) we have

$$\frac{L}{r} = \frac{T}{2\pi}\cdot\frac{\theta_1}{\phi}\,(1+\lambda/2)$$

or $$L = \frac{rT}{2\pi}\cdot\frac{\theta_1}{\phi}\,(1+\lambda/2)$$

for which we can find out the value of L.

• 4.8. ENERGY STORED IN AN INDUCTIVE CIRCUIT

Whenever a current is passed in an inductor, the back e.m.f. induced in it opposes the growth of current. Hence the growing current works against this e.m.f. to attain its final steady value. This work which is provided by the battery in supplying the current is stored as magnetic energy in the inductor.

Let i be the current in the inductor L at any instant 't'. The induced e.m.f. across the inductor is :

$$e = -L\frac{di}{dt}$$

The work required to move a charge dq against this e.m.f. is :

$$dw = -e\,.\,dq = L\frac{di}{dt}\,.\,dq$$

$$= L\,.\,\frac{dq}{dt}\,.\,dt = Lidi$$

$\therefore$ Total work required to build up a steady current i_0 is :

$$w = \int dw = L\int_0^{i_0} i\,di = \frac{1}{2}Li_0^2$$

This work which is supplied to the inductor is stored in the form of potential energy U. Thus

$$U = \frac{1}{2}Li_0^2.$$

This is the required expression.

• 4.9. ENERGY DENSITY IN A MAGNETIC FIELD

Let there is a long solenoid of length l and cross-sectional area A. When a current flows in it, a magnetic field is established which is uniform everywhere inside the inductor. Hence the volume associated with the magnetic field is $A\,l$.

Total energy stored in the solenoid

$$U = \frac{1}{2}Li_0^2$$

We know that the inductance L of the solenoid is

$$L = \frac{\mu_0 N^2 A}{l}$$

(where N is the total number of turns in it)

$$\therefore \qquad U = \frac{1}{2}\cdot\frac{\mu_0 N^2 A}{l}\cdot i_0^2$$

The magnetic field inside the solenoid is

$$B = \mu_0 n i_0$$

or

$$B = \frac{\mu_0 N i_0}{l}$$

Hence $$U = \frac{1}{2}\cdot\frac{B^2}{\mu_0}\cdot Al$$

$\therefore$ energy per unit volume or the energy density u in the magnetic field is

$$\boxed{u = \frac{U}{A\,l} = \frac{1}{2}\frac{B^2}{\mu_0}\ \text{J/m}^3}$$

This is the required result.

• 4.10. SELF INDUCTANCE OF A LONG SOLENOID

Let us consider a long solenoid in which

l = length of the solenoid

n = number of the turns per unit length.

B = magnetic induction inside the solenoid.

A = area of cross reaction.

Flux linked with the solenoid will be

$$N\phi_B = (nl)(BA)$$

We know, in a solenoid

$$B = \mu_0 ni$$

$$\therefore \qquad N\phi_B = (nl)(\mu_0 ni)A$$

$$N\phi_B = \mu_0 n^2 liA$$

We know $$L = \frac{N\phi}{i}$$

$$\therefore \qquad L = \mu_0 n^2 lA$$

The inductance L of a solenoid is proportional to its volume (lA) and to square of turns per unit length (n^2).

• 4.11. SELF-INDUCTANCE OF TOROID

Consider a toroid of mean radius r, total number of turns N and carrying a current i.

The magnetic field induction at a point within its core is given by

$$B = \mu_0 ni$$

$$B = \mu_0 \left(\frac{N}{l}\right) i \qquad \text{where } l = 2\pi r$$

$$\therefore \qquad B = \frac{\mu_0}{2\pi}\left(\frac{Ni}{r}\right)$$

where μ_0 is the permeability constant.

The magnetic flux through each turn of the coil is given by

$$\phi_B = BA = \frac{\mu_0}{2\pi}\left(\frac{Ni}{r}\right)A,$$

where A is the cross-sectional area of each turn.

$\therefore$ The total flux linked through the coil is

$$N\phi_B = \frac{\mu_0}{2\pi} \cdot \frac{N^2 iA}{r}.$$

The self-inductance of the toroid is given by

$$L = \frac{N\phi_B}{i} = \frac{\mu_0}{2\pi} \cdot \frac{N^2 A}{r}.$$

If the toroid is wound on core of permeability μ, then the self inductance of the toroid will be given by

$$\boxed{L = \frac{\mu}{2\pi} \cdot \frac{N^2 A}{r},}$$

where $\mu = \mu_0 \mu_r$, μ_r is the relative permeability of the core.

• STUDENT ACTIVITY

1. What is meant by self induction ?

2. Explain coefficient of self induction.

3. What is inductive circuit ?

4. Obtain an expression for the self inductance of a long solenoid.

5. Calculate the self inductance of a toroid.

• 4.12. MUTUAL INDUCTANCE

When current is passed through a coil then magnetic field is produced around it and magnetic flux begins to flow. When the current passing through coil changes, the magnetic flux due to the current changes. If another coil is placed near to it then the magnetic flux linked through the neighbouring coil also changes. Hence, an induced e.m.f. is set up in the neighbouring coil and this phenomenon is called *mutual induction*. The pair of coils which show it, is said to have mutual inductance. The coil in which the current changes is called the *primary coil* while the other in which the induced e.m.f. is set up is called the *secondary coil*.

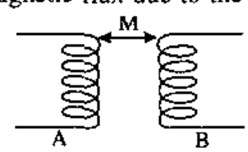

Fig. 7

Hence, "Mutual induction is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced e.m.f.".

Fig. 7 shows two coils A and B. A current i through A produces a magnetic flux Φ_B (say) in each turn of B. If N_s is the total number of turns in B, then the total flux linked through B is $N_s\Phi_B$.

For two given coils situated in fixed relative position, the magnetic flux linked through the secondary is proportional to the current i in the primary, that is

$$N_s\Phi_B \propto i$$

or

$$N_s\Phi_B = Mi$$

where M is a constant called the coefficient of mutual induction or *mutual inductance* between the two coils.

$$\therefore \qquad M = \frac{N_s\Phi_B}{i}. \qquad \text{.... (1)}$$

and the e.m.f. induced in B is given by

$$\epsilon = -\frac{d\,(N_s\,\Phi_B)}{dt}.$$

or

$$\epsilon = -\frac{d}{dt}(Mi) = -M\frac{di}{dt}.$$

$$(\because \; N_s\,\Phi_B = Mi)$$

$$\therefore \qquad M = -\frac{\epsilon}{di/dt} \qquad \text{... (2)}$$

The mutual inductance of the two circuits is defined as follows :

(1) *The mutual inductance of two circuits is numerically equal to the magnetic flux linkages through one circuit when unit current flows through the other.*

(2) *The mutual inductance of two circuits is numerically equal to the e.m.f. in one circuit when the rate of change of current in the other is unity.*

If $\in$ is in volts and di/dt in ampere per sec, then the mutual inductance is in henry.

Mutual Inductance of two given Coils : Fig. 8 shows a long air-cored solenoid (primary coil) of area of cross-section A metre^2 and having n_p turns per metre length of the solenoid. A

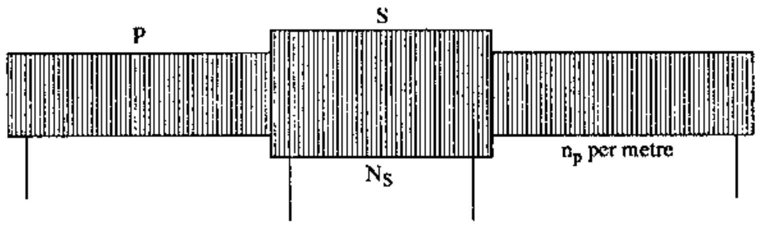

Fig. 8

secondary coil S of total turns N_s is wound closely over the central portion of the primary P. If a current i ampere is flowing in the primary, then the magnetic field induction within the primary is given by

$$B = \mu_0 \, n_p \, i \text{ weber/metre}^2.$$

∴ The magnetic flux through each turn of the primary is

$$\Phi_B = BA = \mu_0 \, n_p \, iA \text{ weber.}$$

The secondary coil is wound closely over the central portion of the primary, therefore the same flux will also be linked with each turn of the secondary.

∴ Total magnetic flux linked with the secondary is

$$N_s \Phi_B = \mu_0 \, n_p \, N_s \, iA \text{ weber-turns.}$$

The mutual inductance of the two coils is given by

$$M = \frac{N_s \, \Phi_B}{i} = \mu_0 \, n_p \, n_s \, A \text{ henry.}$$

Measurement of Mutual Inductance : Fig. 9 shows the circuit diagram. P and S are the two coils whose mutual inductance M is to be measured. A moving-coil ballistic galvanometer B.G. is connected with secondary coil S. C is a four-segment commutator and r is a resistance of the order of 10^{-2} ohm.

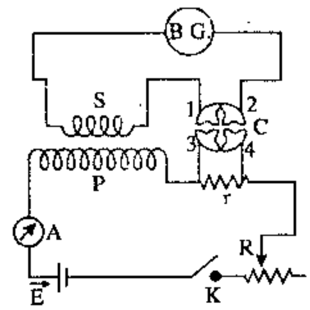

Fig. 9

To begin with, the segments 1 and 2 are connected together so that the ballistic galvanometer and the secondary coil S form a closed circuit. The segments 3 and 4 are connected so that the resistance r is kept short-circuited. The rheostat R is adjusted to give a suitable current through the primary on depressing the key K. When the key K is pressed, the current in the primary P takes some time to grow and the magnetic flux through the secondary S changes during this time. An e.m.f. is induced in the secondary S and a momentary current flows through S. The galvanometer gives a throw.

At any instant, if i is the current through the primary P, then the e.m.f. induced in the secondary S is given by

$$\in = -M \frac{di}{dt}.$$

where M is the mutual inductance of the two circuits.

The instantaneous current i' through the secondary is

$$i' = \frac{\in}{R} = \frac{M}{R} \cdot \frac{di}{dt}$$

where R is the total resistance of the secondary circuit.

∴ The charge dq passing through the galvanometer in time interval dt is given by

$$dq = i' \, dt = \frac{M}{R} \cdot \frac{di}{dt} dt = \frac{M}{R} di$$

As the current in the primary grows from zero to a steady maximum value i_0, the total charge q passed through the galvanometer is obtained by integrating it between the limits 0 to i_0. That is

$$q = \int_0^{i_0} \frac{M}{R} di = \frac{M}{R} i_0.$$

If θ_1 is the first observed throw of the coil of the ballistic galvanometer, then

$$q = \frac{T}{2\pi} \cdot \frac{c}{NBA} \theta_1 \left(1 + \frac{\lambda}{2}\right)$$

where the symbols have their usual meaning.

$$\therefore \qquad \frac{M}{R} i_0 = \frac{T}{2\pi} \cdot \frac{c}{NBA} \theta_1 \left(1 + \frac{\lambda}{2}\right). \qquad \ldots (1)$$

To eliminate i_0 and c/NBA, the contacts between the segments 1 and 2, and between 3 and 4 are broken. Now, the contact between the segments 1 and 3, and between 2 and 4 are made so that the small resistance r is included in the primary circuit. The steady current i_0 is passed in the primary circuit. As r is very smal so i_0 in the primary circuit remains unchanged. But it is equivalent to a potential difference $i_0 r$ and the current $i_0 r/R$ is sent through the galvanometer. If ϕ is the steady deflection, then

$$\frac{i_0 r}{R} = \frac{c}{NBA} \phi.$$

Dividing eq. (1) by eq. (2), we get

$$M = \frac{rT\theta_1}{2\pi\phi}\left(1 + \frac{\lambda}{2}\right).$$

Precautions : (i) To obtain steady currents in primary circuit, an accumulator should be used.

(ii) The connection wires should be thick and as short as possible.

(iii) The primary and secondary coils should be placed away from the galvanometer.

• 4.13. RESULTANT INDUCTANCE WHEN TWO INDUCTANCES ARE IN SERIES

Let L_1 and L_2 inductances are connected in series and a current i is passed through them then the total flux linkage will be the sum of the flux linkage by L_1 and L_2 *i.e.*,

$$\phi_{total} = L_1 i + L_2 i \qquad \ldots (1)$$

if L be the resultant inductance of both series then

$$\phi_{total} = Li \qquad \ldots (2)$$

so eqs. (1) and (2)

$$Li = L_1 i + L_2 i$$

$$\boxed{L = L_1 + L_2}$$

The above conditions are satisfied only when both coils are separated by a large distance. If the separation between the coil is small, then there will be a mutual inductance M between them and hence the resultant induced e.m.f. e in the coils will be sum of the e.m.f. e_1 and e_2 in the coils L_1 and L_2 respectively *i.e.*,

$$e = e_1 + e_2$$

$$= \left[-L_1 \frac{di}{dt} - M\frac{di}{dt}\right] + \left[-L_2 \frac{di}{dt} - M\frac{di}{dt}\right]$$

$$\because \qquad e = -L\frac{di}{dt}$$

so

$$-L\frac{di}{dt} = -L_1\frac{di}{dt} - L_2\frac{di}{dt} - 2M\frac{di}{dt}$$

$$L = L_1 + L_2 + 2M$$

This condition is applicable only when the coils are so placed that the flux linking each coil due to the current in itself is in the same direction as the flux due to the current in the other coil.

If the flux linking each coil due to its own current is opposite in direction to the flux due to the current in other then

$$\boxed{L = L_1 + L_2 - 2M.}$$

• 4.14. RESULTANT INDUCTANCE WHEN TWO INDUCTANCES ARE IN PARALLEL

When L_1 and L_2 are joined parallel then current i is divided between them, then

$$i = i_1 + i_2$$

Differentiating, we get $\dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$.

As the potential difference across each coil is same, the induced e.m.f. must also have the same value e (say). Thus

$$e = -L_1 \frac{di_1}{dt} = -L_2 \frac{di_2}{dt}$$

Let L be the resultant inductance, then

$$e = -L\frac{di}{dt} \quad \text{or} \quad \frac{e}{L} = -\frac{di}{dt}$$

or $$\frac{e}{L} = -\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right) = \frac{e}{L_1} + \frac{e}{L_2}$$

or $$\frac{1}{L} = \frac{1}{L_2} + \frac{1}{L_2} \quad \Rightarrow \quad L = \frac{L_1 L_2}{L_1 + L_2}.$$

This is the required expression.

If the coils are situated near to each other, then mutual induction also develops in between them and then the resultant inductance is given by mutual inductance between the coils then

$$L = \frac{L_1 L_2 + M^2}{L_1 + L_2 + 2M}.$$

• STUDENT ACTIVITY

1. Explain mutual inductance of a pair of coils.

2. Find out resultant inductance when two inductances are connected in a series.

3. Find out resultant inductance when two inductances are connected in parallel.

• 4.15. TRANSFORMER

It is a device which is used to convert low alternating voltage into high alternating voltage and vice-versa.

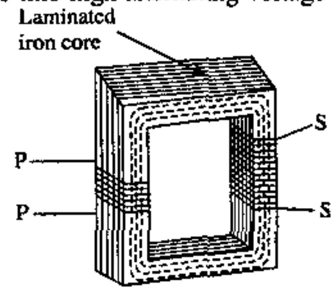

Fig. 10

Principle : It is based upon the principle of mutual induction and works only for a.c. Its principle is "if two coils are inductively coupled and when magnetic flux is changed in one of them, then induced e.m.f. is produced in the other coil".

Construction : It consists of two coils P and S known as *primary and secondary coil respectively,* wound on a soft iron core as shown in the fig. 10. The a.c. input is applied across the primary coil and the transformed output is obtained across the secondary coil.

The transformer are of two types. They are :

Step-up transformer : The transformer which converts low voltage (at high current) into high voltage (at low current) is known as step-up transformer. In this transformer the number of turns in secondary coil is greater than the number of turns in primary coil *i.e.,* $n_s > n_p$.

Step-down transformer : It converts a high voltage (at low current) into low voltage (at high current). In this transformer the number of turns in secondary coil is less than number of turns in the primary coil *i.e.,* $n_s < n_p$.

Now when A.C. is applied to the primary coil, it sets up an alternating magnetic flux in the core which also get linked with secondary. This change in flux linked with the secondary coil induces an alternating e.m.f. in the secondary coil. *In this way power is transferred from one coil to the other coil via the changing magnetic flux in the core.*

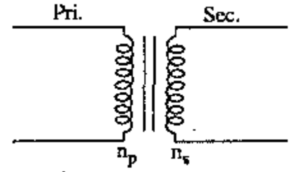

Fig. 11

Theory : When alternating e.m.f. is applied across the primary, a *current flows. This develops a magnetic flux in the core.* Here it is assumed that there are no losses and no leakage of flux. This magnetic flux is linked up with both the primary and the secondary. Hence an induced alternating e.m.f. is produced in primary as well as in the secondary. Now let

n_p = number of turns in primary,

n_s = number of turns in secondary,

ϕ = flux linked with turn of either coil, then by Faraday's law of electromagnetic induction, we have

e.m.f. induced in the primary, $$e_p = -\frac{d(n_p\phi)}{dt} = -n_p\frac{d\phi}{dt}$$

and e.m.f. induced in the primary, $$e_s = -\frac{d(n_s\phi)}{dt} = -n_s\frac{d\phi}{dt}$$

$$\therefore \qquad \frac{e_s}{e_p} = \frac{n_s}{n_p} \qquad ...(1)$$

At no load condition, the induced e.m.f. 'e_p' across the primary is numerically equal to the applied voltage E_p across the primary and induced e.m.f. 'e_s', across the secondary is equal to E_s.

$$\therefore \qquad \frac{E_s}{E_p} = \frac{e_s}{e_p} = \frac{n_s}{n_p} = K$$

where K is known as transformation ratio. Thus

$$K = \frac{\text{voltage obtained across secondary}}{\text{voltage applied across primary}} = \frac{\text{no. of turns in secondary}}{\text{no. of turns in primary}}$$

In this case power output is equal to power input. Suppose I_p *and* I_s be currents in primary and secondary at any instant respectively, then

power in the secondary = power in the primary

$$E_s \times I_s = E_p \times I_p$$

$$\frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{n_s}{n_p} = K. \qquad ...(2)$$

Transformer on load : If the primary circuit has an appreciable resistance R_p, the difference between the applied voltage E_p and the back e.m.f. 'e_p' must be equal to the potential drop $I_p \times R_p$ in the primary coil *i.e.*,

$$E_p - e_p = I_p \times R_p$$

or
$$e_p = E_p - I_p \times R_p \qquad \ldots(3)$$

Again if the secondary circuit is closed having finite resistance R_s, secondary overcomes the potential drop $I_s \times R_s$ and the available potential difference across the secondary is given by

$$E_s = e_s - I_s R_s$$

$$e_s = E_s + I_s R_s \qquad \ldots(4)$$

Hence
$$\frac{e_s}{e_p} = \frac{E_s + I_s R_s}{E_p - I_p R_p} = K$$

or
$$E_s + I_s R_s = K(E_p - I_p R_p)$$

$\because$
$$E_s = KE_p - I_s R_s - KI_p R_p$$

$$= KE_p - I_s R_s - K^2 I_s R_p \qquad [\because\ I_p = KI_s]$$

$$= KE_p - I_s(R_s - K^2 R_p)$$

In this case $\dfrac{E_s}{E_p}$ is not a constant but decreases as more current is drawn from the secondary circuit.

Energy Losses in a Transformer

The following losses occur in a transformer :

(i) Magnetic flux Leakage : Due to some natural leakage of magnetic flux, the entire flux produced by the primary is not linked with the secondary, *i.e.*, a certain amount of electrical energy supplied to the primary is wasted.

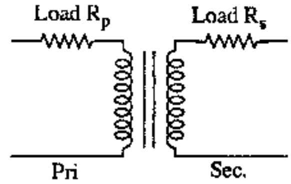

Fig. 12

(ii) Iron losses (eddy currents) : This is due to the eddy currents being produced in the core of the transformer. This is minimised by using a laminated iron core.

(iii) Copper Losses : This is the energy used up in heating effect in the primary coils.

(iv) Hysteresis Losses : This is the energy required to magnetise the core. This loss can be reduced to the minimum by selecting the material of the core which has a thin hysteresis loop.

Uses of Transformer

The following are the main uses of transformers :

(1) The step up and step down transformers are used extensively in A.C. electrical power distribution for domestic and industrial purposes.

(2) An audio frequency (20–16000 cyles/sec.) transformer which is essentially a step up transformer, is used in radio receivers, radio telephony, radio telegraphy and in television.

(3) A radio frequency transformer is used in radio communication at frequency of the order of mega-cycles.

(4) Impedance transformer is used to match the impedance of two independent circuits.

(5) Constant voltage transformer is designed to give a constant output voltage even when input voltage varies considerably. Such transformers known as stabilizers are now commonly used with television, referigerator etc.

(6) Constant current transformer is designed to give a constant output current even when input current varies considerably.

(7) Welding and Furnace transformers are used in welding as well as in induction furnaces.

Use of transformers in long distance power transmission : Suppose we want to transmit a given power $(VI) = 44{,}000$ watts from a generating station to some city at a distance of hundreds of miles. It can be transmitted

(i) at a voltage 400 volts and a current of 110 amperes or

(ii) at a voltage of 11,000 volts and a current of 4 amperes.

Following are the losses which occur in the transmission :

(a) When the current is flowing through the line wires, the energy (I^2Rt) will be lost as heat. This would be greater than the first case because current (110 amps) is higher than the current (4 amperes) in the second case.

(b) The voltage drop along the line wire is equal to (RI). Again this loss is greater in the first case.

• 4.16. CALCULATE THE INDUCTANCE OF LENGTH l, OF COAXIAL CYLINDER

Let radius of outer conductor be b and of inner conductor be a. Consider a coaxial shell of radius r and width dr as shown in Fig. (13). Applying Ampere's law for this shell

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i \qquad \ldots(1)$$

or $$B(2\pi r) = \mu_0 i$$

or $$B = \mu_0 i / 2\pi r$$

where i is the current in the inner conductor.

Therefore energy density for points between the conductors is

$$u = \frac{1}{2\mu_0} B^2$$

$$= \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi r}\right)^2 = \frac{\mu_0 i^2}{8\pi^2 r^2}.$$

The energy dU contained in shell of radius r and width dr is

$$dU = \text{energy density} \times \text{volume}$$

$$dU = u_B (2\pi r l)(dr)$$

$$= \frac{\mu_0 i^2}{8\pi^2 r^2}(2\pi r l dr)$$

$$= \frac{\mu_0 i^2 l}{4\pi} \frac{dr}{r}.$$

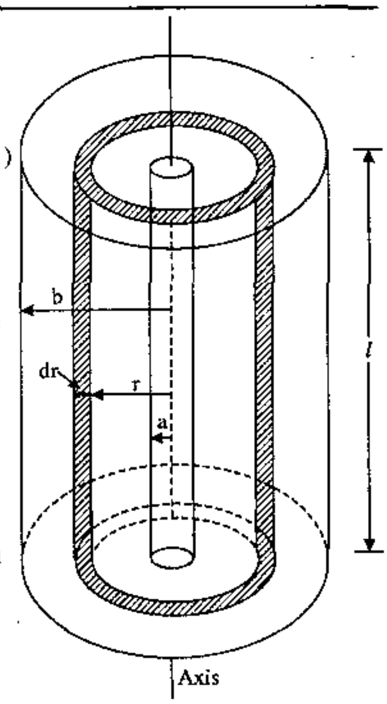

Fig. 13

Total stored magnetic energy between coaxial cylinders will be

$$U = \int dU = \frac{\mu_0 i^2 l}{4\pi} \int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 i^2 l}{4\pi} \log_e\left(\frac{b}{a}\right),$$

which is the desired expression.

(b) The inductance, L, can be found using the expression

$$U = \frac{1}{2} L i^2$$

or $$L = \frac{2U}{i^2}$$

$$L = \frac{2}{i^2} \cdot \frac{\mu_0 i^2 l}{4\pi} \log_e\left(\frac{b}{a}\right)$$

$$= \frac{\mu_0 l}{2\pi} \log_e\left(\frac{b}{a}\right),$$

which is the inductance of a coaxial cable of length l.

• SUMMARY

- Faraday's first law : According to this law, "whenever the amount of magnetic flux linked with a circuit changes then an e.m.f. is induced in the circuit.
- Faraday's second law : According to this law, whenever an e.m.f. is setup by a change of magnetic flux through a circuit, it's direction is always such as to oppose the change that causes it.

- The differential form of Faraday's law,
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
- Units of self-inductance are Henry which is equal to 1 volt/1 ampere-second.
- Transformer is a device which is used to convert low alternating voltage into high alternating voltage.

• TEST YOURSELF

1. State and explain Faraday's laws of electromagnetic induction.
2. Deduce Faraday's law $e = -\frac{d\phi_B}{dt}$.
3. How would you use a ballistic galvanometer and a search coil to measure the magnetic field due to a horse shoe magnet or electromagnet ?
4. What is displacement current ? Show that it is identical to conduction current across a charged capacitor gap.
5. Describe Rayleigh method to determine the self inductance of a coil.
6. Calculate the total magnetic energy stored in an inductor carrying a current i_0.
7. Show that the energy stored per unit volume of a magnetic field B is given by $u = \frac{B^2}{2\mu_0}$ where μ_0 is the permeability of free space.
8. Find an expression for the mutual inductance between two co-axial coils. Describe the method for determining the mutual inductance.
9. Describe the principle, construction and working of a transformer. What are the energy losses in it and how we reduce them ?
10. A train is moving towards south with uniform speed of 10 meters/sec. If the vertical component of earth's magnetic induction is 5.4×10^{-5} web/met^2, calculate the e.m.f. induced in an axis 1.2 metre long.
11. A railway line 0.2 m, wide runs along the magnetic meridian. The vertical component of the earth's magnetic field is 0.3 oersted. Calculate the e.m.f. in volts that exist between the rails when a train runs on the line at a speed of 60 km/hr.
12. Calculate the e.m.f. induced between the ends of an axis 1.75 metre long of a railway carriage travelling on level ground with a uniform speed of 50 km/hr. The vertical component of earth's magnetic field is 5.0×10^{-5} web./met^2.
13. Calculate the self inductance of a coil of 100 turns of a current of 1 amp. produces a magnetic flux of 5×10^{-5} weber through the coil.
14. A ballistic galvanometer of 2.2×10^{-9} amp./cm. current sensitivity and 10 sec. periodic time is connected in a circuit changes by 1000 e.m.u. Calculate the deflection obtained.
15. A solenoid 8 cm. in length, 6.0 cm^2 is cross-sectional area has 96 turns and carries a current of 0.25 amp. A secondary coil of 2 turns is wound around the centre of the solenoid when the magnetic field of the solenoid is switched off within 0.05 sec., what is the e.m.f. induced in that secondary ?
16. A field of 200 oersteds acts at right angles to a coil of 50 turns and of area 100 sq. cm. The coil is removed from the field in 1.0 sec. Calculate the average e.m.f. produced in the circuit. [1 oersted = $(10^3/4\pi)$ amp. turn/meter]
17. A solenoid of length 200 cm, and radius of cross-section 1.5 cm. has five layers of winding 850 turns each, if the solenoid carries a current of 5.0 amperes, calculate the magnetic flux for a cross-section of the solenoid at the centre of the solenoid.
18. Calculate the coefficient of self-induction of a 1 meter solenoid of 500 turns and 5 cm diameter.
19. A 50 cm long solenoid, having 500 turns and radius 2 cm is wound on an iron core of relative permeability 800. What will be the average e.m.f. induced in the solenoid if the current in it changes from 0 to 2 amp. in 0.05 second ?
20. Long solenoid of length 1 meter, radius 2.5 cm having number of turns 500. Calculate the coefficient of self induction of the solenoid.
21. A solenoid of length 16 cm has 1280 turns and its area of cross-section is 10 cm^2. A coil of 1000 turns is wound closely on the middle part of the solenoid. Calculate the mutual inductance between the solenoid and the coil.
22. An induced emf is produced when a magnet is plunged into a coil. The magnitude of induced e.m.f. does not depend upon :

(a) the number of turns in the coil.
(b) the speed with which the magnet is moved
(c) the strength of magnet
(d) the resistivity of the wire of the coil

23. A cylindrical magnet is kept along the axis of a circular coil. If the magnet is rotated about its axis, then :
(a) a current is induced in the coil
(b) no current is induced in the coil
(c) both an emf and a current are induced in the coil
(d) only an emf is induced in the coil

24. The relation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{dt}$ represents :
(a) Ampere's law
(b) Gauss's law
(c) Ohm's law
(d) Faraday's law

25. For a constant inductance L henry, the energy stored in a magnetic field is given by :
(a) $\frac{1}{2}mv^2$ (b) $\frac{1}{2}Lv^2$ (c) $\frac{1}{2}LI^2$ (d) $\frac{1}{2}LV^2$

26. The laws of electromagnetic induction have been used in the construction of :
(a) electric motor
(b) generator
(c) voltmeter
(d) galvanometer

27. In an induction coil, the coefficient of mutual inductance is 4 henry. If the current of 5 A in the primary coil is cut off in 1/1500 sec. the emf at the terminals of the secondary coil will be:
(a) 15 KV (b) 60 KV (c) 10 KV (d) 30 KV

28. The self inductance of a coil is a measure of :
(a) electrical inertia
(b) electrical friction
(c) induced emf
(d) induced current.

29. Two inductance coils, of same self inductance are connected in parallel and the distance between them is large. The resultant self inductance of the coil will be :
(a) $\frac{L}{4}$ (b) $2L$ (c) L (d) $\frac{L}{2}$

30. If the number of turns in a coil is N then the value of self inductance of the coil will become:
(a) N times
(b) N^2 times
(c) N^{-2} times
(d) N^0 times

31. The value of mutual inductance can be increased by :
(a) decreasing N
(b) increasing N
(c) winding the coil on wooden frame
(d) winding the coil on china clay

32. The coefficients of self-induction of two coils are L_1 and L_2. To induce an e.m.f. of 25 volt in the coils change of current of 1 A has to be produced in 5 sec and 50 ms respectively. The ratio of their self-inductances $L_1 : L_2$ will be :
(a) 1 : 5
(b) 200 : 1
(c) 100 : 1
(d) 50 : 1

33. Two coils are made of copper wires of same length. In the first coil the number of turns is $3n$ and radius is r. In the second coil number of turns is n and radius is $3r$. The ratio of self inductances of the coils will be :

(a) 9 : 1
(b) 3 : 1
(c) 1 : 3
(d) 1 : 9

34. A train A is running in north-south direction and another train B is running in east-west direction, the e.m.f. in the axle will be :
(a) induced in A but not in B (b) induced in B but not in A
(c) will not be induced in both (d) induced in both

35. Two circular conducting loops of radii R_1 and R_2 are lying concentrically in the same plane. If $R_1 > R_2$ then the mutual inductance (M) between them will be proportional to :
(a) $\frac{R_1}{R_2}$
(b) $\frac{R_2}{R_1}$
(c) $\frac{R_1^2}{R_2}$
(d) $\frac{R_2^2}{R_1}$

36. The coefficients of self-induction of two coils are $L_1 = 8$ mH and $L_2 = 2$ mH respectively. The current rises in the two coils at the same rate. The power given to the two coils at any instant is same. The ratio of currents flowing in the coils will be :
(a) $\frac{i_1}{i_2} = \frac{1}{4}$
(b) $\frac{i_1}{i_2} = \frac{4}{1}$
(c) $\frac{i_1}{i_2} = \frac{3}{4}$
(d) $\frac{i_1}{i_2} = \frac{4}{3}$

37. In the above problem, the ratio of induced e.m.f.'s is in the coils will be :
(a) $\frac{V_1}{V_2} = 4$
(b) $\frac{V_1}{V_2} = \frac{1}{4}$
(c) $\frac{V_1}{V_2} = \frac{1}{2}$
(d) $\frac{V_1}{V_2} = \frac{2}{1}$

38. The number of turns in a coil of wire of fixed radius is 600 and its self inductance is 108 mH. The self-inductance of a coil of 500 turns will be :
(a) 74 mH
(b) 75 mH
(c) 76 mH
(d) 77 mH

39. Two coils X and Y are lying in a circuit. The change in current X is 2 ampere and change in magnetic flux in Y is 0.4 weber. The coefficient of mutual induction between the coils will be:
(a) 0.2 Henry
(b) 5 Henry
(c) 0.8 Henry
(d) 0.4 Henry

40. When a coil is wound on a core of relative permeability μ then its self inductance becomes :
(a) L
(b) $\frac{L}{\mu}$
(c) μL (d) μL^2

41. $\text{curl } B = \mu_0 \left(J + \varepsilon_0 \frac{\partial E}{\partial t} \right)$

   is known as Gauss law of electrostatics in differential form.
42. The magnetic flux coming out of a surface is equal to number of lines of force passing normally to that surface.
43. Lenz law explains the concept of displacement current.
44. Induced emf always act in such a direction such as to oppose the cause of change of magnetic flux.
45. Faraday's first law is also known as Lenz law.
46. Induced e.m.f. is generated in the rectangular coil moving in a non-uniform magnetic field.
47. The inductance of coil (L) depends on the permeability of the core material inside the coil.
48. When two coils are connected in parallel, then

$$L = \frac{L_1 L_2}{L_1 + L_2}.$$

49. The unit of M is weber/amp.
50. The coefficient of mutual induction between two closely lying coils does not depend upon the current flowing in them.

ANSWERS

10. 6.48×10^{-4} volt, **11.** 1309 volts **12.** 1.575×10^{-2} volt, **13.** 1 mH **14.** 5.7 cm
15. -9.0×10^{-6} volt. **16.** -0.1 volt **17.** 1.885×10^{-5} weber **18.** 0.6 mH **19.** -25.2 volt
20. 0.6 mH **21.** 10.05×10^{-3}
22. (d) **23.** (b) **24.** (d) **25.** (c) **26.** (b) **27.** (d) **28.** (a) **29.** (d) **30.** (b) **31.** (b)
32. (c) **33.** (b) **34.** (d) **35.** (d) **36.** (a) **37.** (a) **38.** (b) **39.** (a) **40.** (c)
41. False **42.** True **43.** False **44.** True **45.** False **46.** True **47.** True **48.** True
49. True **50.** True.

UNIT 5

ELECTRO-MAGNETIC WAVE

STRUCTURE

- Electromagnetic Waves
- Equation for Plane Electromagnetic Waves
- Poynting Theorem
- Poynting Vector
  - Student Activity
- Boundary Conditions
- Reflection and Refraction of E.M.W.
- Reflection and Refraction of Plane Electromagnetic Wave at a Plane Boundary of Two Dielectrics.
- Reflection Coefficient of a Plane Electromagnetic Wave Reflected at a Plane Boundary
- Polarisation by Reflection and Brewster's Law
- Total Internal Reflection at the Boundary of Two non-magnetic Media
- Coulomb's Law
  - Summary
  - Student Activity
  - Test Yourself

LEARNING OBJECTIVES

After going this unit you will learn :

- Electromagnetic waves with their properties.
- Gauss's laws for electrostatics and magnetic field, Maxwell-Ampere's law.
- Poynting vector with properties and applications.
- Boundary conditions for electromagnetic waves, polarisation by reflection and Brewster's law.

• 5.1. ELECTRO-MAGNETIC WAVES

The idea of Electro-magnetic waves was given by Maxwell. According to Maxwell, an electric field changing with time at a point produces a magnetic field there. This means that a change in either field *i.e.*, electric or magnetic field, with time produces the other field. This idea led Maxwell to conclude that the variation in electric and magnetic field vectors perpendicular to each other leads to the production of electromagnetic disturbances in space. These disturbances have the properties of wave and can travel in space even without any material medium. These waves are called **electromagnetic waves**. These waves may also be defined as "The waves which consists time varying electric and magnetic fields acting at right angles to each other as well as at right angles to the direction of propagation of waves are known as "electromagnetic waves".

Production of electromagnetic waves : Maxwell predicated the existence of electromagnetic waves on the basis of the following four basic laws.

1. Gauss's Law in Electrostatic

$$\int \vec{E}.d\vec{S}=\frac{q}{\varepsilon_0}$$

2. Gauss's Law in magnetism :

$$\int \vec{B}.d\vec{S}=0$$

3. Faraday's Law :

$$\int \vec{E}.\vec{dl} = -\frac{d\phi_B}{dt}$$

4. Maxwell-Ampere's Circuital Law :

$$\int \vec{B}.\vec{dl} = \mu_0\left(i + \varepsilon_0 . \frac{d\phi_E}{dt}\right)$$

We know that a moving charge produces both the electric and magnetic fields. If the charge is moving with a constant velocity *i.e.*, if current is not changing with time then the electric and magnetic fields will not change with time and hence no electromagnetic waves can be produced. But if the charge is moving with a non-zero acceleration *i.e.*, charge is accelerated then both the magnetic field and electric field will change with space and time, in this position it produces electromagnetic waves. *Hence we can say that an accelerated charge is responsible for the production of electromagnetic waves.*

For example the L–C circuit is an oscillatory circuit in which the charge is oscillating across the capacitor plates. In L–C circuit the oscillatory charge has a non-zero acceleration due to this it emits electromagnetic waves which have the same frequency as that of the oscillating charge.

In an atom, an electron revolves around the nucleus in a stable orbit. Since electron accelerating, but it does not emit electromagnetic waves. Electromagnetic waves are emitted only when it falls from higher energy orbit to lower energy orbit.

Electromagnetic waves are also produced when fast moving electrons are suddenly stopped by the metal target of high atomic number.

Properties of electromagnetic waves : Some important properties of electromagnetic waves are as follows :

1. The electromagnetic waves are produced by accelerated or oscillating charge.
2. These waves do not require any material medium for propagation.
3. These waves propagate in free space with the speed of light and given by

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

4. The variation in both electric and magnetic field vectors ($\vec{E}$ and $\vec{B}$) occurs simultaneously. Due to this they attain the maxima and minima at the same place and at the same time.

5. The directions of electric and magnetic field vectors are perpendicular to each other and also perpendicular to the direction of propagation of waves. Therefore, the electromagnetic waves are transverse in nature like light waves.

6. The velocity of electromagnetic waves depends on the electric and magnetic properties of the medium in which these waves propagate and is independent of the amplitude of the field vectors.

7. The velocity of electromagnetic wave in dielectric is less than velocity of light.

8. The energy in electromagnetic waves is divided equally between electric field and magnetic field vectors.

9. The electric vector is responsible for the optical effects on an electromagnetic waves and is called the **light vector.**

10. The electromagnetic waves do not affected by electric and magnetic fields.

• 5.2. EQUATION FOR PLANE ELECTROMAGNETIC WAVES

The four basic laws of electricity and magnetism are :

(1) Gauss's law in electrostatics :

$$\int \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

or

$$\text{Div } \vec{E} = \vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

It may be written as

$$\frac{\partial E_x}{\partial x} + \frac{\partial Ey}{\partial y} + \frac{\partial Ez}{\partial z} = \frac{\rho}{\varepsilon_0} \qquad \ldots (1)$$

(2) Gauss's law for magnetic field :

$$\int \vec{B}.\vec{dS} = 0$$

or

$$\text{Div } \vec{B} = \vec{\nabla}.\vec{B} = 0$$

It may be written as

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0 \qquad \dots (2)$$

(3) **Faraday's law in electricity :**

$$\int \vec{E}\cdot d\vec{l}=-\frac{d\phi_B}{dt}$$

or

$$\text{curl } \vec{E}=\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

It may be written as

$$\left.\begin{aligned}\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}&=-\frac{\partial B_x}{\partial t}\\ \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}&=-\frac{\partial B_y}{\partial t}\\ \frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}&=-\frac{\partial B_z}{\partial t}\end{aligned}\right\} \qquad \dots (3)$$

(4) **Maxwell-Ampere's circuital law :**

$$\int \vec{B}\cdot d\vec{l}=\mu_0\left(J+\varepsilon_0\cdot\frac{\partial\phi_E}{dt}\right)$$

or

$$\text{curl } \vec{B}=\mu_0\left(J+\varepsilon_0\cdot\frac{\partial E}{\partial t}\right)$$

It may be written as

$$\left.\begin{aligned}\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}&=\mu_0\left(J_x+\varepsilon_0\cdot\frac{\partial E_x}{\partial t}\right)\\ \frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}&=\mu_0\left(J_y+\varepsilon_0\cdot\frac{\partial E_y}{\partial t}\right)\\ \frac{\partial B_y}{\partial x}-\frac{\partial B_z}{\partial y}&=\mu_0\left(J_z+\varepsilon_0\cdot\frac{\partial E_z}{\partial t}\right)\end{aligned}\right\} \qquad \dots (4)$$

From above equations we can relate $\vec{E}$ and $\vec{B}$ at any point in space to charge density ρ and current density j respectively at that point.

For vacuum we have

$$\rho=0 \quad \text{and} \quad \mathbf{j}=0$$

then above equation reduces to

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=0 \qquad \dots (5)$$

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0 \qquad \dots (6)$$

$$\left.\begin{aligned}\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}&=-\frac{\partial B_x}{\partial t}\\ \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}&=-\frac{\partial B_y}{\partial t}\\ \frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}&=-\frac{\partial B_z}{\partial t}\end{aligned}\right\} \qquad \dots (7)$$

$$\left.\begin{aligned}\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}&=\mu_0\varepsilon_0\frac{\partial E_x}{\partial t}\\ \frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}&=\mu_0\varepsilon_0\frac{\partial E_y}{\partial t}\\ \frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}&=\mu_0\varepsilon_0\frac{\partial E_z}{\partial t}\end{aligned}\right\} \qquad \dots (8)$$

Let us consider the case of plane waves travelling in the x-direction in this position the partial derivatives with respect to y and z must be zero, so above equation becomes :

$$\frac{\partial E_x}{\partial x}=0 \qquad \dots (9)$$

$$\frac{\partial B_x}{\partial x} = 0 \qquad \dots (10)$$

$$\left.\begin{aligned} 0 &= -\frac{\partial B_x}{\partial t} && \dots (a) \\ -\frac{\partial E_z}{\partial x} &= -\frac{\partial B_y}{\partial t} && \dots (b) \\ \frac{\partial E_y}{\partial x} &= -\frac{\partial B_z}{\partial t} && \dots (c) \end{aligned}\right\} \qquad \dots (11)$$

$$\left.\begin{aligned} 0 &= \mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t} && \dots (a) \\ -\frac{\partial B_z}{\partial x} &= \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} && \dots (b) \\ \frac{\partial B_y}{\partial x} &= \mu_0 \varepsilon_0 \frac{\partial E_z}{\partial t} && \dots (c) \end{aligned}\right\} \qquad \dots (12)$$

Equations (9) and (12a) show that the longitudinal components E_x is constant in both space and time. Similarly from equations (10) and (11a), B_x is also constant. For the wave we may take

$$E_x = 0 \quad \text{and} \quad B_x = 0$$

Hence $\vec{E}$ and $\vec{B}$ are both perpendicular to the direction of propagation and hence the wave is transverse.

For simplicity let the wave in which the electric field is along only the y-direction and magnetic field along only the z-direction so we put

$$E_z = B_y = 0$$

so above equations becomes

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad \dots (13)$$

and
$$-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{dt} \qquad \dots (14)$$

differentiating eqn. (13) w.r.t. x and eq. (14) w.r.t. t, we get

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 B_z}{\partial x\, \partial t}$$

and
$$-\frac{\partial^2 B_z}{\partial t\, \partial x} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

eliminating the derivatives of Bz we get

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \qquad \dots (15)$$

In a similar manner we have

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2} \qquad \dots (16)$$

Equations (15) and (16) are similar to the differential equation for plane waves.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 y}{\partial t^2} \qquad \dots (17)$$

Comparing eq. (15) or (16) with eq. (17), we get

$$v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$v = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\because \quad \mu_0 = 4\pi \times 10^{-7}\ \text{N/amp}^2$

$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{Coulomb}^2/\text{Newton-meter}^2$

$$\therefore \quad v = \frac{1}{\sqrt{(4\pi \times 10^{-7}\ \text{NA}^{-2})\,(8.85 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2})}}$$

$$\boxed{v = 3.0 \times 10^8 \text{ mS}^{-1}}$$

Thus, the equations (13) and (14) which are transverse waves travelling together along the x-direction with the speed of light.

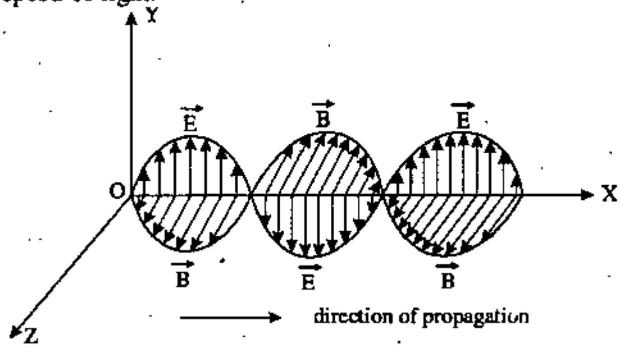

Fig. 1

This led Maxwell to propose that light is an electromagnetic wave. The other examples of electromagnetic waves are radio waves, microwaves etc. therefore we can say that the electric vector is the light vector.

• 5.3. POYNTING THEOREM

Work done against the Coloumb repulsion of like charge is stored in the form of electrostatic energy.

$$U_E = \frac{1}{2}\varepsilon_0 \int E^2 \, d\tau \qquad \ldots (1)$$

where E = resulting electric field.

Similarly work against the back e.m.f. is stored in form of magnetostatic energy

$$U_B = \frac{1}{2\mu_0} \int B^2 \, d\tau \qquad \ldots (2)$$

Therefore total energy stored in electromagnetic field is

$$U_{EB} = \frac{1}{2} \int \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d\tau \qquad \ldots (3)$$

Charge dq feels Lorentz force in Electromagnetic field whose value can be written as

$$F = dq\,(E + v \times B)$$

Therefore work done on charge dq is

$$dW = F \,.\, dl = dq\,(E + \vec{v} \times \vec{B}) \,.\, vdt$$

$$= E\, v\, dr\, dt \qquad dq = \rho d\tau, \text{ and } J = \rho v$$

$$\Rightarrow \qquad \frac{dW}{d}t = \int_v (\vec{E} \,.\, \vec{J})\, d\tau \qquad \begin{array}{l} J = \text{current density} \\ \rho = \text{charge density} \\ v = \text{velocity of charge } dq \end{array} \qquad \ldots (4)$$

From Maxwell electromagnetic wave.

$$\text{Curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + \frac{\partial}{\partial t}(\varepsilon_0 \vec{E})$$

$$\text{Curl } H = J + \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\vec{E} \,.\, \vec{J} = \frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) - \varepsilon_0 \vec{E} \,.\, \frac{\partial \vec{E}}{\partial t}$$

We have

$$\text{div }(\vec{E} \times \vec{B}) = \vec{B} \,.\, \text{curl } \vec{E} - \vec{E} \,.\, \text{curl } \vec{B}$$

$$= \vec{B} \,.\, (\vec{\nabla} \times \vec{E}) - \vec{E}(\vec{\nabla} \times \vec{B})$$

From Faraday law $\qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$E(\vec{\nabla}\times\vec{B}) = \vec{B}\cdot\frac{\partial\vec{B}}{dt} - \vec{\nabla}(\vec{E}\times\vec{B})$$

by using $\frac{1}{2}\cdot\frac{\partial}{\partial t}(B^2) = \vec{B}\cdot\frac{\partial\vec{B}}{\partial t}$ and $\frac{1}{2}\cdot\frac{\partial}{\partial t}(E^2) = \vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$

then $$\vec{E}\cdot\vec{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\nabla(\vec{E}\times\vec{B}) \quad \ldots (5)$$

Now we have

$$\frac{dW}{dt} = \int (\vec{E}\cdot\vec{J})\,d\tau \text{ from equation (5), we get}$$

$$= -\frac{d}{dt}\int_V \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)d\tau - \frac{1}{\mu_0}\int \nabla\cdot(\vec{E}\times\vec{B})\,d\tau$$

$$\Rightarrow \quad \frac{dW}{dt} = -\frac{d}{dt}U_{EB} - \frac{1}{\mu_0}\int (\vec{E}\times\vec{b})\cdot dS$$

$$\frac{dW}{dt} = -\frac{d}{dt}U_{EB} - \oint_S \vec{S}\cdot d\vec{S} \quad \ldots (6)$$

dS surface area

Here, $$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) \quad \ldots (7)$$

$\vec{S}$ = energy per unit time per unit area is known as Poynting vector.

Statement of Poynting theorem is interpreted as *total amount of work done as the charges by the e.m. force is equal to decrease in energy stored in field and loss the energy which emerge out through the surface.*

Equation (6) is simply the statement of Law of conservation of energy.

• 5.4. POYNTING VECTOR

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) \qquad B = \mu_0 H$$

$$\Rightarrow \quad \vec{S} = (\vec{E}\times\vec{H})$$

Poynting vector itself is an vector quantity whose direction is perpendicular to direction of magnetic field induction vector B and also perpendicular to direction of electric field vector $\vec{E}$.

It has same direction as propagation vector k has.

It can be defined as amount of field energy passing through unit area per unit time in a direction perpendicular to plane contains E and H.

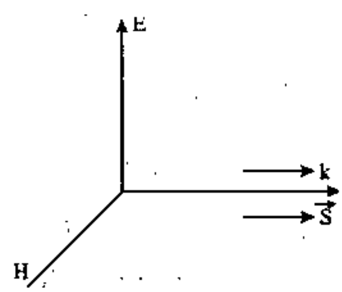

Fig. 2

$$\vec{S} = \vec{E}\times\vec{H}$$
$$= EH\sin 90^\circ$$
$$S = EH \text{ (in magnitude form)}$$

Dimension of Poynting Vector :

$$= \frac{\text{Energy}}{\text{Area}\times\text{time}} = \left(\frac{\text{Joule}}{\text{m}^2\times\text{sec.}}\right)$$
$$= \left(\frac{\text{Joule}}{\text{sec.}}\right)\times\left(\frac{1}{\text{m}^2}\right)$$
$$= \left(\frac{\text{Watt}}{\text{m}^2}\right)$$
$$= \frac{ML^2T^{-2}}{T\times L^2}$$
$$= MT^{-3}$$

Properties of Poynting Vector

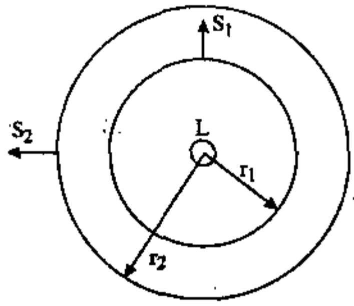

Fig. 3

(i) Poynting vector varies inversely square to distance from the point source of radiation $S \propto \left(\frac{1}{r^2}\right)$. Suppose a source L emits radiation and point of source is P watts. If S_1 and S_2 are two Poynting vector at any point as X and Y.

Then $S_1 \times 4\pi r_1^2 = S_2 \times 4\pi r_2^2 = P$ where X and Y are two concentric spherical surface of r_1 and r_2

$$S_1 = \frac{P}{4\pi r_1^2} = \frac{P}{4\pi r_2^2} = S_2$$

$$\boxed{S \propto \frac{1}{r^2}}$$

(ii) If Poynting vector is zero across a closed surface, then electromagnetic field energy does not flow across the closed surface. Poynting vector may or may not be zero if no net field energy is flowing across a closed surface.

(iii) Average value of Poynting vector over one complete time period *i.e.*,

$$\langle S \rangle = \frac{1}{T}\int_0^{2\pi/\omega} (E \times H)\, dt$$

$$\langle S \rangle = \frac{1}{T}(E_0 \times H_0)\int_0^{2\pi/\omega} \sin^2 \omega t = \frac{1}{T}(E_0 \times H_0) \times \frac{T}{2}$$

$$E = E_0 \sin \omega t \quad \text{and} \quad H = H_0 \cos \omega t$$

$$\boxed{\langle S \rangle = \frac{1}{2}(E_0 \times H_0) = E_{av} \times H_{av.}}$$

Applications of Poynting Vector

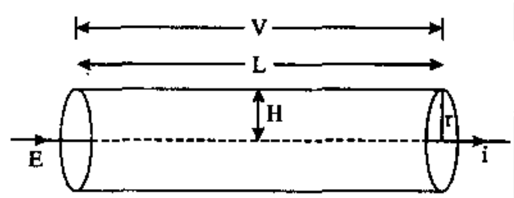

Fig. 4

Poynting vector is applied and its loss is valid when energy flow takes place outside or inside in the volume. Suppose a cylinder resistor of length L, radius r and resistance R. Let a potential difference V is applied.

Energy i^2R is dissipated through the resistor, so it can be proved that

$$\int \vec{S}.\,d\vec{S} = i^2R \qquad \text{... (1)}$$

where S is Poynting vector, dS is the elementary surface area.

$$\because \qquad E = \frac{V}{L} = \left(\frac{iR}{L}\right) \qquad \text{... (2)}$$

We know Ampere law

$$\int H \,.\, dl = i$$

$$H(2\pi r \cos\theta) = i$$

$$H = \frac{i}{2\pi R} \qquad \text{... (3)}$$

$$[\because \ \theta = 0]$$

Now from equation (1), we have

$$\int_{\text{full surface}} S \,.\, dS = \int EH \,.\, dS = \frac{iR}{L} \times \frac{i}{2\pi r} \,.\, 2\pi r^2$$

$$= i^2R.$$

• STUDENT ACTIVITY

1. Write down the basic laws of electricity and magnetism.

2. Find out the dimensional formula of poynting vector.

• 5.5. BOUNDARY CONDITIONS

When an electromagnetic wave passes from one medium to another through a surface then its property changes.

This surface is known as boundary. When electromagnetic wave passes through one medium to another E, H, B and D change, according to following facts :

(i) The normal component of the electric displacement is discontinuous by an amount equal to the free surface density of charge at the boundary *i.e.*,

$$D_{1n} - D_{2n} = \sigma$$

(ii) The normal components of the magnetic induction is continuous across a surface of discontinuty *i.e.*,

$$B_{1n} - B_{2n} = 0$$

(iii) The tangential component of magnetic intensity is discontinuous by an amount equal to the free surface current density J *i.e.*,

$$H_{1t} - H_{2t} = J_S$$

(iv) The tangential component of E is continuous across a surface of discontinuity *i.e.*,

$$\boxed{E_{1t} - E_{2t} = 0}$$

• 5.6. REFLECTION AND REFRACTION OF E.M.W.

We know that when plane electromagnetic waves which are travelling in one medium are incident upon plane surface separating this medium from another have different electromagnetic properties.

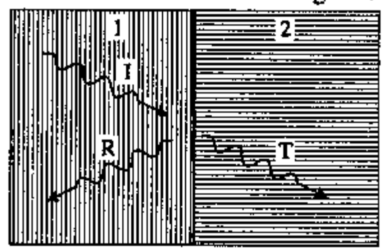

Fig. 5

When an electromagnetic wave is propagated through space then there is an exact balance between the electric and magnetic field *i.e.*, half is electric field energy and other half is magnetic field energy. But if the wave propagates in some different medium then there must be a new distribution of energy due to change in field vectors.

Since no energy can be added to the wave when it passes through the boundary surface so a new balance can be achieved is for some of the incident energy to be reflected. Thus transmitted energy constitutes of the reflected wave and refracted wave.

In this position there are two types of properties of reflection and refraction. They are :

(A) Kinematic properties : The kinematic properties of reflection and refraction are as follows :

(i) **Law of frequency** : The frequency of the wave remains same by reflection or refraction.

(ii) The reflected and refracted waves are in the same plane as the incident wave and the normal to boundary surface.

(iii) **Law of reflection** : In case of reflection the angle of reflection is equal to the angle of incident *i.e.*,

$$\theta_i = \theta_R$$

(iv) **Snell's law** : Snell's law is as follows :

$$n_1 \sin \theta_i = n_2 \sin \theta_T$$

(B) Dynamic Properties : These properties are connected with the :

(i) Intensitises of reflected and refracted waves.

(ii) Phase changes and polarisation of waves.

• 5.7. REFLECTION AND REFRACTION OF PLANE ELECTROMAGNETIC WAVE AT A PLANE BOUNDARY OF TWO DIELECTRICS

Let the medium have permittivity and permeability ε_1 and μ_1 respectively while above it ε_2 and μ_2 $(y = 0)$.

Let an electromagnetic wave travelling in a medium 1 is incident on the plane boundary at angle θ_i. A part of it is reflected and another is refracted. Let θ_R be the angle of reflection and θ_T is the angle of refraction or transmission.

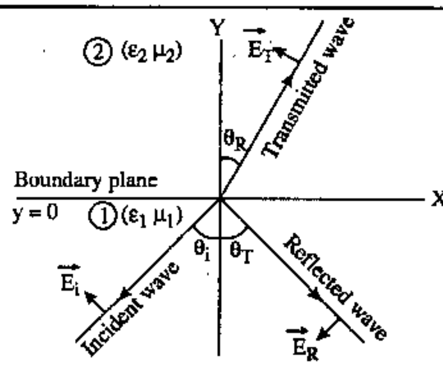

Fig. 6

Let $\vec{E}_i$, $\vec{E}_R$ and $\vec{E}_T$ are the incident, reflected and transmited electric vectors and ω_i, ω_R and ω_T are the frequencies of incident, reflected and transmitted wave respectively. In this position the boundary condition is

$$(E_i)_t + (E_R)_t = (E_T)_t \quad \ldots (1)$$

We have

$$\vec{E}_i = \vec{E}_i' \, e^{-i(\omega_i t - \vec{k}_i \cdot \vec{r})}$$

$$\vec{E}_R = \vec{E}_R' \, e^{-i(\omega_R t - \vec{k}_R \cdot \vec{r})}$$

and

$$\vec{E}_T = \vec{E}_T' \, e^{-i(\omega_T t - \vec{k}_T \cdot \vec{r})}$$

Put these values in eq. (1)

$$E_i' \, e^{-i(\omega_i t - \vec{k}_i \cdot \vec{r})} + E_R' \, e^{-i(\omega_R t - \vec{k}_R \cdot \vec{r})} = E_T' \, e^{-(\omega_T t - \vec{k}_T \cdot \vec{r})} \quad \ldots (2)$$

Equation (2) can only be satisfied if the time and space varying components of the phases are equal *i.e.,*

$$\omega_i t = \omega_R t = \omega_T t$$

$$\therefore \qquad \omega_i = \omega_R = \omega_T = \omega$$

This means that the frequency of the wave remains same after reflection and refraction

and $$\vec{k_i} \cdot \vec{r} = \vec{k_R} \cdot \vec{r} = \vec{k_T} \cdot \vec{r}$$

$$\therefore \qquad k_i r \sin \theta_i = k_R r \sin \theta_R = k_T r \sin \theta_T$$

$$k_i \sin \theta_i = k_R \sin \theta_R = k_T \sin \theta_T \qquad \dots (3)$$

$k_i = k_R$ since they are in the same medium

$$\therefore \qquad \boxed{\theta_i = \theta_R}$$

that is the angle of incidence is equal to the angle of reflection. This is the law of reflection.

Again from equation (3)

$$k_1 \sin \theta_i = k_T \sin \theta_T$$

$$\frac{\sin \theta_i}{\sin \theta_T} = \frac{k_T}{k_i}$$

Since $$v = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon \mu}} \Rightarrow k = \omega \sqrt{\varepsilon \mu}$$

$$\frac{\sin \theta_i}{\sin \theta_T} = \sqrt{\frac{\varepsilon_2 \mu_2}{\varepsilon_1 \mu_1}}$$

If $$v_1 = \frac{1}{\sqrt{\varepsilon_1 \mu_1}} \quad \text{and} \quad v_2 = \frac{1}{\sqrt{\varepsilon_2 \mu_2}}$$

$$\boxed{\frac{\sin \theta_i}{\sin \theta_T} = \frac{v_1}{v_2}} \qquad \dots (4)$$

Let n_1 and n_2 be the refractive indices of media 1 and 2 respectively then

$$\frac{n_2}{n_1} = \frac{C/v_2}{C/v_1} = \frac{v_1}{v_2}$$

Put in eq. (4)

$$\frac{\sin \theta_i}{\sin \theta_T} = \frac{n_2}{n_1}$$

$$\boxed{n_2 = \frac{\sin \theta_i}{\sin \theta_T}}$$

This is Snell's law of refraction.

• 5.8. REFLECTION COEFFICIENT OF A PLANE ELECTROMAGNETIC WAVE REFLECTED AT A PLANE BOUNDARY OF TWO DIELECTRIC MEDIA

Let us consider a plane electromagnetic wave which incident at an angle θ_i on the plane boundary *i.e.,* at $y = 0$. Let θ_R and θ_T are the angles of reflection and refraction respectively. Let ε_1, μ_1 and ε_2, μ_2 are permittiviy and permeability of media (1) and (2) respectively.

Let the electric vector of the wave lie in the plane of incidence. *i.e.,* parallel to the plane of incidence.

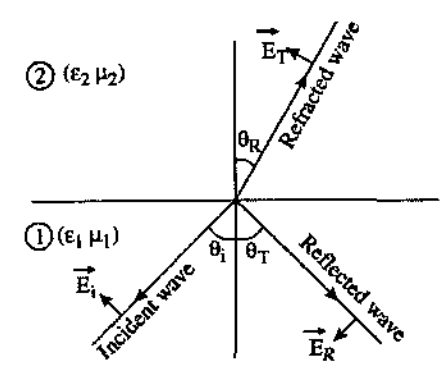

Fig. 7

We have boundary condition

$$(E_i)_t + (E_R)_t = (E_T)_t \quad \dots (1)$$

and

$$(H_i)_t + (H_R)_t = (H_T)_t \quad \dots (2)$$

We have tangential component of magnetic vectors are

$$(H_i)_t = H_i$$

$$(H_R)_t = H_R$$

$$(H_T)_t = H_T$$

and for electric vector

$$(E_i)_t = E_i \cos \theta_i$$

$$(E_R)_t = - E_R \cos \theta_R$$

$$(E_T)_t = E_T \cos \theta_T$$

so eqns. (1) and (2) becomes

$$E_i \cos \theta_i - E_R \cos \theta_R = E_T \cos \theta_T \quad \dots (3)$$

and

$$H_i + H_R = H_T \quad \dots (4)$$

We know

$$\theta_i = \theta_R \quad \text{and} \quad H = \left(\frac{n}{z_0}\right)E$$

So eqns. (3) and (4) reduces to

$$E_i \cos \theta_i + E_R \cos \theta_R = E_T \cos \theta_T \quad \dots (5)$$

and

$$n_1 E_1 + n_1 E_R = n_2 E_T \quad \dots (6)$$

Eliminating E_T from eqs. (5) and (6) then we get

$$(E_i - E_R) \cos \theta_i = \frac{n_1}{n_2} (E_i - E_R) \cos \theta_T$$

i.e.,

$$\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\frac{n_2}{n_1} \cos \theta_i - \cos \theta_T}{\frac{n_2}{n_1} \cos \theta_i + \cos \theta_T}$$

By Snell's law *i.e.,*

$$\frac{n_2}{n_1} = \frac{\sin \theta_i}{\sin \theta_T}$$

So,

$$\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\left(\frac{\sin \theta_i}{\sin \theta_T}\right) \cos \theta_i - \cos \theta_T}{\left(\frac{\sin \theta_i}{\sin \theta_T}\right) \cos \theta_i + \cos \theta_T}$$

$$\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\sin \theta_i \cos \theta_i - \sin \theta_T . \cos \theta_T}{\sin \theta_i . \cos \theta_i + \sin \theta_T . \cos \theta_T}$$

$$\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\sin 2\theta_i - \sin 2\theta_T}{\sin 2\theta_i + \sin 2\theta_T}$$

$$\boxed{\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\tan (\theta_i - \theta_T)}{\tan (\theta_i + \theta_T)}} \quad \dots (7)$$

This is the required expression for the reflection coefficient.

Polarisation by Reflection and Brewster's Law : From expression (7), we have

$$\text{if } \theta_i + \theta_T = \frac{\pi}{2} \text{ then}$$

$$\left(\frac{E_R}{E_i}\right)_{\|} = 0$$

This means that the wave passes through the boundary without suffering reflection or we can say that if an unpolarised wave of light is incident at angle $\theta_i = \frac{\pi}{2} - \theta_T$ then the component of electric vector $\vec{E}$ in the plane of incidence will not be reflected and the reflected wave will be polarised whose vibrations are perpendicular to the plane of incidence as shown in the fig. 8. **This is known as Brewester's law and the angle of incidence *i.e.*, $\theta_i = \frac{\pi}{2} - \theta_T$ is known as Brewester's angle.** It is denoted by θ_B so

$$\theta_B = \frac{\pi}{2} - \theta_T$$

or
$$\theta_T = \frac{\pi}{2} - \theta_B$$

so by Snell's law
$$\frac{\sin\theta_B}{\sin\left(\frac{\pi}{2} - \theta_B\right)} = \frac{n_2}{n_1}$$

$$\tan\theta_B = \frac{n_2}{n_1}$$

$$\boxed{\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)}$$

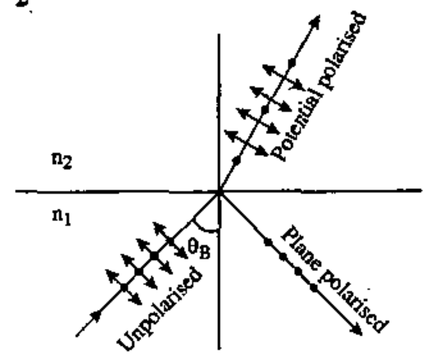

Fig. 8

Thus when the angle of incidence is $\tan^{-1}\left(\frac{n_2}{n_1}\right)$ then the reflected light is plane polarised light.

• 5.9. TOTAL INTERNAL REFLECTION AT THE BOUNDARY OF TWO NON-MAGNETIC MEDIA

When an electromagnetic wave enters in a denser medium from rarer medium *i.e.*, $n_1 > n_2$. Then in this position θ_T is greater than θ_i. We have *Snell's law*

$$n_1 \sin\theta_i = n_2 \sin\theta_T$$

$$\therefore \quad \sin\theta_i = \frac{n_2}{n_1}\sin\theta_T \qquad \ldots (1)$$

i.e.,
$$\sin\theta_i < \sin\theta_T \qquad \left[\because \frac{n_2}{n_1} < 1\right]$$

$$\therefore \quad \theta_i < \theta_T$$

When we change the value of angle of incidence then transmitted angle accordingly for a certain angle say $\theta_i = \theta_C$, angle θ_T becomes 90° then the incident angle is known as critical angle.

Total internal reflection occurs only when light is incident from denser medium to rarer medium.

From equation (1) we have for $\theta_i = \theta_C$, $\theta_T = 90°$ then eqn. (1) becomes

$$\sin\theta_C = \frac{n_2}{n_1}\sin 90° = \frac{n_2}{n_1}$$

$$\boxed{\theta_C = \sin^{-1}\left(\frac{n_2}{n_1}\right)}$$

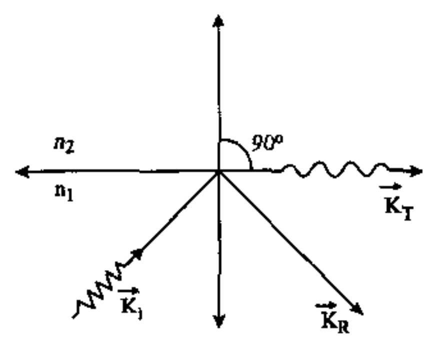

Fig. 8

In this position the reflected wave is propagated parallel to boundary surface. This is known as phenomenon of total internal reflection. This phenomenon is used to produce elliptically and circularly polarised light.

• 5.10. COULOMB LAW

From Maxwell first equation

$$\text{div}\, D = \rho \qquad \ldots (1)$$

Integrating this equation over a sphere of radius r, we get

$$\int_T \operatorname{div} \vec{D}\, d\tau = \int_T \rho\, d\tau$$

or $$\oint \vec{D} . d\vec{S} = q \qquad \text{(for Gauss law)}$$

or $$\varepsilon_0 \oint_S \vec{E} . d\vec{S} = q \qquad [\vec{D} = \varepsilon_0 \vec{E}]$$

or $$\varepsilon_0 E\, 4\pi r^2 = q$$

or $E = \dfrac{1}{4\pi \varepsilon_0} . \dfrac{q}{r^2}$ force experience by its charge q_0 is

$$\boxed{F = q_0 E = \frac{1}{4\pi \varepsilon_0} . \frac{q q_0}{r^2}}$$

Continuity equation from Maxwell third law,

$$\text{Curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

take the div on both the sides

$$\operatorname{div} \text{curl } \vec{H} = \operatorname{div} \vec{J} + \frac{d}{\partial t} (\operatorname{div} \vec{D})$$

$$= \operatorname{div} \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \Rightarrow \boxed{\operatorname{div} \vec{J} + \frac{d\rho}{\partial t} = 0}$$

This is equation of continuity.

• SUMMARY

- The idea of electromagnetic waves was given by Maxwell.
- On the basis of four basic laws, Maxwell predicated the existence of electromagnetic waves.
- When an electromagnetic wave passes from one medium to another through a surface then its property changes. This surface is known as boundary.
- Snell's law of refraction,

$$n_2 = \frac{\sin \theta_i}{\sin \theta_T}$$

- The angle of incidence is $\theta = \tan^{-1}\left(\dfrac{n_2}{n_1}\right)$. In this condition, the reflected light is polarised light.

• STUDENT ACTIVITY

1. Give the boundary conditions satisfied by E.M. field at the interface between two media.

2. Establish Brewester's law.

3. Obtain continuity equation from Maxwell's equations.

• TEST YOURSELF

1. What are electromagnetic waves ? How are they produced ? Give their properties.
2. Establish equation for plane electromagnetic waves and show that they are transverse in nature and travel in free space with the speed of light.
3. State and prove Poynting theorem of E.M. Waves.
4. What do you understand the term Poynting Vector ? Discuss its properties.
5. What do you understand by reflecion and refraction of E.M. waves ?
6. Discuss the reflection and refraction of a plane electromagnetic wave at a plane boundary of two dielectrics and hence establish the laws of reflection and refraction.
7. Explain total internal reflection on the basis of reflection of an E.M. wave at the boundary of two non-magnetic dielectric media.
8. In electromagnetic wave, the phase difference between electric and magnetic field vector is :
   (a) 0
   (b) $\pi/2$
   (c) π
   (d) $\pi/4$
9. The energy of electromagnetic wave in vacuum is given by the relation :
   (a) $\frac{1}{2} \cdot \frac{E^2}{\varepsilon_0} + \frac{B^2}{2\mu_0}$
   (b) $\frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \mu_0 B^2$
   (c) $\frac{E^2 + B^2}{C}$

(d) $\frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$

10. $\sqrt{\mu_0 \varepsilon_0}$ has the dimension of :
(a) LT^{-1}
(b) $L^{-1}T$
(c) LT^{-2}
(d) L^2T^{-1}

11. Electromagnetic waves are :
(a) transverse
(b) longitudinal
(c) may be longitudinal or transverse
(d) neither longitudinal nor transverse

12. Direction of propagation (or Poynting vector) is given by :
(a) $\vec{E}.\vec{E}$
(b) $\vec{E}$
(c) $\vec{B}$
(d) $\vec{E}\times\vec{B}$

13. An electromagnetic wave travels along z-axis. Which of the following pair of space and time varying field generated such a wave :
(a) E_{x_1}, B_y
(b) E_y, B_x
(c) E_z, B_x
(d) E_y, B_z

14. Which of the following has the shortest wavelength :
(a) X-rays
(b) microwave
(c) ultraviolet rays
(d) radio wave

15. If an electromagnetic wave is propagating in a medium with permittivity ε and permeability μ, then $\sqrt{\frac{\mu}{\varepsilon}}$ is :
(a) intrinsic impedance of medium
(b) the square of refractive index of medium
(c) refractive index of the medium
(d) energy density of medium

16. If magnetic monopole existed, then which of the following equation would be modified :
(a) $\oint \vec{E}.\,d\vec{S} = \frac{q}{\varepsilon_0}$
(b) $\oint \vec{B}.\,d\vec{S} = 0$
(c) $\oint \vec{E}.\,d\vec{l} = -\frac{d}{dt}\int \vec{B}.\,d\vec{S}$
(d) $\oint \vec{B}.\,d\vec{l} = \mu_0\varepsilon_0 \frac{d}{dt}(\vec{E}.\,d\vec{S}) + \mu_0 i$

17. Electromagnetic waves are produced by :
(a) charges at rest only
(b) charges in uniform motion only
(c) accelerated or deaccelerated charge only
(d) all above are correct

18. Total internal reflection would take place if light passes :
(a) from air to water (b) from water to water
(c) from glass to diamond (d) from glass to water

19. Critical angle is that angle of incidence in the denser medium for which the angle of refraction in rarer medium is :
   (a) 0°
   (b) 57°
   (c) 90°
   (d) 180°
20. Wave equation satisfied by E and B in the form of :
   (a) $\nabla^2\Psi + \frac{1}{v^2} \cdot \frac{\partial^2\Psi}{\partial t^2} = 0$
   (b) $\nabla^2\Psi - \frac{1}{v^2} \frac{\partial^2\Psi}{\partial t^2} = 0$
   (c) $\nabla^2\psi = v^2 \frac{\partial^2\Psi}{\partial t^2}$
   (d) None of these
21. Poynting vector is :
   (a) amount of energy flow per unit area per unit time
   (b) amount of energy flow per unit area
   (c) amount of energy flow per unit time
   (d) None of these

ANSWERS

Multiple Choice Questions

8. (b) **9.** (d) **10.** (b) **11.** (a) **12.** (d) **13.** (a) **14.** (a) **15** (c)
16. (b) **17.** (c) **18.** (d) **19.** (c) **20.** (b) **21.** (a)